



Daniel Bernoulli found equations for the oscillations of a pendulum, and Leonhard Euler developed corresponding equations for the vibration of a stretched membrane. There are hundreds of millions of order-five magic squares, while the quantity of ordersix magic squares has not yet been calculated. Plato's pupil Aristotle, with his methodical analysis of logical forms, identified the difference between inductive reasoning (such as inferring a rule of thumb from observations) and deductive reasoning (such as inferring a rule of thumb from established premises, or axioms). Ironically, Khayyam is now best known for a work of poetry for which he may not have been the sole author—the Rubaiyat, which was translated into English by Edward Fitzgerald in 1859. Although his theorem was useful, Girard's work was hindered by the fact that he 324 had no concept of complex numbers. The unit, known as the West Area Computers, comprised female African-American mathematicians, including Katherine Johnson. The paradox implied that set theory, which deals with the properties of sets of numbers or functions, and was fast becoming the bedrock of mathematics, but in particular calculus, graph theory, and number theory, were enormous, and he was also influential in standardizing mathematical notation. Leibniz began to devise his version of calculus around 1673, and 269 published it in 1684. The three-circle diagram represents a combination of two functions: (x AND Y) OR Z. After war broke out in 1939, he and others developed the Bombe, an electromechanical device that deciphered enemy messages. See also: Zeno's paradoxes of motion • Syllogistic logic • Logarithms • Euler's number • Catalan's conjecture 175 176 INTRODUCTION Throughout the Middle Ages, the Catholic Church wielded considerable political power across Europe, and had a virtual monopoly of learning, but in the 1400s, its authority was being challenged. He proposed what became known as the Poincaré conjecture: "every simply connected, closed 3-manifold is homeomorphic to the 3-sphere." A "3-manifold" is a shape that appears 3-D when its surface is enlarged but exists within higher dimensions, and "simply connected" means that it has no holes—like an orange but not a doughnut. Picking up on earlier work by al-Karaji, Khayyam also develops ideas about binomial coefficients, which determine how many ways there are to select a number of items from a larger set. Taimina attended a geometry workshop led by David Henderson in which he demonstrated how to make a paper model of a hyperbolic surface. She knows the daily average order, and decides to prepare n potatoes where there is at least 90 percent certainty that n will match demand. Euclid is known to have written an account of conic sections, but this work has not survived. Their main focus was prime numbers. If the shape is represented in 3-D, it has to intersect itself, which is where it starts to look like a bottle. Alan Turing Alan Turing is often cited as the "father of digital computing," yet the Turing machine that earned him that accolade was not a physical device but a hypothetical one. Desargues worked initially as a tutor and later as an engineer and architect. 1904 In his book The Fourth Dimension, British mathematician Charles Hinton coins the term "tesseract" for a four-dimensional cube. In this work, Ptolemy further developed the ideas of Hipparchus on triangles and chords of circles, building formulae that would allow the prediction of the Sun and other "heavenly bodies" based on the assumption of circular orbits around Earth. It used strokes for single digits and a different symbol for each power of 10. Fibonacci found that, starting with 5, every second number in his sequence (13, 34, 89, 233, 610, and so on) is the length of the hypotenuse of a right-angled triangle when the lengths of the two shorter sides are integers. Advances in engineering would have been severely restricted. Brahmagupta died in 668, only a few years after completing his second book. Russell taught in the US in the 1930s, although his appointment at a college in New York was revoked due to a judicial declaration that his opinions rendered him morally unfit. Aged just 14, Carl Gauss found a rough answer, and he was soon able to find an improved version of the prime counting function that could predict the number of primes between 1 and 1,000,000 as 78,628, which is accurate to 0.2 percent. One of several government positions he held was Master of the Royal Mint, where he oversaw the switch of the British currency from the silver to the gold standard. 281 IN CONTEXT KEY FIGURE Isaac Newton (1642–1727) FIELD Applied mathematics BEFORE c.330 BCE Aristotle believes it takes force to maintain motion. If the key were -6, then the cipher would turn the same plaintext "HELLO" into "BZFFI." This simple substitution system is known as the Caesar cipher (or Caesar shift) after the Roman dictator Julius Caesar, who used it in the 1st century BCE. Shortly after Sylvester introduced the term, his friend and colleague Arthur Cayley formalized the rules for manipulating matrices. 1320 French philosopher and mathematician Levi ben Gershon (Gersonides) shows that the only powers of 2 and 3 that differ by 1 are 8 = 23 and 9 = 32. At the age of 20, he explored the binomial theorem. The model predicted the way the atmosphere would evolve in terms of three data points, such as air pressure, temperature, and wind speed. RAFAEL BOMBELLI Born in Bologna, Italy, in 1526, Rafael Bombelli was the eldest of six children; his father was a wool merchant. A field contains two operations; complex numbers 372 (with the operations of addition and multiplication) are a field. The chef must be at least 90 percent sure that demand will be met, so n will be 7 here. 233 See also: Pythagoras • Conic sections • Trigonometry • Rhumb lines • Viviani's triangle theorem • The complex plane • Quaternions 234 IN CONTEXT KEY FIGURES Bonaventura Cavalieri (1598-1647), Gilles Personne de Roberval (1602-75) FIELD Applied geometry BEFORE c. 300 BCE. 122 A source of fascination Diophantine equations are vast in number and form, and mostly very difficult to solve. Archimedes' method used in ancient Greece for calculating in a number is the sum of the previous two: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, and so on - a sequence that became known as the Fibonacci sequence. Moving right, the number of grains would then be doubled, so the second square had two grains, the third had four, and so on, moving left to right along each row to the 64th square at the top right. The Pythagoreans, a semi-mystical group of mathematicians and philosophers associated with Pythagoras of Samos (570-495 BCE) had the pentagram, or fivepointed star, as their symbol. A cipher is an algorithm (a systematic, repeatable method)—in this case, to substitute each letter with one in another position in the alphabet. Alamy Stock Photo: Pictorial Press Ltd (tl). Alamy Stock Photo: James Davies (bc). 240 BCE In Method of Mechanical Theorems, Archimedes uses indivisibles to estimate the areas and volumes of curvilinear shapes. All numbers are in essence complex. Prime numbers that can only be divided by themselves or 1—have fascinated scholars since the ancient Greeks of Pythagoras's school first studied them. not least because they can be thought of as the building blocks of all natural numbers (positive integers). A "closed" shape is finite, with no boundaries—like a sphere. Realizing its significance, Weil had the letter typed up and circulated among number theorists through the late 1960s and '70s. Two are for calculation, one for the result. For example, 23 means that 1 will be multiplied by 2 three times: 1 × 2 × 2 × 2 = 8, while 21 means that 1 will be multiplied by 2 just once: 1 × 2 = 2. 546 See also: Non-Euclidean geometries • Group theory • Quaternions • Matrices RICHARD DEDEKIND 1831-1916 Dedekind was one of Carl Gauss's students at the University of Göttingen, Germany. AFTER 1999 The study of "allometric scaling" applies fractal growth to metabolic processes within biological systems, leading to valuable medical applications. Getty Images: John Pratt / Stringer / Hulton Archive (cla). This has led to topology being called "rubber-sheet geometry." For more than 2,000 years, from the time of Euclid, c. The area of a circle is also πr<sup>2</sup>. This raises the question of how Euclid's five postulates can be applied to the sphere. Khayyam anticipated the work of 16th-century Italian mathematicians, who solved cubic equations without direct recourse to geometry. Sophie Germain French mathematician Geometric solutions Up until the era of the Arab "algebraists"—from al-Khwarizmi in the 9th century to the death of the Moorish mathematician al-Qalasadi in 1486—the key developments within algebra were underpinned by geometrical representations. If it had been constructed, it would have been the first "Turing-complete" device. Yet 300 years later when Leonardo of Pisa (better known as Fibonacci) introduced Hindu-Arabic numerals to Europe, he was still wary of zero and treated it as an operator like + and - rather than a number. Islamic algebraists wrote equations as text with accompanying diagrams, as in the 14thcentury Treatise on the Question of Arithmetic Code by Master Ala-El-Din Muhammed El Ferjumedhi. Alamy Stock Photo: Art Collection 2 (cr). In contrast, the mathematics of ancient Greece was based on geometry and geometrical magnitudes, or their ratios. He worked with triangles defined by the cylinders' height, diameter, and a diagonal from top to bottom. To have the same chance of rolling a double ace as you would from getting one ace when four dice are thrown, the pair should be thrown 6 × 4 = 24 times. In Babylon, fractions were expressed using their base-60 number system, but in writing, it was difficult to distinguish which numbers represented integers and which were part of the whole. AFTER 1854 George Boole uses binary arithmetic to develop Boolean algebra. Russian mathematical genius Nicolas Bourbaki was one of the most prolific and influential mathematicians of the 1900s. The shortest distance across the globe is not a rhumb, however, but a "great circle"—any circle that centers on the center of the Earth. We need a super-mathematics is the Theory of Groups. This multiplication resembles the way the volume of a cube is calculated, by multiplying its length × height × depth. In doing so, they began to identify certain patterns and rules in the concepts of numbers, sizes, and shapes. Carl Gauss Legacy of the theorem The proofs by Gauss and Argand established the validity of complex numbers as roots of polynomials. In Newton's fluxion model, he considered a point moving along a curve as being divided into two perpendicular components. Key work 1806 Essai sur une manière de représenter les quantités imaginaires dans les constructions géométriques (Essay on a method of representing imaginary quantities geometrically) See also: Quadratic equations • Cubic equations • Cubic equations • Coordinates • The fundamental theorem of algebra 338 IN CONTEXT KEY FIGURE Joseph Fourier (1768-1830) FIELD Applied mathematics BEFORE 1701 In France, Joseph Sauveur suggests that vibrating strings oscillate with many waves of different lengths at the same time. Output The result when an input is combined with a function. The tablet's original purpose is unknown, but it may have been used as a practical manual for measuring dimensions. He studied at the universities of Pavia and Padua in what is now Italy, was awarded a doctorate in medicine, and worked as a physician before becoming a teacher of mathematics. Alamy Stock Photo: sciencephotos (tl). AFTER 1905 Albert Einstein presents his theory of relativity, which challenges Newton's view of the force of gravity. See also: The fundamental theorem of algebra • Elliptic functions • Group theory JOSEPH LIOUVILLE 1809-82 Born in northern France, Liouville graduated from the École Polytechnique, Paris, in 1827 and took up a teaching post there in 1838. Alamy Stock Photo: Granger Historical Picture Archive (tr). At its most basic, it is concerned with counting and assigning numerical values to things, but also the operations, such as addition, subtraction, multiplication and division, that can be applied to numbers. Key works 1749-1786 Histoire naturelle (Natural History) 1778 Les époques de la nature (The Epochs of Nature See also: Calculating pi • Probability • The law of large numbers • Bayes' theorem • The birth of modern statistics 322 IN CONTEXT KEY FIGURE Carl Gauss (1777-1855) FIELD Algebra BEFORE 1629 Albert Girard states that a polynomial of degree n will have n roots. A form of the Tower of Hanoi is a popular toy for small children. For example, starting at the fourth 1 on the left edge and going down diagonally to the right, if you stop at the number spassed so far (1 + 4 + 10) can be found by going one diagonal step down to the left: 15. AFTER 1202 Leonardo of Pisa (Fibonacci) commends the Hindu-Arabic number system in Liber Abaci. Meridian An imaginary line on Earth's surface joining the North Pole and South Pole through any given locality. Key works 1609 New Astronomy 1615 New Solid Geometry of Wine Barrels 1619 Harmonies of the World 1621 Epitome of Copernican Astronomy See also: Euclid's Elements • Calculating pi • Trigonometry • Coordinates • Calculus • Newton's laws of motion 221 IN CONTEXT KEY FIGURE René Descartes (1596-1650) FIELD Geometry BEFORE 2nd century BCE Apollonius of Perga explores positions of points within lines and curves. Euler published numerous books and treatises, and worked in every field of mathematics, often recognizing the links between apparently separate concepts of algebra, geometry, and number theory, which were to become the basis for further 279 fields of Mathematical study. If two people, Alice and Bob, wish to communicate in secret, Alice can send Bob her public key. The most recent two volumes were published in 1998 and 2012. The result cannot be the original number, as any number multiplied by zero equals zero. See also non-Euclidean geometries. As the time intervals at which interest is calculated get smaller, the amount of interest yielded at the end of a year approaches A = Per. As binomial expansion leads to ever longer expressions, the coefficients continue to match a corresponding line on the triangle. Representing concepts The Egyptian number system was the first decimal system. When Euler raised the constant e to the power of the imaginary number i multiplied by II, he discovered that it equals -1. In mathematical contexts, they are written (for a 2-D case) in the form (x,y), where x is the horizontal position. Significant contributions to calculus were also made in Switzerland by the brothers Jacob and Johann Bernoulli, who coined the term "integral" in 1690. The world believed that Wiles had resolved Fermat's last theorem, and it was waiting for the finished, published proof. Great ideas are the ones that lie in the intersection of the Venn diagram of 'is a good idea' and 'looks like a bad idea.' Sam Altman American entrepreneur See also: Syllogistic logic • Probability • Calculus • Euler's number • The logic of mathematics 411 IN CONTEXT KEY FIGURE Édouard Lucas (1842-91) FIELD Number theory BEFORE 1876 Édouard Lucas proves that the Mersenne number 2127 - 1 is prime. Right angle An angle that is 90 degrees (a quarter turn), such as the angle between vertical and horizontal lines. This means that enlargement of a portion of the shape reveals smaller replicas with equal detail. Thomas L. Their work inspired a re-examination of the nature of mathematics itself, how it works, and what its limits are. 2015 Fractal analysis is applied to electrical power networks, leading to the modeling of the frequency of power failure. A weighted graph is also called a network. Although it was human curiosity and intuition that recognized the underlying principles of mathematics, and human ingenuity that later provided various means of recording and notating them, those principles themselves are not a human invention. As the orders increase, so do the quantities of magic squares. 830 The Compendious Book on Calculation by Completion and Balancing See also: Quadratic equations • The Rhind papyrus • Diophantine equations • Cubic equations • The algebraic resolution of equations • The fundamental theorem of algebra 149 IN CONTEXT KEY FIGURE Al-Karaji (c. Zeno wrote 40 paradoxes to show the absurdity of the pluralist view. Grothendieck's radical political activities included delivering math lectures just outside Hanoi while the city was being bombed during the Vietnam War. Constant population growth can be calculated with the formula P = P0ert where P0 is the original population number, r is the growth rate, and t is time. In 1931, Austrian mathematician Kurt Gödel demonstrated that mathematician Kurt Gödel demonstrated that mathematics based on formal axioms could not prove everything that was true according to those axioms. F. See also: The Turing machine • Cryptography KATHERINE JOHNSON 1918- A child math prodigy, Katherine Johnson (born Coleman) was a pioneer of computing and the American space program. His ideas have influenced many of the greatest scholars in history, from Plato to Nicolaus Copernicus, Johannes Kepler, and Isaac Newton. He is best known, however, for his triangle theorem, which states that the sum of the distances between any point in an equilateral triangle. Zero is used as a placeholder in the calculation and in the final solution— 70,072,626. Applications of the FTA Research on the fundamental theorem of algebra has led to breakthroughs in other fields. The 2017 world record for Flash Anzan was 15 numbers added together in 1.68 seconds. This was despite steadily losing his sight from 1738 and becoming blind in 1771. The discovery of calculus opened up a field of analysis that later became particularly relevant to, for example, the theories of quantum mechanics and chaos theory in the 1900s. The altitude in an equilateral triangle, such as the above, is always equal to the combined length of lines drawn from any point in the triangle perpendicular to its three sides. A square matrix of size n × n with the value 1 along the diagonal starting top left, and the value 0 everywhere else, is called the identity matrix (In). Dating back to at least 200 BCE, it is one of the oldest bead-counting devices, although the Romans used something similar. They are also used in engineering, particularly in the development of polymer and ceramic materials. The other seven are rotations of this combination. The Chinese abacus, or suanpan, matched the design of the Roman version, but rather than use pebbles set in a metal frame, it employed wooden counters on rods—the template for modern abaci. 477 Networks are used to model relationships between objects or people in many disciplines, including computer science, particle physics, economics, cryptography, biology, sociology, and climatology. 350 BCE Posterior Analytics c. Other mathematicians The Bernoullis and Euler tended to eclipse the achievements of the 1700s. Some people believe in imaginary friends. Because solar transit times could vary by 24 hours, months were between 29 and 32 days long, but their length could differ from year to year. 825 CE The Persian astronomer and mathematician al-Khwarizmi develops algebra. In terms of computing ability, the algorithms at work in a Turing machine are the strongest type ever devised. The growth curve Compound interest is an example of exponential growth. 1905 French scientist Henri Poincaré has the idea of making time the fourth dimension in space. Devised in the 1970s (where it was called Number Place), Sudoku took off in Japan in the 1980s, acquiring its now-familiar name, which means "single digits." A Sudoku puzzle is a nine-by-nine Latin square with the added restriction that subdivisions of the square must also contain all nine numbers. An irrational number that is not algebraic (such as pi or e) is called a transcendental number. He was the chief librarian at the Library of Alexandria, the greatest academic institution of the ancient world. The number 6 and all its multiples have been removed from the list of potential 97 primes, as they are even multiples of 3. Al-Karaji created a table to work out the coefficients of binomial equations. Extending the concept further, the expression Ta(j, k, n) seeks the smallest positive integers (j), each to any power (k) in n distinct ways. At the sound of the starting signal, Achilles runs 100 meters to reach the tortoise's starting point, while the tortoise runs 1 meter, giving it a 1 meter lead. This meeting point is called a vanishing point. In classical geometry, if a pair of shapes has equal corresponding lengths and angles, and you can slide, reflect, or rotate one of the shapes into the other, they are congruent— a mathematical way of saying they are identical. 1958 British mathematician Alfred Goldie proves that Noetherian rings can be understood and analyzed in terms of simpler ring types. Voltaire French philosopher 22 The first people known to have used an advanced numeration system were the Sumerians of Mesopotamia, an ancient civilization living between the Tigris and Euphrates rivers in what is present-day Iraq. It contains two mathematical problems, one of which is a quadratic equation. This notation is yet to become universal. In modern terms this is known as a "brute 514 force" technique. Hippasus is usually credited with discovering irrational numbers, an idea that would have been considered heresy by the sect. Studies indicate that facial symmetry plays a major role in determining a person's perceived attractiveness. Euler and others also used Bessel functions to find solutions to find solutions to find solutions to the "three-body problem," concerned with the motion of a body, such as a planet or moon, being acted upon by the gravitational fields of two other bodies. An undercurrent of thought was going on in my mind which gave at last a result... An electric circuit seemed to close; and a spark flashed forth, the herald of many long years. In the course of their research, mathematicians showed that any string will support a potentially infinite series of vibrations, starting from the fundamental (the string's lowest natural frequency) and including its harmonics (integer multiples of the fundamental). 1835 Adolphe Quetelet advocates the use of the bell curve to model social data. This created a quaternion, with a basic structure of a + bi + cj + dk, where a, b, c, and d are real numbers. In what is sometimes known as Cournot's principle, Cournot suggested that an event with a very small probability will not happen. AIs who progress to the final take it in turns to communicate with one of four judges. Essentially self-taught, he became a civil engineer, designing fortifications. The birth of calculus One of the time, Gottfried Leibniz and Isaac Newton. The product of a debt and a fortune is a debt. The Langlands Program has been described as a mathematical Rosetta Stone, helping to translate ideas from one area of mathematics into another. A pH of 2 is 10 times more acidic than a pH of 3 and 100 times more acidic than a pH of 3 and 100 times more acidic than a pH of 2 is 10 times more acidic than a pH of 3 and 100 times more acidic than a pH of 2 is 10 times more acidic than a pH of 2 is 10 times more acidic than a pH of 3 and 100 times more acidic than a pH of 3 methods for analyzing and studying such data. 440 BCE The Greek mathematician Hippocrates of Chios writes the first systematically organized geometry textbook, Elements. Negative solutions Indian, Persian, and Arab scholars thus far had used only positive numbers. (Used when comparing geometrical shapes.) Conjecture A mathematical statement or claim that has not yet been proved or disproved. Gauss pointed out that each of these earlier proofs had assumed part of what they were trying to prove. Refined by Austrian physicist Ludwig Boltzmann, this technique became known as statistical 345 mechanics. When he died in 1665, French mathematician Pierre de Fermat left behind a wellthumbed copy of Arithmetica by the 3rd-century CE Greek mathematician Diophantus, its margins marked with Fermat's ideas. 144 Al-Khwarizmi showed how to find x in the equation x2 + 10x = 39. This idea had already been mooted by ancient Greeks such as Archimedes. The origins of algebra— a mathematical method for calculating unknown quantities—can be traced back to ancient Babylonians and Egyptians, as equations on cuneiform tablets and papyri reveal. The motivation among Arab scholars for developing trigonometry was not just for astronomy, but also for religious purposes, since it was important that Muslims knew the position of the holy city of Mecca from anywhere in the world. Lovelace anticipated many of the key 351 mathematical aspects of computer programming and foresaw how the machine could be used to analyze any kinds of symbol. One such shape was the "Klein bottle," devised in 1882 by German mathematician Felix Klein. For centuries, the sexagesimal system also had no place value holders, and nothing to separate whole numbers from fractional parts. As the range of applications grows, along with ever-increasing computational capacity, fractals are becoming integral to our understanding of the seemingly chaotic world in which we live. Although he won renown as an astronomer and mathematician, when his patron Sultan Malik Shah died in 1092, he was forced 158 into hiding. It can only work if the Universe follows a predictable mechanical path, so that everything from the 344 spin of galaxies to the tiny atoms in nerve cells controlling thoughts could be mapped out into the future. The result shows how to adjust the number to give the correct solution the equation. The 437 second generation will be closer in height than the first, an example of regression to the mean. But it is also the story of the explorers, the mathematicians who set out with a definite aim in mind, to find answers to unsolved problems, or to travel into unknown territory in search of new ideas— and those who simply stumbled upon an idea in the course of their mathematical journey, and were inspired to see where it would lead. The Soroban Championship Japanese schoolchildren still use the soroban (Japanese abacus) in mathematics lessons as a way of developing mental arithmetic skills. Eventually, this "white noise" overwhelms the original message. The worldline angle of an object moving at the 446 speed of light is 45°. Stevin correctly predicted that a decimal system would eventually be universal. He investigated how, if the diagonal was fixed, like the merchant's rod, changing the barrel height would change its volume. Such scales use the logarithm of a value instead of the actual value of whatever is being measured. Across a mere two pages this book has to cover 3,000 years of history ending with geomagic squares in 2001. The Greeks were not successful with this method, and in the 1800s, squaring the circle was proved to be impossible, due to n's irrational nature. In the Islamic world, too, the development of algebra meant that, rather than spelling out a calculation millions of times for all numbers to infinity. This describes how the time it takes for a planet to complete one orbit of the Sun is related to its average orbital distance. The equation of the circle is r2 = x2 + y2. All transactions are encrypted but public. Pine cones and pineapples, for example, display Fibonacci numbers in the spiral formation of their exterior scales. In decimal terms 60 The positional decimal system of Hindu-Arabic numbers in the spiral formation allowed further study of irrational numbers, which can be shown as an infinite series of digits after the decimal point with no recurring pattern. Weierstrass was a pioneer in the development of mathematical analysis and in the modern theory, differential geometry, mathematical physics, and astronomy, and in 1844 he was the first to prove the existence of transcendental numbers. Art and architecture While the golden ratio can be found in music and poetry, it is more often associated with the art of the Renaissance in the 15th and 16th centuries. It proved that, because it was impossible to know if the machine would ever halt or not, then the answer to the the answer to the the art of the Renaissance in the 15th and 16th centuries. It proved that, because it was impossible to know if the machine would ever halt or not, then the answer to the the art of the Renaissance in the 15th and 16th centuries. decision problem was No: there was no universal test for the validity of algorithms. Points on the line of the quarter circle can be counted as a success. A number that represents the absence of something is a difficult concept, which may be why zero took so long to become widely accepted. Degree (1) A measure of angle in geometry: rotating a full circle involves turning 360 degrees. This is still the largest prime ever found without using a computer. Proving the unprovable Ribet's proof electrified Wiles. 321 Independently wealthy, Buffon was able to write and study tirelessly, corresponding with many of the scientific elite of his day. FIBONACCI Born Leonardo Pisano, probably in Pisa, Italy, in 1170, Fibonacci did not become known as Fibonacci ("son of Bonacci") until long after his death. It is left in root form as it cannot be simplified or written exactly as a decimal. Jean d'Alembert 327 Jean transformations (rotational and reflexive) associated with a regular 2-D or 3-D shape can be arranged into a type of simple group known as a symmetry group. 568 Postulate In mathematics, a statement whose truth is taken for granted or thought to be obvious, but is not supported by a proof. Dana Mackenzie American science writer GRIGORI PERELMAN 534 Born in 1966 in St. Petersburg, Grigori Perelman developed a passion for mathematics from his mother, who taught the subject. She also continued his project of preserving and expanding the classic texts, in particular providing commentaries on Diophantus's 13-volume Arithmetica, and Apollonius's work on conic sections. 100 BCE Mathematician and astronomer Aglaonike of Thessaly wins renown for her ability to predict lunar eclipses. In her work on number theory, Germain also corresponded with Adrien-Marie Legendre (see above) and Carl Gauss, and her ideas on Fermat's last theorem helped Legendre to prove the theorem where n 544 = 2. Alamy Stock Photo: Painting (t). At first, he used a separate mechanism for each carryover, but that proved too complicated. It outlined the Hindu-Arabic decimal place value system and clergyman William Oughtred invents the slide rule using logarithmic scales. The superscript number, the exponent, shows how many times the other number, in this case 2, is multiplied by itself. 1770 Swiss mathematician Leonhard Euler shows that Fermat's last theorem is true when n = 3. By using Cartesian coordinates, scholars could visualize mathematician Leonhard Euler shows that Fermat's last theorem is true when n = 3. By using Cartesian coordinates, scholars could visualize mathematician Leonhard Euler shows that Fermat's last theorem is true when n = 3. 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In 1970, Matiyasevich provided the final proof that the tenth problem is unsolvable because there is no general method of determining whether Diophantine equations have a solution. This is because a circle can be thought of as all the points that lie at an equal distance from a central point (that distance being the radius of the circle). 431 Hilbert was renowned for his broad understanding of many areas of mathematical problems. Although little is known of his life, he achieved fame as an astronomer from the studies he carried out while living on the island of Rhodes For example, an equilateral triangle with the vertices A, B, and C can be rotated in three ways (through 120°, 240°, or 360°) around its center, and be reflected in three different lines. This graph also shows the tangent to the curve. He approximated the area of a circle by enclosing it within polygons with increasing numbers of sides. With the number 10, partitions include 3 and 7, with a product of 21; or 6 and 4 to produce 24; or 5 and 5 to give 25, which is the maximum product for a partition of 10 using two numbers. See also: Euclid's Elements • Conic sections BHASKARA II 1114-85 One the greatest of the medieval Indian mathematicians, Bhaskara II was born in Vijayapura, Karnataka, and is believed to have become the head of the astronomical observatory at Ujjain in Madhya Pradesh. generate numerals up to 19. As a result, many mathematicians kept their methods secret rather than sharing them for the common good. AFTER Early 1800s Carl Friedrich Gauss and French mathematicians kept their methods secret rather than sharing them for the common good. conjecture about the density of primes. New technologies Not all mathematics in this period was purely theoretical—and even some of the shape of a 2-D surface, we need to look down on it in three dimensions. f' Invented by Lagrange for differentiation He also developed an early understanding of graphical curves, articulating the significance of maximum and minimum values. His political beliefs led to his expulsion from a number of academic posts. Alamy Stock Photo: Granger Historical Picture Archive (b). Peter Ludlow American philosopher Asymmetric encryption With symmetric encryption, messages are only as secure as the key. He also created Napier's Bones, a calculation device using numbered rods. Without that accurate tool and knowledge, the roads, canals, ziggurats, and pyramids of ancient Mesopotamia and Egypt could not have been built. The polar version of the Cartesian point (0,1) is (1,90°). The fact that we work in 10s as opposed to any other number is purely a consequence of our anatomy. Pythagoras is thought to have traveled widely, assimilating ideas from scholars in Egypt and elsewhere in the Middle East before establishing his community of around 600 people in Croton, southern Italy, around 518 BCE. It investigated the arithmetic of negative numbers, and included some economical notation that represented a major advance on what had gone before. Simple symmetries— 369 such as those of a regular polygon—are intuitively graspable. In 1593, Flemish mathematician Adriaan van Roomen (Romanus) used a polygon—are intuitively graspable. In 1593, Flemish mathematician Adriaan van Roomen (Romanus) used a polygon with 230 sides to compute n to 17 decimal places; three years later, German- Dutch professor of mathematics Ludolph van Ceulen calculated π to 35 decimal places. For example, 3/ can be expressed as 0.075, and this value is exact because 40 is only 40 divisible by the primes 2 and 5. Proposition 4 If two sides of one triangle are equal in length to two sides of another triangle, and if the angles contained by each pair of equal sides are equal, then the base of one triangle will be equal to those in the other, the two triangles will be equal to those in the other triangle. Correlation does not imply a causal relationship between the two variables; it simply means they vary together. Catalan was a republican from an early age and a participant in the 1848 revolution. Bertrand Russell British philosopher and mathematician 18 19 INTRODUCTION As early as 40,000 years ago, humans were making tally marks on wood and bone as a means of counting. Powers of 10, particularly those inscribed on stone, were depicted as hieroglyphs—picture symbols. Working independently, they developed a method for calculating the arc length, or length of a section, of any ellipse. Today, the equations are considered to be one of the most interesting areas of number theory. In mathematics, Khayyam is best remembered for his work on cubic equations, but he also produced an important commentary on Euclid's fifth postulate, known as the parallel postulate. Transformations, rotations, rotat BCE In Central Africa, numbers are recorded on bone as carved marks. Two numbers, one from each axis, "coordinate" to pinpoint an exact position. It is a positional system, but instead of multiplying by 10, each column is multiplied by 2, also expressed as 21, 22, 23 and upward. Lorenz remained physically and mentally active until just before his death in 2008. AFTER 1733 Abraham de Moivre proposes what becomes the central limit theorem—as a sample size increases, the results will more closely match normal distribution, or the bell curve. In 1900, David Hilbert suggested that the question of whether or not they could all be solved was one of the greatest challenges facing mathematicians. 561 Differentiation In calculus, the process of working out the rate of change of a given mathematical function. With certain exceptions, m and n can be any two integers, such as 7 and 4, which produce the triple 33, 56, 65 (332 + 562 = 652). According to al-Khwarizmi, by using his completion and balancing methods, all quadratic equations—those in which the highest power of x is x2 — can be simplified to one of six basic forms. G. Games of chance were the focus of such studies even after the deaths of both Pascal and Fermat, although their letters on the subject contributed much to subsequent theory. He also created a design model for modern computer architecture, and worked in quantum and nuclear physics, contributing to the atomic bomb during World War II. Here, the Sumerians elaborated on the concept of tally marks, using different symbols to denote different quantities, which the Babylonians then developed into a sophisticated numerical system of cuneiform (wedge-shaped) characters. Getty Images ullstein bild Dtl. 258 Proving the theorem Starting with an equilateral triangle of base (side) a, and an altitude of h, a point is the rate of acceleration (any change in velocity). The straight section of the curve has the equation y = x. This is not a prime number, as 2,047 = 23 × 89. Deduction A process by which a problem is solved by drawing on known or assumed mathematical principles. Boolean algebra calculations. The coefficients in a column are calculated by adding together pairs of numbers in the preceding column. For Newton, calculus was a practical tool for his work in physics and especially on the motion of planets, but Leibniz recognized its theoretical importance and refined the rules of differentiation. It evolved from the very practical business of describing the physics and construction projects but Leibniz recognized its theoretical importance and refined the rules of differentiation. measuring and apportioning plots of land, and astronomical observations for navigation and compiling calendars. 384 The arrays found in Mayan relics suggest to some historians that the Maya used matrices to solve linear equations. Platonic solid One of the five polyhedra that form completely regular and symmetrical shapes: each face is an identical polygon and all the angles between the faces are the same. He proposed a form of scientific determinism, saying that with the relevant knowledge of moving particles, the behavior of everything in the Universe could be predicted. But I hope that we are now improving the diversity of mathematicians and encouraging all humans to discover and learn about mathematics. Science Photo Library: (bl). Today, for instance, in the decimal system, the position of a digit in a number indicates whether its value is in ones (less than 10), tens, hundreds, or more. 1873 French mathematician Charles Hermite proves that e is transcendental. UCLA's Terence Tao, winner of the Fields Medal in 2006 and the Breakthrough Prize in mathematics in 2015, published a rigorous proof of a weak Goldbach conjecture in 2012. THOMAS BAYES The son of a Nonconformist minister, Thomas Bayes was born in 1702 and grew up in London. Each judge is also communicating with a human and must decide whether the AI or the human is most humanlike. set of all countable ordinals, such as  $\omega + 1$ ,  $\omega + 1 + 2$ ,  $\omega + 1 + 2 + 3$ ..., will contain  $\omega 1$  items. When they joined, members were obliged to give all their possessions and wealth to the brotherhood, and also swore to keep its mathematical discoveries secret. Leibniz outlined his ideas on the binary system in 1703 in Explanation of Binary Arithmetic, showing how 0s and 1s could represent numbers and so simplify even the most complex operations into a basic binary form. An object thrown up into the air will fall back down again as a result of gravity. In 1733, de Moivre was satisfied that he had found a simple way of approximating binomial probabilities using normal distribution, thus creating and so simplify even the most complex operations into a basic binary form. An object thrown up into the air will fall back down again as a result of gravity. bell curve for binomial distribution on a graph. During the Great Plague in 1665-1666, the university was forced to close, and it was during this period that he formulated his ideas on fluxions (rates of change at a given point in time). By 1637, René Descartes had published a solution to the quartic equation (involving x4), reducing it to a cubic equation and then to two quadratic equations to solve it. As this graphic Pythagoreans emigrated to other parts of southern shows, 9 plus 16 equals Italy, spreading their knowledge of triples across the 25. Hannah Fry British mathematician Against the golden ratio 184 In the 1800s, German psychologist Adolf Zeising argued that the perfect human body aligned with the golden ratio; it could be found by measuring the person's total height and dividing this by the height from their feet to their navel. 1877 Austrian physicist Ludwig Boltzman develops the link between entropy (measure of randomness) and probability. Vertex (plural vertices) A corner or angle, where two or more lines, curves, or edges meet. Cardano published Ferro's solution in his book Ars Magna in 1545. Axiom A rule, especially one that is fundamental to an area of mathematics. They compared the sizes of the squares. Charles Babbage 353 This replica of the demonstration model Babbage made in 1832 of Difference Engine No. 1 has three columns, each with its numbered cogwheels. Public key cryptography—the use of two large primes to encrypt a message—is the basis of all internet security. The correspondence between the two laid down basic principles of probability theory, and games of chance would continue to prove fertile ground for early theorists. Proposition 14 If with any straight lines and at a point on it, two straight lines will be in a straight line with one another. Segment (1) Part of a line, with definite end points. Van Schooten translated La Géométrie into Latin and also popularized the use of coordinates as a mathematical technique. c. Egyptian interest is evident in the Rhind papyrus, a scroll that contains a set of tables relating to fractions. 254 On a standard roulette wheel, there is a 1/37 chance of the ball landing on any given number for a single spin of the wheel. He had discovered the phenomenon of "regression to the mean," a tendency for measurements to even out, always drifting toward the mean over time. The most ancient references to Pascal's triangle, however, come from India. He found that he could break down the cubic equation into two simpler equations: one for a circle, and the other for a parabola. Average The typical or middle value of a set of data. Nothing takes place in the world whose meaning is not that of some maximum or minimum. W. Setting down a geometric sequence (such as 1, 2, 4, 8...) and above it an arithmetic s raised to arrive at the series below. He died a few months later, the proof unverified. 133 An abax, a table or board covered in sand, was used by the Greeks to count. The pattern of such tessellations usually repeats. For 10, a different symbol was used by the Greeks to count. same vein, the 7/1,000 became a 7 followed by a 3 inside a circle. The question was whether there was a way of knowing which algorithms (or virtual machines), would halt and which would not; if Turing could find out, he would answer the decision problem. Binary numbers are written as 1s and 0s, using a base-2 system. Some types of data fit the law better than others. 1972 Hewlett Packard invents an electronic scientific calculator for personal use. He reported difficulties in analyzing the movements of three bodies bound together by gravity—even when those bodies were as seemingly stable as the Earth, Moon, and Sun. Roots and permutations In his Reflections on the algebraic resolution of equations (1771), French-Italian mathematician Joseph-Louis Lagrange introduced a general approach for solving polynomial equations. They married five years later, but George died soon after the birth of their fifth child. 44 An order-four magic square appears beneath the bell in Melencolia I by the German artist Albrecht Dürer and wittily includes the engraving's date of 1514. Hilbert was interested in whether logic could be made more rigorous by being simplified into a set of rules, or axioms, in the same way that arithmetic, geometry, and other fields of mathematics were thought possible to simplify at the time. Hardy declared that they had to be the work of a mathematician "of the highest class," and had to be true, because no one could invent them. Gerolamo Cardano heard of this and persuaded Tartaglia to share his methods with him. Aryabhata also correctly believed pi to be irrational. Tesseract A 4-D shape with four edges at every vertex, whereas a cube has three edges at every vertex, and a square has two. For four items, the number of possible permutations is 24, but to find the derangements of 1, 2, 3, 4, all other arrangements beginning with 1 must first be eliminated. For example, in the function  $y = x^2 + 3$ , the value of y is calculated by squaring x and then adding 3. He believed that complex analysis, a branch of mathematics in which ideas of function are applied to complex numbers (combinations of real numbers, such as 1, and imaginary ), would lead to a resolution. Slicing through a cone horizontally creates an ellipse (and then open curves called a parabola and a hyperbola). In Prior Analytics, he describes the propositions as being of broadly four types, in the form of "all S are P," "no S are P," and "some S are not P," where S is a subject, such as sugar, and P the predicate—a quality, such as sweet. He utilized this new set of logical principles in the momentous Principia Mathematica, written with Alfred North Whitehead and published in three volumes from 1910 to 1913. With the advent of electronic computers, mathematics was in demand to provide methods of designing and programming computer systems. Exponential growth and decay relate to many aspects of everyday life. René Descartes 225 This edition of La Géometrie (in Latin because that was the language of scholars) was printed in 1639. First graph theorem Euler began from the premise that each bridge could be crossed only once and each time a land area was entered it also needed to be exited, which required two bridges in order to avoid crossing any bridge twice. He became head of the leading astronomical observatory at Ujjain, and incorporated new work on number theory and algebra into his studies on astronomy. If the first step was to find them, and they are frequent among the small numbers, the next step was to identify a pattern to describe their distribution. The suanpan is traditionally a 2:5 abacus—each column has two "heaven" beads, each with a value of 5, and 5 "earth" beads, each with a value of 1, giving a potential value of 15 units. Group A mathematical set, together with an operation which, when performed on members of the set, yields an answer that is still a member of the set. Fuzzy logic recognizes a continuum of truth values instead of the Boolean binary values of "yes" (1) or "no" (0). At Bletchley, she became one of the leading cryptanalysts, working closely with Alan Turing, to whom she was engaged for a short time. Many of these notes covered systems that would become part of modern computing. In 1922, Philip Franklin proved that any map with 25 regions or fewer was four-colorable. 412 Solving the puzzle With just three disks, the Tower of Hanoi can be solved in just seven moves. In number theory conjectured the quadratic reciprocity law and the prime number theorem. A more sophisticated program would, like a human, take the weight of the egg into account. With this idea, he established the principle of axiomatic truths as the basis for a logical progression of ideas—the model for mathematical theorems from Euclid onward. In much the same way as algebra and arithmetic are tools for working with numerical or generalized quantities, calculus has its own rules, notations, and applications, and applications, and physics. There are also Möbius strip roller-coasters, such as the Grand National at Blackpool Pleasure Beach in northern England. Gauss published four proofs of the FTA, but did not fully resolve the problem. Bayes managed to solve the problem. Bayes managed to solve the problem. Bayes managed to solve the problem. providing a better understanding of the behavior of phenomena, calculus can be used to predict and influence their future state. In 1649, he was invited by Christina, Queen of Sweden, to tutor her and to launch a new academy. Natural number Any of the positive whole numbers. Alamy Stock Photo: IanDagnall Computing (bl). Cauchy's formalization allowed calculus to 271 be regarded as a unified whole, dealing with infinitesimals in a consistent way using universally agreed notation. The machine (or tape, switching a 0 to 1 or vice versa. Galton was a rigorous scientist, determined to analyze data to show mathematically how probable outcomes are. Its status was enhanced by the acquisition and translation of manuscripts from earlier cultures, including works by the Greek mathematicians Euclid. Apollonius, and Diophantus, as well as Indian scholars such as Brahmagupta. Denominator The lower number in a fraction, such as the 4 in 3/4. Stevin tutored the prince in mathematics and also advised him on military strategy, leading to some significant victories over the Spanish. It is an algorithm for sorting primes from composite (not prime) numbers. He died from unexplained causes in 1662, aged just 39. Modern uses Although φ's historical use is debated, the golden ratio can still be traced in modern works, such as Salvador Dalí's Sacrament of the Last Supper (1955), in which the shape of the painting itself is a golden rectangle. He was also President of the Royal Society. In about 300 BCE, for example, the Greeks were starting to develop a more sophisticated form of mathematics based on geometry, with quantities being represented by the lengths of lines. See also: Positional numbers • Pythagoras • Calculating pi • Algebra • Decimals 42 IN CONTEXT KEY CIVILIZATION Ancient Chinese I Ching (Book of Changes) lays out trigrams and hexagrams of numbers for use in divination. The first of these emerged in the sixth millennium BCE, in Mesopotamia, western Asia, home to the world's earliest agriculture and cities. In 1860, however, he gave up his lucrative law practice to take up a pure math professorship at Cambridge, on a far more modest salary. When a binomial expression is raised to a power, for example (x + y)3, the result when multiplied out gives (in this case) x3 + 3x2y + 3x2y + 3xy2 + y3. By then, the king's granary was running low, and he realized that the next square alone, number 33, would need 4 billion grains, or one large field's worth. The study of geometry was important in the Islamic designs were based on geometric patterns. See also: The fundamental theorem of algebra • Group theory • Boolean algebra MARY EVEREST BOOLE 1832-1916 Mary Everest's love of math began young when she studied the books in the study of her clergyman father, whose friends included polymath Charles Babbage, the inventor of the Difference Engine. Arthur Eddington British astrophysicist Group theory in physics The ATLAS detector at the CERN accelerator is designed to study subatomic particles, including those predicted by group theory, when early humans found ways to count and quantify things. Fourier analysis of the way materials vibrate allows engineers to construct buildings that resonate at different frequencies from a typical earthquake and thus avoid the kind of damage that occurred in Mexico City in 2017. Starting from the basics, they met in the 1930s and 40s, rigorously formalizing all branches of mathematics in terms of set theory. Taimina has since embarked on a second career as a mathematician-artist. Galton measured how many beads were in each tube and described the resulting distribution as "normal." His work—along with that of Karl Pearson —popularized the use of the term "normal" to describe what was also known as a "Gaussian" curve. Among the reviewers was Wiles's friend Nick Katz. Hypatia is not known to have contributed any original research, but she is credited with editing and writing commentaries, in 1990 she became the first woman since Emmy Noether to give a plenary speech at the International Congress of Mathematicians in ancient Greece and India, but the man credited with its discovery is the Persian mathematician alKaraji, one of many scholars who flourished in Baghdad from the 8th to the 14th century. 360 BCE Timaeus See also: Pythagoras • Euclid's Elements • Conic sections • Trigonometry • NonEuclidean geometries • Topology • The Penrose tile 70 IN CONTEXT KEY FIGURE Aristotle (384-322 BCE) FIELD Logic BEFORE 6th century BCE Pythagoras and his followers develop a systematic method of proof for geometric theorems. For instance, the bold capital N is used for natural numbers from the set {0, 1, 2, 3, 4...}, enclosed in curly brackets to denote a set. In 323 BCE, after Alexander's death, Athens again became fiercely anti-Macedonian, and Aristotle retired to his family estate in Chalcis, on Euboea. 1764b Leonhard Euler analyzes a vibrating membrane using what are later understood to be Bessel functions. Simple groups have been described as algebra's atoms. The Riemann hypothesis, derived from the Riemann zeta function concerning complex numbers, is as yet unsolved. to prove properties of parallel lines. The Age of Enlightenment, as this period came to be known, was a time of significant sociopolitical change, and produced an enormous increase in the spread of knowledge and education during the 1700s. integers. In 1973, French mathematicians Jean Guillaud and Martin Bouyer achieved 1 million decimal places, and in 1989, a billion decimal places, and in 1989, a billion decimal places were computed by Ukrainian-American brothers David and Gregory Chudnovsky. and predict the behaviors of more complex systems. By around 300 BCE, however, the Babylonians used two wedges to indicate no value, much as we use a placeholder zero today; this was possibly the earliest use of zero. LEONHARD EULER Born in 1707, in Basel, Switzerland, Euler grew up in nearby Riehen. Artificial intelligence A humanoid robot using AI works at the front desk of a Henn-na hotel in Tokyo, which claims to be the world's first hotel with robotic staff. Equations contain variables), and operators (symbols such as the plus and equals sign). For much of his life, Nash fought paranoid schizophrenia, as dramatized in the film A Beautiful Mind (2001). The set 3, 4, 5 is called a "primitive" Pythagorean triple because its components share no common divisor larger than 1. Chord A straight line that cuts across a circle, but does not go through its center. 355-415 CE) FIELDS Arithmetic, geometry BEFORE 6th century BCE Pythagoras's wife Theano and other women actively participate in the Pythagorean community. The same function of x." Geometry The branch of mathematics that studies shapes, lines, points, and their relationships. For example, n = 1 gives a value for e of 2, n = 10 gives a value for e of 2, of 2.5937... and n = 100 gives a value for e of 2.7048.... Self-similarity on infinite scales is explored in philosophy and the arts, often to produce a meditative effect. 1937 Claude Shannon shows how Boolean algebra could be implemented using electronic circuits and binary code. the Zurich Polytechnic, Switzerland. This was the first decimal place value system. 469 We need to feed [information] through a processor. He also looked at "polynomials" (expressions with multiple terms), such as 6y2 + x3 - x + 17. The larger the matrix used, the more secure the encryption is. In France, it is called the d'Alembert-Gauss theorem, acknowledging the influence of d'Alembert on Gauss. Jacob Bernoulli's book Ars Conjectandi (1713) showed that as the number of identically distributed, randomly generated variables increases, so their observed average gets closer to their theoretical average gets closer to their theoretical average. q, and r, the areas of the triangles add up to 1/2 (p + q + r)a. For example, he was able to show that the areas of circles relate to each other according to the cubes of their radii; and that the volume of a cone is one-third that of a cylinder of the same height. These Venn diagrams represent three of the most basic functions in Boolean algebra: the functions for AND, OR, and NOT. Yet it is thanks to Islam that these and other ideas from India's Golden Age (which continued until the 12th century) went on to influence the history of mathematics. He had been influenced by correspondence with missionaries in China, who introduced him to the I Ching, an ancient Chinese book of divination. These books inspired a "scientific revolution" that would accompany the cultural rebirth known as the Renaissance. Some would be periodic, repeating a path of whatever complexity over and over again. For example, an irrational number can be found between the recurring numbers 0.124124... and 0.125125... by changing 1 to 3 in the second cycle of 124, to give 0.124324..., and doing so again at the fifth, then ninth cycle, increasing the gap between the replacement 3s by one cycle each time. For example, the British coastline could, in theory, be measured with a one-dimensional rope, but inlets would require string, and crevices require thread. The quadratic function is generally written as y = ax2 + bx + c, which, on a graph, produces a curve called a parabola. Exchange of ideas in addition to drawing on theorems formulated by the ancient Greeks, Descartes exchanged ideas with other French mathematicians, among them Pierre de Fermat, with whom he frequently corresponded. such as The quadratic formula is a way to solve quadratic formula is a way to solve quadratic formula is a way to solve quadratic equations. The concept of infinity is central to calculus. In 1989, she was appointed leader of the Sonic Boom Team, working on designs to reduce noise pollution and other negative effects of supersonic flight. Although little is known of Bayes' life as a mathematician, in 1736 he anonymously published An Introduction to the Doctrine of Fluxions, and a Defence of the Mathematicians Against the Objections of the Analyst, in which he defended Isaac Newton's calculus foundations against the Criticisms of the Analyst, in which he defended Isaac Newton's calculus foundations against the Criticisms of the Analyst, in which he defended Isaac Newton's calculus foundations against the Criticisms of the Analyst, in which he defended Isaac Newton's calculus foundations against the Criticisms of the Analyst, in which he defended Isaac Newton's calculus foundations against the Criticisms of the Analyst, in which he defended Isaac Newton's calculus foundations against the Criticisms of the Analyst, in which he defended Isaac Newton's calculus foundations against the Criticisms of the Analyst, in which he defended Isaac Newton's calculus foundations against the Criticisms of the Analyst, in which he defended Isaac Newton's calculus foundations against the Criticisms of the Analyst, in which he defended Isaac Newton's calculus foundations against the Criticisms of the Analyst, in which he defended Isaac Newton's calculus foundations against the Criticisms of the Analyst, in which he defended Isaac Newton's calculus foundations against the Criticisms of the Analyst, in which he defended Isaac Newton's calculus foundations against the Criticisms of the Analyst, in which he defended Isaac Newton's calculus foundations against the Criticisms of the Analyst, in which he defended Isaac Newton's calculus foundations against the Criticisms of the Analyst, in which he defended Isaac Newton's calculus foundations against the Criticisms of the Analyst, in which he defended Isaac Newton's calculus foundations against the Criticisms of the Analyst, in which he defended Isaac Newton's calculus foundations against the Criticisms of the Analyst, in which he defended Isaac Newton's calculus foundations against the Criticisms of the Analyst, in which he defended Isaac Newton's calculus foundatio the FTA was made in 1746 by French mathematician Jean le Rond d'Alembert in his "Recherches sur le calcul intégral" ("Research on integral calculus"). 194 BCE) FIELD Number theory BEFORE c. Since heat in the form of radiation is a wave, Fourier's discoveries about heat distribution had applications to the study of sound. He developed an approach that would allow him to calculate the temperature at any location within an object, at any time after a source of heat had been applied to one of its edges. See also: Pythagoras • Euclid's Elements • Trigonometry HIPPOCRATES OF CHIOS C. It says that if a straight lines (A, B) creates interior angles on one side that total less than two right angles (180°), lines A and B will eventually cross on that side, if extended indefinitely. Some were sufficiently accurate for practical purposes, but several Chinese mathematicians sought more precise methods for determining n. Meanwhile, in drawings and calculations, Babbage had also been working on his idea for an Analytical Engine. Kempe received plaudits for this work, and two years later became a Fellow of the Royal Society partly on the strength of his proof. Galois showed that this group, must have one kind of structure if the equation is algebraically solvable, and a different kind of structure if it is not. 276 Bacon's cipher English philosopher and courtier Francis Bacon (1561-1626) was a great dabbler in cryptography, or the science of deciphering codes. Convergence A property of some infinite mathematical series where not only is each term smaller than the last, but the terms, when added up, approach a finite answer. Alamy Stock Photo: Painters (tr). He conceptualized the ring—a set of elements with two operations, such as addition and multiplication. The truth of Catalan's conjecture could now be tested by checking each of these powers. If interest is calculated more often than annually, the calculation changes. Place value—the Babylonian legacy—remains a powerful, easily understood, and efficient way to represent large numbers. At 60 (60 × 1), the original symbol for one was reused but placed further to the left than the symbol for 1. In 1847, a friend, British logician Augustus De Morgan, became involved in a dispute with a philosopher about who deserved the credit for a particular idea. Getty Images: Fine Art / Corbis Historical (clb). AFTER 1202 In his book Liber Abaci, Leonardo of Pisa (Fibonacci) introduces zero to Europeans. Dorling Kindersley: H. In the Bible, for instance, Psalm 90 talks of a human lifespan being "threescore years and ten" or as great as "fourscore years." From around 500 BCE until the 16th century when Hindu-Arabic numbers were officially adopted in China, the Chinese used rod numerals to represent numbers. It is written as a collection of 246 practical problems and their solutions. De Méré consistently lost the bet and was compelled to deduce that a double ace from 24 throws of a pair of dice was less likely than one ace from four throws of a single die. of algebra to generalize relationships. As in the Babylonian system, the position of a digit indicates its value, and the smallest value digit is always to the right. She met Charles Babbage when she was 17 and was fascinated by his work. For example, 4 is a polygonal number as 4 dots can form a square, and 10 is a polygonal number as 10 dots can form a triangle with 4 dots at the base, 3 dots on the next, and 1 dot at the top of the triangle (4 + 3 + 2 + 1 = 10). In 335 BCE, Aristotle returned to Athens and founded the Lyceum, a rival institution to the Academy. Correlation measures the degree to which two random variables, such as height and weight, correspond. 42 Getty Images: DEA Picture Library / De Agostini (br). In 1886, he was appointed as chair of mathematical physics and probability at the Sorbonne in Paris, where he spent the rest of his career. However, in 1813, another Swiss mathematician, Simone L'Huilier, noted that Euler's formula was not true for all polyhedra; it was false for polyhedra with holes and for nonconvex polyhedra—shapes with some diagonals (linked by vertices) not contained within or on the surface. Newtons can be converted into measurements of mass, such as pounds. Algebra A branch of mathematics that involves the use of letters to stand for unknown or variable numbers in calculations. Four dimensions Hamilton's solution was to add a fourth nonreal unit, k. René Descartes Curved lines La Géométrie sets out what soon became the foundation of the coordinate system. By 19, he had won the Paris Grand Prix for mathematics, and at 23, he became a professor at the University of Bonn. Composite number that is not prime, but can be created by multiplying together smaller numbers. For example, Ta(4, 2, 2) requires the smallest number that is the sum of four squares (or two fourth powers) in two different ways: 635,318,657. The oldest surviving abacus is the Salamis Tablet, a marble slab made c. Two of the earliest estimates for π are given in the ancient Egyptian documents known as the

Rhind and Moscow papyri. Islamic scholars not only 129 preserved and translated the ancient Greek texts, but provided commentaries on them and developed their own original concepts. Placeholder A numeral, usually zero, used in a place value system to differentiate 1 from 100, for example, but that does not necessarily imply an exact measurement as in phrases such as "about 100 miles away." Plane A flat surfaces, which are twodimensional surfaces, which are twodimensional surfaces, but the Klein bottle can only exist properly in four). Graduating from Trinity College, Cambridge, he embarked on a career as a conveyancing lawyer. He worked as a tutor before becoming a member of the Munich Academy in 1759 and the Berlin Academy in 1759 Jacobi was initially tutored by an uncle. 319 Using pi, the probability of a randomly chosen point falling within the quarter circle in this square can be calculated as roughly 78 percent. This means that the second particle is never going to reach the end of its journey. For example, if you toss a coin for long enough, the number of times it comes up heads will get closer and closer to half the total of tosses —a probability of 0.5. In 1718, Abraham de Moivre grappled with the mathematics underpinning probability. The Enigma machine was a formidable encryption device. In a geometric sequence, each number after the first term is determined by multiplying the previous number by a fixed amount, called the "common ratio." For example, the sequence 1, 2, 4, 8, 16 has a common ratio of 2. Probability The branch of mathematics that studies the likelihood of different outcomes occurring in the future. He held the post until his death in Paris in 1788 390-C. Binomial An expression consisting of two terms added together, e.g. x + y. AlKhwarizmi died around 850. This tetrahedron is a 3-D structure made of Sierpinski triangles. Algebra evolved from the need to solve practical problems, often of a geometrical nature, requiring the determination of a length, area, or volume. This diagram shows two numbers: 3 + 5i and 7 + 2i. Wellcome Collection (bl). (2) The degree or order of a polynomial is the highest-power term within it: for example, a polynomial is "of degree 3" or "of order 3" if it contains a cubed term, such as x3, as its highest power. Matrices have applications in mathematics, physics, and computer science, such as in computer graphics and describing the flow of a fluid. Automatic calculation In the 17th and 18th centuries, mathematicians such as Gottfried Leibniz and Blaise Pascal had created mechanical calculating aids, but these were limited in power and also prone to error as human input was needed at every step. After many iterations, run for long periods, that region becomes a beautiful looping surface. 470-C. Wolfgang Bolyai Father of János Bolyai Artistic explorations Hyperbolic geometry also features in art. Interest in computing had begun in the search for a solution to Hilbert's Entscheidungsproblem (decision problem) and the possibility of an algorithm to determine the truth or falsity of a statement. Although polygons had long been used to estimate the circle) regular polygons to find upper and lower limits for n. For example, factorial 5, also written 5! (with an exclamation mark) is 5 × 4 × 3 × 2 × 1 = 120. 1801 French weaver Joseph-Marie Jacquard demonstrates the first programmable machine - a loom controlled by a punchcard. The result, called relative frequency, gives the probability of throwing a six, which can be expressed as a fraction, a decimal, or a percentage. Quadratic equations are used to model the relationship between time, speed, and distance, and in calculations with parabolic objects such as lenses. When a referee flips a coin, there is no advantage, according to the law of large numbers, in a team captain basing a heads or tails choice on what has been called in previous games. In 1653, he was struck down by plague but survived to do some of his best work. Hilbert possessed a substantial grasp of most fields of mathematics. This continues, and Achilles never catches up. Although Poincaré had laid the foundations of chaos theory, Edward Lorenz was able to establish the principles more firmly with the aid of computer models. finished its job. Key works Glorious on algebra Vonderful on calculation 154 See also: Positional numbers • Diophantine equations • Zero • Algebra • Probability • Calculus • The fundamental theorem of algebra 155 IN CONTEXT KEY FIGURE Omar Khayyam (1048-1131) FIELD Algebra BEFORE 3rd century BCE Archimedes solves cubic equations using the intersection of two conics. However, Pythagoras is believed to have been the first to prove the truth of the formula that states this relationship, and its validity for all right-angled triangles, which is why the theorem takes his name. He established a set of procedures for performing arithmetic on polynomials— expressions that contain a mixture of algebraic terms. This held that souls are immortal and at death transmigrate to occupy a new body. 1963 Edward Lorenz describes the Lorenz attractor, a model that produces chaotic results with every tiny change to the initial parameters. The two-dimensional complex plane extends the onedimensional number line by combining real numbers with imaginary units. It is illustrated in the 1303 book by Zhu Shijie entitled Precious Mirror of the Four Elements. In 1766, at Euler's recommendation, he was made Director of Mathematics at the Berlin Academy, and in 1787 he moved to the Académie des Sciences in Paris. Laplace's "intellect" would be capable of capturing and analyzing all of their movements; it would create a single formula that uses present movements to ascertain past and predict future ones. John B. Monge was active in public life, embracing the ideals of the French Revolution. I was ready to... remove the flaw from geometry [but] turned back when I saw that no mar can reach the bottom of this night. Algebraic geometry The use of graphs to plot lines and curves that represent the change of one quality with respect to another—for example, how temperature changes with distance. Key work 1915 Contributions to the founding of the theory of transfinite numbers • Calculus • The logic of mathematics • The infinite monkey theorem 409 IN CONTEXT KEY FIGURE John Venn (1834-1923) FIELD Statistics BEFORE c. This has not, however, deterred mathematicians in their resolute efforts to prove Gödel wrong. 1859 Bernhard Riemann outlines a possible proof for the prime number theorem, but the necessary mathematicians' fascination with primes has led them to seek a formula that would predict the values of this function. Briggs also helped with the calculation of logarithms of ordinary numbers based on the logarithm of 10 being 1 and spent several years recalculating the tables. It began with a summary and criticism of d'Alembert's proof, among others. An encoded plaintext might read "buy lemons on Thursday," where "buy" is code for "kill" and "lemons" is code for a particular target on a hit list—perhaps with all targets encoded as fruits. A self-confessed mistake threw his calculations for the stable orbit into doubt, but in turn paved the way for the study of "chaos theory." He died in 1912. By the early 1960s, he was attracting attention for the unexpected results of a toy climate model ("toy" meaning that it was a simplistic model made to demonstrate processes concisely). He died of a heart attack in 1918. , which does not Incommensurable Something that cannot be measured exactly in terms of something else. Allied codebreakers used the Turing Bombe, an electromechanical device that mimicked Enigma machines to break the encryption by brute force, using shortcuts developed by British mathematicians in the 1800s also found new purposes for Cartesian coordinates. Mathematicians since Argand have continued to work on proving the FTA using new methods. As the number of sides increases, the polygons (of known area) more closely resemble the circle. In 1509, Italian priest and mathematical and geometric underpinnings of perspective in architecture and the visual arts. 680 Little is known about Bhaskara I, although he may have been born in the Saurastra region on India's west coast. Mathematical proof is a vital part of modern algebra and one of the tools of proof is called mathematical induction. For example, a cube, which has six faces and eight vertices, and an octahedron (eight faces and six vertices) form a dual pair. Each month was worked out by the passage of the sun into the 161 corresponding zodiac region, which required both computations and actual observations. While such ideas may seem obvious to us today, they were profound insights for their time. To describe rotation, complex numbers are multiplied together. The plus or minus symbol is included because two negative numbers multiplied together make a positive number. Other counting systems In Mesoamerica, on the other side of the world, the Mayan civilization developed its own advanced numeration system in the 1st millennium BCE—apparently in complete isolation. It is this that makes Euclid unique; his writings are the earliest surviving example of fully axiomatized mathematics. Hippasus realized, however, that the square root of 2 could not be written as a fraction, as there is no rational number that can be multiplied by itself to produce precisely 2. At any instant there is a unique correspondence between the positions of the two particles. 250 BCE Archimedes finds an approximate value for  $\pi$  using a polygon algorithm method. Three centuries later, the Roman 453 philosopher Cicero argued that this was so unlikely that it was essentially impossible. 566 Number line A horizontal line with numbers written on it that is used for counting and calculating. PIERRE DE FERMAT Born in 1601 in Beaumont-de-Lomagne in France, Pierre de Fermat moved to Orléans in 1623 to study law and soon began to pursue his interest in mathematics. The good, of course, is always beautiful, and the beautiful never lacks proportion. ANCIENT AND CLASSICAL PERIODS Exploring pi is like exploring the Universe David Chudnovsky, Ukrainian-American mathematician The very flower of arithmetic Regiomontanus, German mathematician and astronomer An incomparable star in the firmament of wisdom Martin Cohen, British philosopher THE MIDDLE AGES Algebra is a scientific art Omar Khayyam, Persian mathematician and poet The ubiquitous music of the spheres Guy Murchie, American writer The power of doubling Ibn Khallikan, Islamic scholar and biographer THE RENAISSANCE The geometry of art and life Matila Ghyka, Romanian novelist and mathematician Like a large diamond 576 Chris Caldwell, American mathematician THE ENLIGHTENMENT for incelli, Italian physicist and mathematician THE ENLIGHTENMENT One of those strange numbers that are creatures of their own Ian Stewart, British mathematician No theory is perfect Nate Silver, American statistician Simply a question of algebra Robert Simpson Woodward, American engineer, physicist, and mathematician Algebra often gives more than is asked of her Jean d'Alembert, French mathematician and philosopher THE 19TH CENTURY The imp that knows the positions of every particle in the Universe Steven Pinker, Canadian psychologist An indispensable tool in applied mathematics Walter Fricke, German astronomer and mathematician A new kind of function 577 W. The image on the right shows how the presence of three other celestial bodies— perhaps nearby planets or other stars—complicates the planet's path, making it unpredictable, or chaotic. Problem solving and theory building go hand in hand. 1801 Carl Friedrich Gauss introduces the symbol for congruence—equal size and shape. See also: The law of large numbers • Bayes' theorem • Buffon's needle experiment • The birth of modern statistics 257 IN CONTEXT KEY FIGURE Vincenzo Viviani (1622-1703) FIELD Geometry BEFORE c. In 629, he also found a remarkably accurate approximation of the sine function. Italian polymath Galileo gave the cycloid its name (from the Greek for "circular") and tried to calculate its area by cutting up models of a cycloid and a circle, weighing the pieces, and comparing the results. HIPPARCHUS Hipparchus was born in Nicaea (now Iznik in Turkey) in 190 BCE. Networks are used to model relationships between objects in many disciplines—including computer science, particle physics, economics, cryptography, sociology, biology, and climatology— usually with a view to optimizing a particular property, such as the shortest distance between two points. This number may decrease with the growth of social media. In a key paper in 1921, Idealtheorie in Rings), she studied ideals in a particular set of "commutative rings," in which with a view to optimizing a particular set of "commutative rings," in which with a view to optimizing a particular set of "commutative rings," in which with a view to optimize the shortest distance between two points. the numbers can be swapped around when they are multiplied without affecting the result. Whether or not he was responsible for all the many achievements attributed to him in math, science, astronomy, music, and medicine, there is no doubt that he founded an exclusive community that lived for the pursuit of mathematics and philosophy, and regarded numbers as the sacred building blocks of the Universe. It was so called because it is the exact sum of all the divisors less than itself. Finding primes in random ways The RSA algorithm and other public key encryption systems require a large collection of primes in random ways axis and horizontal x-axis on a graph. This allowed Allied codebreakers to try frequently used phrases, such as "Heil Hitler" and "Weather Report" to attempt to figure out that day's key. Taking each endpoint as a center, he then drew two intersecting circles, so that each had the radius AB. Once formed, the Möbius strip has only one surface—an ant crawling along that surface would be able to cover both sides of the paper in one continuous movement without crossing the edge of the paper. Polynomial equations involving numbers and a single unknown quantity (x, and powers of x such as x2 and x3) are a powerful tool for solving real-world problems. One sequence of numbers occurs time and again in the natural world. He called such negative solutions "fictitious" and "false" and described the intellectual effort involved in finding them as "mental torture." His Ars Magna by 5 shows his use of the negative square root. Zeno was inducted into the school of Eleatic thought, founded by Parmenides. For much of the past 2,000 years, the prevailing assumption has been that most natural objects—mountains, trees, and so on—can be deconstructed into Latin in the 12th century by Robert of Chester, and soon after, complete translations of Euclid's Elements and other important texts began to appear in Europe. 225 BCE On Spirals Squaring the circle," was a popular challenge for mathematicians in ancient Greece. Their solution to the problem was the theorem of calculus, a set of rules for calculating using infinitesimals. Weldon, followed by the world's first university 440 IN CONTEXT KEY FIGURE Bertrand Russell (1872-1970) FIELD Logic BEFORE c. More commonly, coordinates are used to describe points on twodimensional space, which also has depth. However, many other flowers have four or six petals, so while numbers from the sequence are common, other patterns are also found. Fermat also located maxima and minima, the greatest and least values of a curve. A single invariant property, such as collinearity, needs only to be proved for a single case, rather than tested on each conic. AFTER 1937 Claude Shannon designs electrical switching circuits that use Boolean algebra to make digital circuits that follow rules of logic. Fractals, a word derived from the Latin fractus, meaning "broken," would eventually lead to the topic of fractal geometry. By modern convention, quadratic equations include a number, a multiplied by x2; a number, b, multiplied by x2; and a number, c, on its own. By arranging the segments of a circle in a near-rectangular shape, it can be shown that the area of a circle is mr2. Polar coordinate system is often used to calculate the movement of objects around, or in relation to, a central point. 1995 Andrew Wiles publishes his final proof of Fermat's last theorem. A mathematical argument uses the rules of logic to ensure that if a basic proposition 390 is true, then any and all statements constructed from that proposition will also be true. Books I to IV tackle plane geometry—the study of flat surfaces. Oakland University runs the Erdős Number Project, which analyzes collaboration among research mathematicians. Imaginary and complex numbers were new tools in this endeavor, and Bombelli's Algebra advanced understanding of how these and other numbers work. As a result, the two parts of any complex number can be plotted on a surface or plane. He also recognized its application in statistics. Leonhard Euler 294 Derangements The various ways in which a set of items can be ordered are called permutations. GEORGE BOOLE Born in Lincoln in 1815, George Boole was the son of a shoemaker who passed his love of science and mathematics on to him. 1915 Wacław Sierpinski describes the fractal pattern of triangles later known as Sierpinski triangles. In fact, the character may already have been in use for some time. The first verified instance of the use of matrices comes from ancient China. The particle on the infinite line traveled with uniform motion, so it covered equal distances in equal times. This idea of monkeys typing the works of Shakespeare captured people's imagination and Borel's law came to be known as the infinite monkey theorem. In 1823, he managed to persuade the British government to part-fund the project, with the promise that it would make producing official tables much quicker, cheaper, and more accurate. But in 1907, German mathematician Hermann Minkowski delivered a lecture in which he added time, an invisible fourth dimension, to create the concept of spacetime. The figure zero first became known in Italy from the Arte dell' Abbaco (Art of Calculation, also known as The Treviso Arithmetic), published anonymously in 1478 and the first printed mathematician Vincenzo Viviani studied under Galileo in Florence. Using this diagram, Khayyam drew a perpendicular line from the first printed mathematics textbook in Europe. point where the circle and parabola intersected down to the x axis. This was soon resolved in 1900 by German-born American mathematician Eudoxus of Cnidus establishes a strong mathematical foundation for irrational numbers. A third kind would fly off to infinity—perhaps right away, or perhaps after a period of apparent stability. Each pair of axes creates its own coordinate plane; these intersect at right angles to each other, thus dividing the space into eight zones called octants. The numbers 3, 5, 7, and many others are similarly nonsquare and in each case, their square root is irrational. Using algebra as his starting point, George Boole devised a form of logic based on a binary system, and using the operators AND, OR, and NOT. Now known as Buffon's needle experiment, it was one of the earliest probability calculations. Reckoning by twos, that is, by 0 and 1... is the most fundamental way of reckoning for science, and offers up new discoveries, which are... useful, even for the practice of numbers. Fibonacci made several assumptions: no rabbit ever died; rabbit pairs mated every month, but only after they were two months old, the age of maturity; and each pair produced one male and one female offspring every month. It is evident that primes decrease in frequency as numbers. get larger. 1639 In France, 16-year-old Blaise Pascal asserts that where a hexagon is inscribed in a circle, the opposite sides of the hexagon meet at three points on a straight line. If the game is interrupted before an ace has been thrown, Pascal seems to suggest that the stakes should be allocated according to the players' expectations of winning. For Pythagoras and his successors, the duty of a philosopher was the pursuit of wisdom. A piano keyboard scale from C to C spans 13 keys, eight white and five black. This phenomenon is popularly known as the "butterfly effect," after the frequently cited example of the massive effect a single butterfly can theoretically have on a weather system when it causes a small disturbance by flapping its wings. Today, normal distribution is widely used to model statistical data, with applications ranging from population studies to investment analysis. His famous drawing of the "Vitruvian Man"—a "perfectly proportioned" man inscribed in a circle and square—for Divina Proportione is also said to contain many instances of the golden ratio in the proportions of the ideal human body. Euler's number can help to find both the maximum product, as e(10/e) = 39.598..., and number of partitions: 10/e = 3.678.... If these conditions apply, the chef can find the value of n—how many potatoes to pre-bake. The theorem has two parts. 310 In 1763, Richard Price, a Welsh minister and mathematician, published a paper called "An Essay Towards Solving a Problem in the Doctrine of Chances." Its author, the Reverend Thomas Bayes, had died two years earlier, leaving the paper to Price in his will. Later, during the Sui dynasty (581-618 CE), the Chinese also used triangular rods for positive quantities and rectangular rods for negative quantities. 2008 In a bid to stimulate major new mathematical breakthroughs, the US Defense Advanced Research Projects Agency (DARPA) announces its list of 23 unsolved problems. He is said to have experimented with notes produced by lyre strings of different lengths. ZENO OF ELEA Zeno of Elea was born around 495 BCE in the Greek city of Elea (now Velia, in southern Italy). In ancient Greece, Zeno's paradoxes of motion, a set of philosophical problems devised by the philosophilosophical problems devised by the philosophical pro mathematician François Viète promoted the use of symbols in algebra (which had previously been described in words), while Flemish mathematician Simon Stevin initiated the concept of 266 mathematical limits, whereby the sum of amounts could converge to a limiting value, much like the area of Archimedes' polygons converged to the area of a circle. Key works 1711 De Mensura Sortis (On the Measurement of Chances (1st edition) 1756 The Doctrine of Chances (3rd edition) 1756 The Doctrine of Chances (3rd edition) 1756 The Doctrine of Chances (3rd edition) 1756 The Doctrine of Chances (1st edition) 1756 The Doctrine of Chan distribution • The birth of modern statistics 301 IN CONTEXT KEY FIGURE Leonhard Euler (1707-83) FIELDS Number theory, topology BEFORE 1727 Euler develops the constant e, which is used in describing exponential growth and decay. Laplace died in Paris in 1827. In mathematics, there are different kinds of infinity: the set of natural numbers theory, topology BEFORE 1727 Euler develops the constant e, which is used in describing exponential growth and decay. for example, is countably infinite (countable one by one, even though the end is never reached), while the real numbers. Key works 1921 Idealtheorie in Ringbereichen (Ideal Theory in Rings) 1924 Abstrakter Aufbau der Idealtheorie im algebraischen Zahlkörper (Abstract Construction of Ideal Theorem • The binomial theorem • The binomial theorem • The binomial theorem • The fundamental theorem • The binomial theorem theory, algebra BEFORE 1637 René Descartes creates coordinate geometry, allowing points on a flat surface to be described. Nevertheless, de Moivre impressed and befriended many eminent scientists of the time, including Isaac Newton, and was elected as a fellow of the Royal Society in 1697. Manual and electronic methods have, as yet, failed to find any even number that does not conform to the original strong conjecture. It is a key tenet of Buddhist meditation and mandalas (symbols used to suggest the infinite nature of God in Islamic decoration, such as tilework. The next prime is 7, and removing its multiples eliminates 49, 77, and 91. AFTER 1997 In the US, Neil Robertson, Daniel P. His revolutionary understanding of space was used by Einstein in developing relativity theory. Fuzzy logicians call these crisp sets: each egg either does not belong. Expansion In algebra, the expansion of an expression is the opposite of factorization. In particular, Aristotle looked at a type of argument known as a syllogism, consisting of three propositions. This illustration of Kepler's Platonic solid model of the Solar System appeared in a book published in 1596. Aristotle identified the structure of syllogisms that are logically valid, those where the conclusion follows from the premises, and those that are not, where the conclusion does not follow from the premises, providing a method for both constructing and analyzing logical arguments. ARCHIMEDES Born in c. Institute for Advanced Study: Randall Hagadorn (bl). He later proposed a test of artificial intelligence. The description "simply connected" means that the figure has no holes, unlike a bagel or hoop shape (torus), and "closed" means the shape is limited by boundaries, unlike the open endlessness of an infinite plane. Newton recognized that at the maxima and minima, the gradient of the curve was zero, because when something is at its highest or lowest point, it is momentarily not changing. Dreamstime.com: Millafedotova (crb). This makes the square root of 2 an irrational termed nonsquare or square-free. Benefits of decimals The same processes of addition, subtraction, multiplication, and division of whole numbers, resulting in a far simpler way of performing basic arithmetic than the previous method, which relied on learning a different set of rules for calculations with fractions. It was at least partly Bourbaki's work on algebraic geometry that led British mathematician Andrew Wiles to finally prove Fermat's last theorem; he published his proof in 1995. Abu Kamil embraced irrational numbers as possible solutions to quadratic equations, rather than rejecting them as awkward 143 anomalies Logical deductions Euclid was not writing in a vacuum; he built upon foundations laid by a number of influential Greek mathematical terms, logarithms are the inverse of exponentials (powers of a number) and can be to any base. Key works Elements Conics Catoptrics Phaenomena Optics World of proof The title of Euclid's work has a particular meaning that reflects his mathematical approach. 1854 George Boole publishes The Laws of Thought, his second book on algebraic logic. Notice anything strange about that? Rijksmuseum, Amsterdam: (bl). Radian A measure of angles that is an alternative to degrees and is based on the length of the radius and circumference of a circle. If it were possible to set the start conditions precisely, it would be possible to create every conceivable path. Key works 1860 Traité élémentaire des séries (Elementary Treatise on Series) 1890 Intégrales eulériennes ou elliptiques (Eulerian or Elliptic Integrals) See also: Pythagoras • Diophantine equations • The Goldbach conjecture • Taxicab numbers • Proving Fermat's last theorem 382 IN CONTEXT KEY FIGURE James Joseph Sylvester (1814-97) FIELDS Algebra, number theory BEFORE 200 BCE The ancient Chinese text The Nine Chapters on the Mathematical Art presents a method for solving equations using matrices. Gottfried Leibniz The teaching and commentaries on the I Ching of ancient Chinese philosopher Confucius (551-479 BCE) influenced the work of Leibniz and other 17th-18th-century scientists. As the sun eclipses the stars by its brilliancy, so the man of knowledge will eclipse the fame of others in assemblies of the people if he proposes algebraic problems, and still more if he solves them. In (a + b)2, n = 2. Three years later, he published the first textbook on infinitesimal calculus: Analyse des infiniment petits pour l'intelligence des lignes courbes (Analysis of the Infinitesimally Small for the Understanding of Curved Lines). Recorde suggested that symbols would save mathematicians from having to write out calculations in words. He then put forward five postulates: any two points can be joined with a straight line segment, a circle can be drawn having the segment as its radius and one endpoint as its center; all right angles are equal to one another; and a postulate about parallel lines (see Euclid's five postulates). The building blocks of codes now used to program computer software are based on the logic formulated by Boole. 190-120 BCE) FIELD Geometry BEFORE c. 57 The legacy of Pythagoras's status as the most famous mathematician from antiquity is justified by his contributions to geometry, number theory, and music. Aristotle's ideas about logic were unrivaled and unchallenged in Western thought for more than 2,000 years. Hitherto unsolved lengthy calculations, which could now be done quickly and accurately by computer. It is my hope that Six Degrees [a philanthropic project] will... [bring] a social conscience to social networking. 1806 Jean-Robert Argand publishes a geometrical interpretation of complex numbers, leading to the Argand diagram. Littlewood, with whom she would have a long academic collaboration, especially on the study of functions and differential equations. Mayan numeral system The Dresder Codex, the oldest surviving Mayan book, dating from the 13th or 14th century, illustrates The Mayans, who lived in Central America from around 2000 BCE, used a base-20 (vigesimal) number system from around 570 BCE on the Greek island of Samos in the eastern Aegean Sea. Pythagorean community After 20 years of traveling, Pythagorean community After the golden ratio (the ratio of the length of the head to its width, for instance) are often cited as being more attractive than those whose faces do not. These differ from irrational numbers, which do not terminate and have no pattern of recurrence. Alongside rates of change, another important aspect of calculus is summation (the process of adding things), which developed from the need to calculate areas. For example, a square matrix can be repeatedly multiplied by itself. The first and final numbers are always 1. The graph of y = ex (the exponential function) is a curve whose tangent (the straight line that touches but does not intersect the curve) at the coordinates (0,1) also has a gradient (steepness) of precisely 1. Euler analyzed derangements of 10 numbers for Frederick the Great of Prussia, who hoped to create a lottery to pay off his debts. It only became practical to follow a great circle course with the invention of GPS. Fibonacci employed a horizontal bar to divide the numerator and denominator (bottom number), but followed the Arabic practice of writing the fraction to the left of the integer, rather than to the right. The probabilities are calculated by dividing the number of objects: for example, in a family of three children (the total number of objects), the probability of one girl and two boys is 3/8 (the sum of all the coefficients in the third row of the triangle is 8, and there are three ways of having one girl in a family of three children). [The golden proportion] is a scale of proportions which makes the bad difficult [to produce] and the good easy. The number 4 (2 × 2) has already had its multiples removed, since they are all even Swiss giants Building on the work of Newton and Leibniz, whose ideas were finding practical application in physics and engineering, the brothers Jacob and Johann Bernoulli further developed the theory of calculus in their "calculus of variations" and several other mathematical concepts discovered in the 1600s. So the binary number 1011 is not 1,011 but 11 (from right to left: one 1, one 2, no 4s, and one 8). An ellipse is a closed curve that is defined as the set of all points in a plane, the sum of whose distances from two fixed points —each one called a focus—is always the same number. The set of all points in a plane, the sum of whose distances from two fixed points —each one called a focus—is always the same number. The set of all points in a plane, the sum of whose distances from two fixed points —each one called a focus—is always the same number. mathematician Siméon Poisson, and based on the work of Abraham de Moivre, it can help to forecast a wide range of possibilities. Only Alice can find out using her private key, z, because Cz modn = M. The RSA algorithm was an early development in building asymmetric encryption, where a sender and receiver use two keys: one private and the other public. Imprisoned when the Germans invaded France in World War II, he was released and fought for the Resistance, earning himself the Croix de Guerre. The same number is also significant in a curve called the golden spiral, which gets wider by a factor of 1.618 for every quarter turn it makes. More renowned for findings such as Pascal's triangle and Fermat's last theorem, the two men took the mathematics of probability to a new level, laying the foundations for probability theory. Desargues then considered what happens as the vertex of the projection cone moves further away. This translates as x + x/7 = 19. Blaise Pascal Number theory There are also many more complex patterns hidden within the triangle. Scottish mathematician John Napier compiled tables of logarithms to base 2.718..., which worked particularly well for calculations involving exponential growth. For example: "All French people are European. Quaternion A mathematician John Napier compiled tables of logarithms to base 2.718..., which worked particularly well for calculations involving exponential growth. For example: "All French people are European. Quaternion A mathematician John Napier compiled tables of logarithms to base 2.718..., which worked particularly well for calculations involving exponential growth. For example: "All French people are European. Quaternion A mathematician John Napier compiled tables of logarithms to base 2.718..., which worked particularly well for calculations involving exponential growth. For example: "All French people are European. Quaternion A mathematician John Napier compiled tables of logarithms to base 2.718..., which worked particularly well for calculations involving exponential growth. For example: "All French people are European. Quaternion A mathematician John Napier compiled tables of logarithms to base 2.718..., which worked particularly well for calculations involving exponential growth. For example: "All French people are European. Quaternion A mathematician John Napier compiled tables of logarithms to base 2.718..., which were apprecision of the people are European. Quaternion A mathematician John Napier compiled tables of logarithms to base 2.718..., which were apprecision of the people are European. Quaternion A mathematician John Napier compiled tables of logarithms to base 2.718..., which were apprecision of the people are European. Quaternion A mathematician John Napier compiled tables of the people are european. Quaternion A mathematician John Napier compiled tables of tables are european. Quaternion A mathematician John Napier compiled tables are european. Quaternion A mathematician John Napier compiled tables are european. Quaternion A mathematician John Napier com components added together, rather than just two. Exponents can be viewed as instructions for how many times 1 should be multiplied by a given number. After graduating with a first-class degree in mathematics from the University of Cambridge in 1934, he went on to study at Princeton in the US. The exponential function Later in the 1600s, logarithms revealed something of further significance. Mathematicians now also seek "cabtaxi" numbers: based on the taxicab formula, these allow calculations using both positive and negative cubes. Modern computers and the programs that run on them are effectively working as Turing machines, and so are said to be "Turing complete." As a leading figure in mathematics and logic, Turing made important contributions to the development of real computers, not just virtual ones. Sir Thomas Heath British historian of mathematics that was governed by definite rules rather than the tables of numbers relied on by the earliest mathematicians. The idea that every mathematician from Fermat's time could have missed a proof that a mathematician from Fermat's time could have missed a proof that a mathematician from Fermat's time could have missed a proof that a mathematician from Fermat's time could have missed a proof that a mathematician from Fermat's time could have missed a proof that a mathematician from Fermat's time could have missed a proof that a mathematician from Fermat's time could have missed a proof that a mathematician from Fermat's time could have missed a proof that a mathematician from Fermat's time could have missed a proof that a mathematician from Fermat's time could have missed a proof that a mathematician from Fermat's time could have missed a proof that a mathematician from Fermat's time could have missed a proof that a mathematician from Fermat's time could have missed a proof that a mathematician from Fermat's time could have missed a proof that a mathematician from Fermat's time could have missed a proof that a mathematician from Fermat's time could have missed a proof that a mathematician from Fermat's time could have missed a proof that a mathematician from Fermat's time could have missed a proof that a mathematician from Fermat's time could have missed a proof that a mathematician from Fermat's time could have missed a proof that a mathematician from Fermat's time could have missed a proof that a mathematician from Fermat's time could have missed a proof that a mathematician from Fermat's time could have missed a proof that a mathematician from Fermat's time could have missed a proof that a mathematician from Fermat's time could have missed a proof that a mathematician from Fermat's time could have missed a proof that a mathematician from Fermat's time could have missed a proof that a mathematician from Fermat's time could have missed a proof that a mathematician from Fermat's time could have missed have missed a proof that a mathem to the work of Liu Hui in revising and expanding the classic texts of Chinese mathematical knowledge of the time, they were not seen as works of scholarship. The Persian mathematican and poet al-Khwarizmi (c. The 12-faced dodecahedron was associated with the heavens and its constellations. Cipher Any systematic method of coding messages so that they cannot be understood without being deciphered first. His acceptance of these as valid solutions was an important step in algebra. A vital point in the proof contained an error that undermined Wiles's method. BERTRAND RUSSELL The son of a lord, Bertrand Russell was born in Monmouthshire, Wales, in 1872. BRAHMAGUPTA Born in 598 CE, astronomer and mathematician Brahmagupta lived in Bhillamala, northwest India—a center of learning in those fields. When an equation is true of all the values of a variable (for example, the equation y × y × y = y3), it is called an identity. 2560 BCE, which has a base to height ratio of 1.5717. 1545 Italian mathematician Gerolamo Cardano publishes his Ars Magna, setting out the rules of algebra. In 1905, French mathematician Maurice Fréchet devised the idea of a metric space—a set of points along with a "metric" that defines the distance between them. AFTER 1713 Jacob Bernoulli's Ars Conjectandi (The Art of Conjecturing) develops Pascal's triangle. When it comes to a real physical system, there is no way to be absolutely precise about the starting conditions. In the 10th century, Egyptian scholar Abu Kamil Shuja ibn Aslam made use of negative numbers and algebraic irrational numbers (such as the square root of 2) 34 as both solutions and coefficients (numbers multiplying an unknown quantity). M then reaches an answer of Yes or No. Turing then imagined a modified version of this machine (M\*), which would do the opposite—it would loop forever (and not halt). AFTER 1638 Pierre de Fermat circulates his Method for determining Maxima and Minima and Tangents for Curved Lines. See also: Coordinates • Topology • The butterfly effect • Proving Fermat's last theorem • Proving Fermat's last theorem • Proving the Poincaré conjecture 464 IN CONTEXT KEY FIGURE Alan Turing (1912-54) FIELD Computer science BEFORE 1837 In the UK, Charles Babbage designs the Analytical Engine, a mechanical computer using the decimal system. Since antiquity, civilizations have appreciated the need for right angles in architecture. See also: The birth of modern statistics ARTHUR CAYLEY 1821-95 Born in Richmond, Surrey, Cayley was probably the leading British pure mathematician of the 1800s. Badly wounded, he died the next day, leaving behind just a handful of mathematical papers which contain the foundations of group theory, finite field theory, and what is now called Galois theory, and what is now called Galois theory, finite field theory, and what is now called Galois theory. variable—are also rational numbers. In this way, Lovelace argued, the machine could solve a system of linear equations or generate extensive tables of prime numbers. Pi ( $\pi$ ), which is the ratio of the circumference of a circle to its diameter, is irrational. In 1751, Leonhard Euler investigated the imaginary unit," or i. In 13 volumes, only six of which have survived, he explored 130 problems involving equations and was the first person to use a symbol for an unknown quantity—a cornerstone of algebra. Buffon classed needles as "successful" (blue) if they didn't, then calculated the probability of "successful" (blue) if they crossed a line when dropped, or "unsuccessful" (blue) if they crossed a line when dropped, or "unsuccessful" (blue) if they classed needles as "successful" (blue) if they classed needles as yield analysis, the predicted energy output of a wind farm during its lifetime is calculated, giving different levels of uncertainty, by using Monte Carlo probability methods. In the fifth month, both the original pair and their first offspring produce baby rabbits, although the second pair of offspring is still too young. Since the Renaissance, the mathematical analysis of art by means of the "golden mean"—or, as Pacioli called it, the Divine Proportion—has come to symbolize geometrical perfection. The title of the lecture was not chosen by Lorenz himself, but by physicist Philip Merilees, the convener of the American Association for the Advancement of Science's annual meeting in Boston. Ancient Greek thinkers and many later European mathematicians regarded negative numbers—and the concept of something being less than nothing—as absurd. Perhaps the greatest insight in her notes was Lovelace's vision of machines as mechanical brains with wide applications. He was not the first to study the probability of unlikely events. 1693 De Roberval's solution to the area of a cycloid is published more than 60 years after its discovery and 18 years after his death. Determinants The determinant of a matrix was named by Gauss, due to the fact that it determines whether the system of equations represented by the matrix has a solution. Pi ( $\pi$ ) The ratio of a circle's circumference to its diameter, approximately 22/7, or 3.14159. By the 19th and 20th centuries, mathematics had become a global discipline, with its practitioners involved in all the sciences. Returning to France in 1623, he sold his property there in order to secure a lifelong income, then moved back to the Netherlands to study. 1874 Georg Cantor creates set theory, describing how sets and their subsets interrelate. AFTER 350 BCE Aristotle produces his treatise Physics, in which he draws on the concept of relative motion to refute Zeno's paradoxes. The triangle is most commonly named after French philosopher and mathematician Blaise Pascal, who explored it in detail in his Treatise on the Arithmetical Triangle in 1653. Surprising the math world Perelman had achieved success quietly. To achieve a perfectly cooked egg, however, the boiling time must be adjusted to match the weight of the egg. She keeps a private key, z, to herself. He allegedly used the ratio in the design of the Parthenon in Athens. This pattern can be made without Pascal's triangle by breaking an equilateral triangles into ever smaller triangles by connecting the midpoints of each of the triangles' three sides. He continued to reach a logical arguments, and how they can be combined to reach a logical conclusion. Series A list of mathematical terms added together. The Egyptians knew that a triangle with sides of 3, 4, and 5 (the first Pythagorean triple) would produce a right angle, so their surveyors used ropes of these lengths to construct perfect right angle for their building projects. Adding a negative number gives the same result as subtracting its equivalent positive number, so 169 + -130 = 169 - 130 = 39. The message is converted into a code made of 0s and 1s - every 0 is a low voltage and every 1 is a high voltage. In 2015, President Obama awarded Johnson the Presidential Medal of Freedom. 358 The "squashed circle" of an ellipse is one of the most recognizable curves in math. This outcome is highly improbable, but mathematically, over an infinite time (or with an infinite number of monkeys), it must happen. They did so by using imaginary numbers in algebra. To achieve this, more than one number line is needed—each starting at the same zero point, or origin. He did so by drawing a straight line, removing the middle third (leaving two lines and a gap), and then repeating the process ad infinitum. First, 1 is crossed out. Quartic Referring to equations or expressions of the fourth degree, where the highest power contained in them is 4—for example, x4. Exhausted by anemia, he retired in 1930, and Göttingen's math faculty soon declined after the Nazi purges of Jewish colleagues. Any complex number, z, can be written in the form a + bi, where a is the real part of z and bi is the imaginary part. He introduced some preliminary concepts of calculus; established that dividing by zero yields infinity; found solutions to quadratic, cubic, and quartic equations (including negative and irrational solutions); and suggested ways to unlock Diophantine equations of the second order (to the power of two), which would not be solved in Europe until the 1700s. AFTER 2000 The Clay Institute issues a list of seven Millennium Prize mathematical problems, offering a million dollars for each problem solved. Then, in 1841, British mathematician William Rutherford computed 208 digits of π using arctan series. They were inspired by the 265 trigonometry of Arab astronomer al-Battani (858-929 CE) and the logic and physics of Aristotle. A path that travels each arc once and only once is called an Eulerian path (or a semi-Eulerian path if the start and end are at different nodes). He developed a loom that used cards punched with holes to tell it how to weave complex patterns in silk. 168 Although it is often associated with the arts, the Fibonacci sequence has also proved a useful tool in finance. 493 Langlands' letter In a 17-page handwritten letter to number theory and harmonic analysis. Returning to the UK in 1938, Turing joined the Government Code and Cypher School at Bletchley Park. Seven letters exchanged in 1654 reveal the two men's thoughts on the points problem, which they examined in different scenarios. This method is effective for small amounts of data, but is too time-consuming for large amounts. He introduced the Indian decimal place-value system to the Islamic world, which later led to the adoption of the Hindu-Arabic numeral system widely used today. This led Daniel Gorenstein to propose a more difficult task: the classification of every finite simple group. The former arrives at its views slowly, but they are... rigid; the latter is endowed with greater flexibility. Correlation and regression 435 It was first Galton, then Pearson, who began to draw these threads together. This study of basic mathematics, yet an essential adjunct to every other field of modern mathematics. Getty Images: Bletchley Park Trust / SSPL (bl). Although he considered himself French, Catalan won recognition in Belgium, where he lived from his appointment as professor of analysis at the University of Liège in 1865 until his death in 1894. There are three regular shapes that tessellate on their own, without the need for another shape: the square, equilateral triangle, and regular hexagon 1503 French mathematician Charles de Bovelles gives the first description of a cycloid in Introductio in geometriam (Introduction to Geometry). In the 1300s, it was exported to Japan, where it was called the soroban. Arts and analysis The Fibonacci sequence can also be found in poetry, art, and music. 574 TOM JACKSON A writer for 25 years, Tom Jackson has written about 200 non-fiction books for adults and children and contributed to many more on a wide range of science and technology topics. If more than two nodes have an odd number of arcs, then a route using each bridge only once is impossible. Key works 1798-1828 Celestial Mechanics 1812 Analytic Theory of Probability 1814 A Philosophical Essay on Probabilities See also: Probability • Calculus • Newton's laws of motion • The butterfly effect 346 IN CONTEXT KEY FIGURE Siméon Poisson (1781-1840) FIELD Probability BEFORE 1662 English merchant John Graunt publishes Natural and Political Observations upon the Bills of Mortality, marking the birth of statistics. 1800 BCE The columns of cuneiform numbers on the Plimpton 322 clay tablet from Babylon include some numbers related to Pythagorean triples. Prime number itself. No doubt, new applications for mathematical theorems will be found in the future too—and with numerous problems still unsolved, it seems that there is no end to the mathematical discoveries to be made. 2000 BCE The Babylonians develop positional numbers. It provides a shorthand summary of what happens when you multiply out a binomial, which is a simple algebraic expression consisting of two known or unknown terms added together or subtracted. Fermat 255 notes that what matters is the number of throws remaining when the game stops. When scientists at NASA's Jet Propulsion Laboratory in California wanted to know how much hydrogen might be available beneath the surface of Europa, one of Jupiter's moons, they estimated the hydrogen might be available beneath the surface area, which is 4mr2, as it is for any sphere. I learnt with as much astonishment as satisfaction that two young geometers...succeeded in their own individual work in considerably improving the theorem," smiled, and added, "I think I'll leave it there." Some mathematics problems look simple. Calculus has many important applications today; it is used, for instance, in search engines, construction projects, medical advances, economic models, and weather forecasts. Initially developed by the Arab mathematician al-Kindi in the 9th century, this technique made use of the frequency of each letter of the alphabet in a particular language. I believe in imaginary numbers. Today, projective geometry is used by architects and engineers in CAD technology, and in computer animation for films and gaming. Book XIII of the Elements is actually attributed to another author—Athenian mathematician and disciple of Plato, Theaetetus, who died in 369 BCE. Whether the Roman or Chinese abaci came first is unclear, but their similarities may be a coincidence, inspired by the way people count using the five fingers of one hand. This function is used to represent exponential growth—where the rate of growth of a quantity is proportional to its size at any particular moment, so the bigger it is, the faster it grows—which is relevant to fields such as finance and statistics, and most areas of science. The first is the identity axiom; it states that a unique element exists that does not change any element in the group when combined with it. Finally rehabilitated 20 years later, he lived quietly and died in 1131. 1683 Jacob Bernoulli uses e in his work on compound interest. Taimina went on to make her own models of hyperbolic surfaces using crochet to assist in her teaching. That, however, is the "gambler's fallacy"—where a person assumes that the outcomes of each game (toss) are connected. Modular arithmetic Also called clock arithmetic, a form of arithmetic where, after counting up to a certain point, 0 is reached, and the process is repeated. This, however, is an observed finding, based on actual experiments. Only eight of these produce a magic square, where the same. The beauty of number theory [is] related to the contradiction between the simplicity of the integers and the complicated structure of the primes. In the second century BCE, the textbook The Nine Chapters on the Mathematical Art described how to set out a counting board and use a matrixlike method to solve linear simultaneous equations with several unknown values. In 1901, he founded the statistical journal Biometrika with Francis Galton and evolutionary biologist Walter F. When the cylinder is cut from top to bottom and rolled out, it becomes a twodimensional map. He died in 1675. As well as pure mathematics, he made significant discoveries in theoretical physics, including his proposed principle of relativity. Quetelet Index (now called the BMI) to indicate body mass. English mathematician John Wallis (1616-1703) gave some meaning to negative numbers then enables the calculation of motion and rotation in two dimensions. In 1600, non-Catholics were expelled from Graz and Kepler moved to Prague, where his friend Tycho Brahe lived. Its values of V = 8, F = 6, and E = 12, when fed into the formula, produce the calculation 8 + 6 - 12 which equals 2. The subject was taken up by Nicolai Lobachevsky and János Bolyai, who independently developed theories of hyperbolic geometry and curved spaces, resolving the problem of Euclid's parallel postulate. His own work, Algebra, was a more accessible version, and was a thorough and innovative survey of the subject. ANDREW WILES The son of an Anglican priest who later became a professor of divinity, Wiles was born in Cambridge in 1953, and was a passionate problem-solver in mathematics from an early age Stephanie Strickland American poet To a topologist, a coffee mug is identical in shape to a doughnut, because by pulling, stretching, and bending one, you could mold one into the shape of the other. While studying number series, Italian mathematician Pietro Mengoli showed that the alternating series 1 - 1/2 + 1/3 - 1/4 + 1/5 - ... has a value of around 0.693147, which he demonstrated to be the natural logarithm of 2. Applying pi Astrophysicists use n in their calculations to determine the orbital paths and characteristics of planetary bodies such as Saturn. When measuring physical variables, such as sound, flow, or pressure, where values may change exponentially, rather than by regular increments, a logarithmic scale is often used. AFTER 1846 Galois' work is published posthumously by fellow Frenchman Joseph Liouville. When mathematicians first showed that the angles of any triangle in a flat plane when added together come to 180°, a straight line, this was not their invention: they had simply discovered a fact that had always been (and will always be) true. Pascal attended these meetings, but he and Fermat had never met. This can be shown as e = 1 + 1 + 1/2! + 1/3! + 1/4! in factorial notation. Douglas Hofstadter Cognitive scientist 199 A series of cups shows blue food dye being dripped over an ice cube (left). Jacobi was, like Riemann, multi-talented, often linking different fields of mathematics in new ways. Descartes presented his coordinate geometry in La Géométrie (Geometry, 1637), one of three appendices to his philosophical work Discourse on the Method), in which he proposed methods for arriving at truth in the sciences. While waiting for a drainage recommence, he embarked on his major work, Algebra, which laid out a primitive but thorough arithmetic of complex numbers for the first time. 6th century BCE Greek philosopher Thales of Miletus proposes a nonmythological explanation of the Universe— pioneering the idea that nature can be interpreted by reason. Scipione del Ferro produced the first algebraic solution to cubic equations, discovered in his notebook after his death. In his astronomical manual Almagest, written in the 2nd century CE, the Greco-Roman scholar Ptolemy used a circular symbol positionally between digits and at the end of a number, but did not consider it a number in its own right. For example, the statement A AND B can only be considered true if both A and B are true. Pearson's aim was to determine whether the difference between observed values and expected values and expected values and expected values are the building blocks for numbers in mathematics, just as the elements are for compounds in chemistry. So something like the short section on magic squares covers thousands of years and the span of the globe. L'Huilier devised a system 415 whereby every shape had its own "Euler characteristic" – (V - E + F)—and shapes with the same Euler characteristic were related regardless of how much they might be manipulated. Some things held Apollonius back: he did not use negative numbers, nor did he explicitly work with zero. Divergence A term applied mainly to infinite series that do not approach closer and closer to an end-number. The law works for ordinary addition and 558 multiplication, but not for subtraction or division. A male bee, or drone, develops from the unfertilized egg of a queen bee. This seems reasonable; the higher the numbers that could be divisors exist below it. A similar story emerges in the slightly later civilization of the ancient Egyptians. 1564 Gerolamo Cardano writes Liber de ludo aleae (The Book on Games of Chance), the first work on probability. Of the numbers between 1 and 100, 25 are prime (1 in 4); between 1 and 100,000, 9,592 are prime (about 1 in 10). He was also involved in the development of calculus, taking the side of Gottfried Leibniz against Isaac Newton in their rival claims to have invented a new mathematical field. Partial differential equation A differential equation containing several variables, in which the differentiation is applied to only one of the variables at a time. 4000-3000 BCE Babylonians use a small clay cone for 1 and a large cone for 60, along with a clay ball for 10, as their base-60 system evolves. Why they used 60 as a number base is still not known for sure. He dominated the study of mathematics for much of the first half of the century, making contribution, Gaussian function, Gaussian decimal places. It turned out that the maximum volume is held in short, squat barrels with a height just under 1.5 times the diameter—like the barrels at his wedding. In 1644, he was appointed professor of geometry at the University of Oxford and became a champion of arithmetic algebra. Desargues made several visits to Paris and, through Marin Mersenne, became friends with Descartes and Pascal. Fractals Self-similar curves or shapes of different sizes that form complex patterns that have the same general appearance at any magnification. One of the great challenges of modern number theory has been establishing whether there are more rational or irrational numbers. Defining impossibility Over the past two millennia, various thinkers have probed the balance between the improbable and the impossible. As with many mathematical problems, it is based on a hypothetical situation: Fibonacci's assumptions about how the rabbits behave are unrealistic. 311 If a disease affects 5 percent of the population (event A) and is diagnosed using a test with 90 percent accuracy (event B), you might assume that the probability (P) of having the disease if you test positive—P(A|B)—is 90 percent. These processes require new symbols for abbreviation. Nelson began a career of teaching and research at McMaster University after obtaining her doctorate there in 1970. Aristotle later described him as the inventor of the dialectical method (a method starting from two opposing viewpoints) of logical argument. Euclid lived in Alexandria, Egypt, in around 300 BCE, when the city was part of the culturally rich Greek-speaking Hellenistic world that flourished around the Mediterranean Sea. As an astronomer, he helped to construct a highly accurate calendar that was used until the 1900s. 1975 American logician Harvey Friedman develops the "reverse mathematics" program, which starts with theorems and works backward to axioms. Sexagesimal A number system used by the ancient Babylonians based on the number 60, and still used in a modified form for time angles, and geographic coordinates. With the support of her father, a railroad clerk, she graduated from Howard University, Washington DC, in 1935, and, after teaching briefly in New Orleans, continued her studies at the University of Michigan, gaining her doctorate in 1949. Marcus du Sautov British mathematician 28 Ebisu, the Japanese god of fishermen and one of the seven gods of fortune, uses a soroban to calculate his profits in The Red Snapper's Dream by Utagawa Toyohiro. Uncovering links 494 Langlands' ideas involve highly technical mathematics. Gauss's proof In 1799, at the age of 21, Carl Friedrich Gauss's proof In 1799, at the age of 21, C logic began in an unconventional way. These fields seem fundamentally different: while sine waves are continuous, integers are discrete. Alamy Stock Photo: Wenn Rights Ltd (tr). Here n(x) denotes the prime counting function (how many primes) and is unrelated to the number pi, and ln(x) is the natural logarithm of x. The field of complex numbers is unrelated to the number pi, and ln(x) is the natural logarithm of x. The field of complex numbers is unrelated to the number pi, and ln(x) is the natural logarithm of x. The field of complex numbers is unrelated to the number pi, and ln(x) is the natural logarithm of x. The field of complex numbers is unrelated to the number pi, and ln(x) is the natural logarithm of x. The field of complex numbers is unrelated to the number pi, and ln(x) is the natural logarithm of x. The field of complex numbers is unrelated to the number pi, and ln(x) is the natural logarithm of x. The field of complex numbers is unrelated to the number pi, and ln(x) is the natural logarithm of x. The field of complex numbers is unrelated to the number pi, and ln(x) is the natural logarithm of x. The field of complex numbers is unrelated to the number pi, and ln(x) is the natural logarithm of x. The field of complex numbers is unrelated to the number pi, and ln(x) is the natural logarithm of x. The field of complex numbers is unrelated to the number pi, and ln(x) is the natural logarithm of x. The field of complex numbers is unrelated to the number pi, and ln(x) is the natural logarithm of x. The field of complex numbers is unrelated to the number pi, and ln(x) is the natural logarithm of x. The field of complex numbers is unrelated to the number pi, and ln(x) is the natural logarithm of x. The field of complex numbers is unrelated to the number pi, and ln(x) is the natural logarithm of x. The field of complex numbers is unrelated to the number pi, and ln(x) is the natural logarithm of x. The field of complex numbers is unrelated to the numbers is unrelated to the number pi, and ln(x) is the natural the territory in which solutions to polynomial equations are found. Models devised by Henri Poincaré inspired many graphic works by M. Each square had a value, and that square's value was "on" or "off" depending on whether a counter was placed on the square. He quickly realized its usefulness and efficiency for both recording and calculating with whole numbers. In particular, he applied the idea of infinitesimals to what is known as the exponential function, ex. As Pascal showed, the number of ways various occurrences can combine to produce a given result. Al-Tusi was more interested in determining types of cubic equation than al-Khwarizmi and Khayyam had been. (tr). The symmetry group of the equilateral triangle is a member of a small group called D3. The first written record of the wheat on a chessboard problem was made in 1256 by Muslim historian Ibn Khallikan, though it is probably a retelling of an earlier version that arose in India in the 5th century. It was a much more sophisticated version of this scheme that lay at the heart of the tables of logarithms developed by Scottish landowner John Napier. In 1859, German mathematician Bernhard Riemann worked toward a proof in his paper On the Number of Primes Less Than a Given Magnitude. As well as his work on calculus, he made important contributions to number theory, following in the footsteps of Gauss. In 1632, de Roberval became professor of mathematics at the Collège Gervais, and two years later he won a competition for a highly prestigious post at the 237 Collège Royale. Numbers that act as a name or label, such as bus or telephone numbers, do not fit. Had he worked with others rather than alone, the error might have became professor of mathematics at the 237 Collège Royale. been identified earlier. Alamy Stock Photo: Science History Images (cr). Only a few individuals have single-digit EBS numbers. This produces a result of 2 + 1/4 + 1/8 (not 23/8, as Egyptian multiplication was based on doubling and halving fractions), which is the scaling factor that should be applied. 15 Since the Classical period, mathematics had been concerned with static situations, or how things are at any given moment. For example, his approach to the seemingly simple problem of planning a route through the city of Königsberg, crossing each of its seven bridges only once, uncovered much deeper concepts of topology, inspiring new areas of research. Euler calculated e to 18 decimal places, but noted that the decimals continued indefinitely. Several of the works attributed to Euclid have been lost, but at least five have survived to the 21st century. With the computer facilities available at IBM, he was 505 able to analyze the Julia set in great detail, noting that some values of the constant (c) gave "connected" sets, in which each of the points is joined to another, and others were disconnected. Continuing the "Big Ideas" series' trademark combination of authoritative, clear text and bold graphics, The Maths Book uses an innovative visual approach to make the subject accessible to everyone, whether you're an avid student or just curious about maths. Omar Khayyam Toward modern algebra Mathematicians who continued the guest for cubic equation solutions included Rafael Bombelli. Over the next 2,000 years, mathematicians would stake their reputations on attempts to resolve the issue. Persian powerhouse After the death of the Prophet Mohammed in 632, Islam rapidly became a major political as well as religious power in the Middle East and beyond, spreading from Arabia across Persia and into Asia as far as the Indian subcontinent. He proved this by showing that the probability of f being greater or less than W by a specified amount approached 0 (meaning impossible) as the game was repeated. He became a private mathematics tutor in London. The second part of the theorem states that if values are inserted into the anti-derivative F(x), the result—the definite integral of the function f(x)—makes it possible to calculate areas under the curve of the function f(x). In a barrel, for example, each laver is a shallow cylinder. Early 1700s The longitude problem is resolved by British clockmaker John Harrison and others -using springs rather than pendulums. In 1960-61, Gorenstein attended a nine-month program in group theory at the University of Chicago, which inspired him to propose a classification of finite simple groups. There are three dimensions in our familiar view of the world—length, width, and height—and they can largely be described mathematically by the geometry of Euclid. Mathematicians are continually finding new surprises in it. He represented movement between positions as a theoretical line, a "worldline," which could be plotted on a graph, with space and time as the axes. However, the elongated shape of the 220 second barrel means it has a smaller volume, containing less wine but for the same price as the first. The set 6, 8, 10 is not primitive as its components share the common divisor 2. For example, (x + 2)(x + 3) can be expanded to  $x^2 + 5x + 6$ , by multiplying each term in the second pair of parentheses. 205 In Spain, the decimal separator is a comma, as seen in the prices at this market stall in Catalonia. The PP was controversial among the ancient Greeks, since it did not seem as self-evident as Euclid's other postulates, nor was there an obvious way of verifying it. S. The symbol that is placed between the whole-number part and the fractional part of a number is called the decimal separator. Chinese rod system 113 The earliest ideas of negative quantities seem to have arisen in commercial accounting: the seller received money for what had been sold (a positive quantity), and the buyer spent the same amount, resulting in a deficit (a negative quantity). By using combinations of dots over bars they could 29 Mayan number symbols and glyphs. Japan still uses the soroban today: there are even contests in which young people demonstrate their ability to perform soroban calculations mentally, a skill known as anzan. After graduating with a masters degree in mathematics in 1976, Perlman worked for a government contractor that developed software. formula to surfaces that are joined to form a single surface. Pierre-Simon Laplace Estimating the odds One popular game in 17th-century France involved two players taking turns to throw four dice in a bid to obtain at least one "ace," or six. This book uses the historical journey of humans discovering math as a way to classify it and wrangle it into a linear progression, which is a valiant but difficult effort. One of the first to tackle the problem was Alan Turing, who went on to develop codecracking machines during World War II that were the forerunners of modern computers. Each has a probability of occurring—P(A) and P(B)— with P for each being a number between 0 and 1. The hunt was on for the smallest number that could be expressed as the sum of two positive cubes in three, four, or more different ways. 2003 Scientists at Plymouth University in the UK test Borel's theory with real monkeys and a computer keyboard. This may be why he is credited with discovering the link between mathematical ratios and harmony. British mathematician Roger Penrose investigated whether any polygons could only lead to nonperiodic tessellations. Different systems The decimal point has never become universally accepted. Of the many pioneering mathematicians produced by ancient Greece, Apollonius of Perga was one of the most brilliant. Mary Everest Boole British mathematicians An equation asserts that one quantity is equal to another, and provides a means of determining an unknown number. 1890 David Hilbert refines the concept of the ring. A logarithm to the base e (2.71828...) is called a natural logarithm and is indicated by the prefix ln or loge. The same year, Blaise Pascal calculated the area of any vertical slice of a cycloid. Getty Images: Jamie Cooper / SSPL (bc). Even light is not fast enough to escape the hole's immense gravitational pull. Alamy Stock Photo: Artokoloro Quint Lox Limited (tr); NMUIM (clb). Coefficient A number or expression, usually a constant, that is placed before another number (especially a variable) and multiplies it. Composed entirely of corners, the Weierstrass function has no smoothness anywhere, however much it is magnified. The 8th square had 128 grains, the 24th had more than 8 million, and the 32nd, the last square on the chessboard's first half, had over 2 billion. Jacobi's elliptic functions are complex, but a more simple form, the p-function, was introduced in 1862 by German mathematician Karl Weierstrass. He was evidently prepared to experiment with the method, but appears not to have been convinced. This used his third postulate. Greatly impressed by a copy of Diophantus's Arithmetica found in the Vatican library, Bombelli helped to translate it into Italian - work that led him to revise Algebra. In the 3rd century, Wang Fau stated that a circle with that God had made the Universe according to a mathematical plan. He has also composed extensively for contemporary choreographers and designed sound installations for architects. As hunter-gatherers turned to trade and farming, and societies became more sophisticated, arithmetical operations and a numeral system became essential tools in all kinds of transactions. His mother died shortly after his birth, and he was sent to live with his grandmother. D'Alembert's proof argued that if a polynomial P(x) with real coefficients has a complex root, x = a + ib, then it also has a complex root, x = a + ib, then it also has a complex root, x = a + ib, then it also has a complex root, x = a + ib, then it also has a complex root, x = a + ib, then it also has a complex root, x = a - ib. It is thought that he studied at the Academy in Athens, which had been founded by Plato. In 2015, Stanford mathematics professor Keith Devlin argued that the golden ratio is a "150-year scam." He blamed Zeising's work for the idea that the golden ratio has historically had a relationship to aesthetics. Alamy Stock Photo: INTERFOTO (bl). Two years later, she married William King, Earl of Lovelace, with whom she had three children, but she continued to study mathematics and follow the progress of Babbage, who called her "the Enchantress of Number." Lovelace wrote exhaustive notes on Babbage's Analytical Engine. During the 9th century, Arab he simplified these calculations to π = , and 62,832/20,000 as values for π, mathematician al-Khwarizmi used 31/7, attributing the first value to Greece and the other two to India. (Anything to the power of 0 is 1.) To multiply the numbers 16 and 32 in the lower row, their exponents (4 + 5) can be added together to produce 2 9 (= 512). The Mayan system employed symbols: a dot representing one and a bar representing five. 780 CE near what is now Khiva, Uzbekistan, Muhammad Ibn Musa al-Khwarizmi moved to Baghdad, where he became a scholar at the House of Wisdom. Euclid Collection of vorks The Elements is a collection of 13 books that range widely in subject matter. APOLLONIUS OF PERGA Little is known about the life of Apollonius. Plots of complex numbers— or Argand diagrams—make complicated polynomials

easier to solve. Key work 1687 Philosophiae Naturalis Principia Mathematica (Mathematical Principles of Natural Philosophy) See also: Syllogistic logic • The problem of maxima • Calculus • Emmy Noether and abstract algebra 285 IN CONTEXT KEY FIGURE Jacob Bernoulli (1655-1705) FIELD Probability BEFORE c. In the 1600s, Galileo recognized that sounds are 339 produced by vibrations: the higher the frequency of the vibrations, the higher the pitch of the sound we perceive. Pierre de Fermat's analysis of maxima and minima, which quickly followed Kepler's, opened the way for the development of calculus by Isaac Newton and Gottfried Leibniz later in the 17th century. Thereafter, the population grows more quickly. See also: Positional numbers • Irrational numbers • The Fibonacci sequence • Binary numbers 208 IN CONTEXT KEY FIGURE John Napier (1550-1617) FIELD Number systems BEFORE 14th century The Indian mathematician Madhava of Kerala constructs an accurate table of trigonometric sines to aid calculation of angles in right-angled triangles. Algorithm A defined sequence of mathematical or logical instructions, or rules, devised to solve a class of problems. Unconventionally, he posted his first 39page paper on the subject online in 2002, emailing a summary to 12 mathematical or logical instructions, or rules, devised to solve a class of problems. algorithm to rewrite an input (the data on the tape) into an output. In 2016, he founded the competition Ritangle for students of mathematics. In essence, the Turing machine (M) is an algorithm that tests another algorithm that tests another algorithm that tests another algorithm (A) to see if it is solvable. It can also be stated that all polynomials of degree n containing complex coefficients have n complex roots. They may have met in Egypt. Following a move to Göttingen in 1902, Minkowski became fascinated by the mathematics of physics, especially the interaction of light and matter. EUGENE CATALAN Born in Bruges, Belgium, in 1814, Eugène Catalan studied under French mathematician Joseph Liouville at the École Polytechnique in Paris. When taking a graph reading, these two numbers are now presented as a tuple—a strictly ordered sequence listed inside brackets. Had this book been much the same up until about page 280. This means that the population continues to grow at the same rate, leading to ever-larger totals. Because it was a base-60 system, two such symbols signified 61, while three such symbols indicated 3,661, that is, 60 × 60 (602) + 60 + 1.1995 American mathematician Ted Hill proves that Benford's law can be applied to statistical distributions. 250 BCE, the Greek scholar Archimedes developed an algorithm for determining the value of π based on constructing regular polygons that exactly fit within (inscribed), or enclosed (circumscribed), a circle. See also: Coordinates • Huygens's tautochrone curve • Graph theory • NonEuclidean geometries 190 IN CONTEXT KEY FIGURE Robert Recorde (c. She coined the word "bug" for a computer glitch after a moth flew into circuits on which she was working. Although no part of the tiling matches another part exactly, the pattern does repeat on a larger scale in a similar way to a fractal. In Classical Greece, there was no clear distinction between mathematics and philosophy; the two were considered interdependent. eastern Mediterranean. Briggs visited Napier in 1616 and again in 1617. JEAN-ROBERT ARGAND Little is known of Jean-Robert Argand's early life. The length of this hypotenuse is found because 32 + 42 = 52 (9 + 16 = 25). The value of numbers such as pi can be estimated using convergent series. Calculus is the mathematics of continuous change, and maxima and minima are the turning points, or limits in any change—the peak and trough of any graph. So when observations are plotted on a graph, they create a bellshaped curve with a peak created by the most likely result, or "norm," in the 433 middle. 1668 In Logarithmo-technia, German mathematician Nicholas Mercator first uses the term "natural logarithms." For thousands of years, most calculations were carried out by hand, using devices such as counting boards or the abacus. Alamy Stock Photo: Chris Pearsall (clb, cb, cb/blue). Medieval Italian mathematician Leonardo of Pisa (also known as Fibonacci) came across the Indian place-value number system while he was traveling in the Arab world. To appreciate the shape of the ball, it has to be viewed in 3-D space. 274 Binary choices are black and white; in any column there is only ever 1 or 0. If a game would normally end only when one player had won a certain number of rounds, for instance, de Méré wanted to know if the division of the stakes should reflect how many rounds each player had won. He then trained as a teacher, ultimately becoming a professor of mathematics at the Humboldt University of Berlin. But the mathematical groups they used included a combinatorics A branch of mathematics that studies the ways in which sets of numbers, shapes, or other mathematical objects can be combined. Some, however, see the mask—used as a template for plastic surgery—as an unethical, unfounded use of mathematics. 1920 Alexander Ostrowski proves the remaining assumptions in Gauss's proof of the FTA. Al-Khwarizmi approached more complex problems too, producing a geometrical method for solving quadratic equations that used the technique known as "completing the square". Euclid showed how to construct the golden ratio using a ruler and compass. When his father's business collapsed, the 16-year-old George took up a post as an assistant schoolmaster to support his family. Devlin argues that Zeising's ideas have led people to look back at historical art and architecture and retrospectively apply the golden ratio. It makes no sense to find the product of two piles by apples). The figure of 25 was slowly increased; Norwegian mathematician 509 Øystein Ore and American mathematician Joel Stemple together achieved 39 in 1970, and Frenchman Jean Mayer lifted the figure to 95 in 1976. Khayyam had also explored the properties of conic sections, and had deduced that a solution to the cubic equation could be found by giving the circle in the diagram a diameter of 4. 377 Hamilton's expertise in Newtonian mechanics enabled him to calculate the paths of heavenly bodies. To prove Proposition 1, Euclid drew a line with endpoints labeled A and B. It has two generators (elements that can be combined to give any other element of the group). In 1545, Italian scholar Gerolamo Cardano published his Ars Magna (The Great Art, or the Rules of Algebra) in which he explored the problem: "What pair of numbers have a sum of ten and product of 40?" He found that the problem led to . 63 Zeno of Elea belonged to the Eleatic school of philosophy that flourished in ancient Greece in the 5th century BCE. 387 Modern applications Matrices can store vast amounts of data compactly, making them essential across math physics, and computing. Others would be unstable initially but would settle into a limit cycle eventually. The path the imaginary body takes in each case is called the geodesic path. 502 In 1883, another German mathematician, Georg Cantor, built on work by British mathematician Henry Smith to demonstrate how to create a line that is nowhere continuous and has zero length. See also: Euclid's Elements • Trigonometry • Cubic equations ARYABHATA 476-550 CE A Hindu mathematician and astronomical to astronomical astro calculations. While this is easy enough for a single dice, with multiple dice, or 52 playing cards, the calculations become complicated. In 1975, he coined the term "fractal," and in 1980 he unveiled the Mandelbrot set, a structure that became synonymous with the new science of fractal geometry. Strength of mind rests in sobriety; for this keeps your reason unclouded by passion. Möbius died in Leipzig in 1868. AFTER 1978 Belgian mathematician Pierre Deligne receives the Fields Medal for his work on number theory, including the proof of a conjecture in the theory of modular forms that was first made by Ramanujan. Dutch artist M.C. Escher created a notable woodcut of ants endlessly patrolling the shape. Key works 1494 Summa de arithmetica, geometria, proportionalita (Summary of arithmetic, geometry, proportionality) 1509 Divina Proportionality) 1509 Divina Proportionality) 1509 Divina Proportionality (The Divine Proportionality) 1509 Divina Proportionality) 1509 Divina Proportionality) 1509 Divina Proportionality (The Divine Proportionality) 1509 Divina Proportionality) 1509 Divina Proportionality) 1509 Divina Proportionality (The Divine Proportionality) 1509 Divina Proportionality) 1509 Divina Proportionality) 1509 Divina Proportionality (The Divine Proportionality) 1509 Divina Proportionality) 1509 Divina Proportionality (The Divine Proportionality) 1509 Divina Proportionality) 1509 Divina Proportionality (The Divine Proportionality) 1509 Divina Proportionality) 1509 Divina Proportionality (The Divine Proportionality) 1509 Divina Proportionality) 1509 Divina Proportionality (The Divine Proportionality) 1509 Divina Proportionality) 1509 Divina Proportionality (The Divine Proportionality) 1509 Divina Proportionality) 1509 Divina Proportionality (The Divine Proportionality) (The Divin c. In 1921, he was awarded the Nobel Prize for his contribution to physics, and he continued to develop the understanding of quantum mechanics in the years that followed. Getty Images: Boston Globe / Rubik's Brand Ltd www.rubiks.com (bc). Hermann Minkowski HERMANN MINKOWSKI Born in Aleksotas (now in Lithuania) in 1864, Minkowski moved with his family to Königsberg in Prussia in 1872. A vital part of any axiomatic system is to have enough axioms, and postulates in the case of Euclid, to derive every true proposition, but to avoid superfluous axioms that can be derived from others. Mathematicians later used Bessel functions to make breakthroughs in various fields. Descartes, however, was primarily interested in finding out how coordinates could help him use algebra to better understand lines, especially curved lines. The height of the "rectangle" is approximately equal to the radius r of the circle, and the width is half of the circumference (half of 2nr, which is nr). For example the first 10 tosses could be seven heads and three tails. Alamy Stock Photo: Flhc 80 (clb); RayArt Graphics (tr). The earliest known evidence for such arrays comes from the ancient Mayan civilization of Central America, c. Mathematics could be seen as the attempt to find the simplest explanations for the biggest ideas. The ratio can be found by dividing a straight line into two parts, so that the ratio of the longer length (a) to the smaller length (b) is the same as the ratio of the whole line (a + b) divided by the longer length (a). In 1970, Russian mathematician Yuri Matiyasevich settled Hilbert's query, which he and three others had studied for years, concluding that no general algorithm to solve a Diophantine equation exists. 1848 Russian mathematician Pafnuty Chebyshev presents the first significant study of the prime counting function  $\pi(n)$ . Alamy Stock Photo: The History Collection (cra). Guy Murchie American writer Plant life The Fibonacci sequence can also be seen in the arrangement of leaves and seeds in some plants. Working to the very end, he died in 1783 in St. Petersburg. His verse treatise Aryabhatiya contains sections on algebra and trigonometry, including an approximation for pi (π) of 3.1416, accurate to four decimal places. The parallel postulates from which Euclid deduced his theorems of geometry in his Elements. 284 CE) FIELD Algebra BEFORE c. However, this is one of two correct solutions to the problem; -13 is the other. French mathematician Rafael Bombelli in 1572. The long and difficult Book X deals with incommensurables. Chaos theory has practical relevance to areas such as population dynamics, chemical engineering, and financial markets, and helps in the development of artificial intelligence. For him, it was not simply about solving the technical problems of transmitting information efficiently. Newton studied at Trinity College, Cambridge, where he showed a fascination for science and philosophy. Two centuries later, Italian polymath Leonardo da Vinci (1452-1519) proposed making a rectangle whose length was the same as a circle's circumference and whose height was half its radius to determine the area of the circle. Among them are Henri Cartan (standing far left) and André Weil (standing fourth from left). This set cannot be counted, making this infinity larger than countable ones, so it is said to have a cardinality of \$1. Borel noted that, while it cannot be mathematicians should consider it impossible. Others, however, doubt these patterns represent actual matrices. He devised axioms for natural numbers (Peano axioms), developed natural logic and set theory 548 notation, and contributed to the modern method of mathematical induction, used as a proof technique. SuperStock: fototeca gilardi / Marka (crb). From 1958 when Jackson became NASA's first female black engineer—to 1963, she worked on Project Mercury, the program that put the first Americans into space. For example, in logic, two statements may be connected by AND, as in "this animal is covered in hair" AND "this animal is covered in hair" AND "this animal feeds its young with milk," or by OR, as in "this animal can swim" OR "this animal has feathers." The statement "A AND B" is true when A and B are both individually true, whereas the statement "A OR B" is true if one or both of A and B is true. He is our master in everything. 1905 Albert Einstein states his theory of special relativity. The latter comprised a real part and an imaginary part, and were to be of great significance in the development of mechanical and electrical engineering. 14 Another major field in mathematics is algebra, which is the study of structure, the way that mathematics is organized, and therefore has some relevance in every other field. She achieved all this despite being barred from a university education in her native Russia because of her gender. Series usually follow a mathematical rule, and even if the series is infinite, it may add up to a finite number. Any point on that plane, which could stretch to infinity, can be described exactly using a pair of numbers. The king was so delighted with the game of chess that he offered to grant Sissa any reward that he wanted. Erdős has an amazing ability to match problems with people. Two centuries later, Aristotle suggested that sound traveled through the air in waves, although the incorrectly thought that higher-pitched ones. The smallest pair are 220 and 284. She later faced similar discrimination in Göttingen, where her colleagues had 459 to fight to have her officially included in the faculty. Desargues' theorem states that lines extending from the corresponding sides of each triangle will always meet on a line known as the axis of perspectivity. The value of this ratio is a mathematical constant denoted by the Greek letter  $\phi$  ("phi"). The theory provided insights into complex systems in daily life such as economics, computing, and the military. Since Abel died at 26, just months after making his major discoveries, many of these applications were developed by Jacobi. Dutch mathematician Christiaan Huygens—who coined the name catenary from the Latin catena ("chain") in 1690—showed that, unlike a parabola, a catenary curve could not be given by a polynomial equation. Hobson British mathematician See also: The problem of maxima • Calculus • The law of large numbers • Euler's number • Fourier analysis 350 IN CONTEXT KEY FIGURES Charles Babbage (1791-1871), Ada Lovelace (1815-52) FIELD Computer science BEFORE 1617 Scottish mathematician John Napier invents a manual calculating device. This ratio starts at 0 and varies with the size of the angle, repeating its pattern after 360 degrees. Kepler wondered whether this worked equally well for all shapes of barrel and, concerned that he may have been cheated, decided to analyze the issue of volumes. the astroid Ceres), cartography, the study of electromagnetism, and the design of optical instruments. Stevin's zero inside a circle later evolved into a dot, now called the decimal point. 256 In the late 1700s, Pierre-Simon Laplace extended the scope of probability theory to science, and introduced his mathematical tools for predicting the probability of many incidents, including natural phenomena. This has played a key part in understanding the nature of the Universe. When we count from 10 onward, we put a 1 in the "tens" column and so on, adding columns for hundreds, thousands, and beyond. In 18th-century Switzerland, Leonhard Euler also became interested in them, and devised a form that he named Latin squares. If the system relies heavily on too few primes, then it is possible for attackers to figure out 518 Lava lamps can be hooked up to computers in order to generate a selection of random numbers based on their movements. Recorde's sign was noticeably longer than the modern form. References by later scholars suggest that he was responsible for the first systematic compilation of geometrical knowledge. Lovelace became renowned for her talents in mathematics and languages. As Pascal's triangle can continue forever, this works with any powers. Adrien-Marie Legendre New tools Just as the mathematics of a circle could be used to model and predict natural phenomena that varied and repeated in a rhythmic (or periodic) way, such as the up-and-down motion of a simple sound wave, the mathematics of the ellipse can 359 be used to do the same for phenomena that follow more complex periodic patterns, such as electromagnetic fields or the orbital motion of a simple sound wave, the mathematics of the ellipse can 359 be used to do the same for phenomena that follow more complex periodic patterns, such as electromagnetic fields or the orbital motion of a simple sound wave, the mathematics of the ellipse can 359 be used to do the same for phenomena that follow more complex periodic patterns, such as the up-and-down motion of a simple sound wave, the mathematics of the ellipse can 359 be used to do the same for phenomena that follow more complex periodic patterns, such as the up-and-down motion of a simple sound wave, the mathematics of the ellipse can 359 be used to do the same for phenomena that follow more complex periodic patterns, such as the up-and-down motion of a simple sound wave, the mathematics of the ellipse can 359 be used to do the same for phenomena that follow more complex periodic patterns, such as the up-and-down motion of a simple sound wave, the mathematics of the ellipse can 359 be used to do the same for phenomena that follow more complex periodic patterns, such as the up-and-down motion of a simple sound wave, the mathematics of the ellipse can 359 be used to do the same for phenomena that follow more complex periodic patterns, such as the up-and-down motion of a simple sound wave, the mathematics of the ellipse can 359 be used to do the same for phenomena that follow more complex periodic patterns, such as the up-and-down motion of a simple sound wave, the up aspects of modern life, from comprehending economic behavior to planning new transportation links and improving public health services. This character could be anything, but the simplest version used 1s and 0s. These may have an infinite number of roots, but in some cases, there are a finite number. To arrive at this figure, he found the mean, which is the sum of all the values divided by how many values there are. From just two such propositions an argument can be constructed and a conclusion deduced. Expert soroban users can usually do such calculator. 1718 Abraham de Moivre publishes The Doctrine of Chances, the first textbook on probability. Practically, we have relied on math to advance our species, with early arithmetic and geometry providing the foundations. Nick Katz and the wider mathematical community were now convinced there were no mistakes, and Wiles emerged for a second time as the conqueror of Fermat's last theorem—this time on solid ground. The length of the recurring cycle (two numbers in the ) can be predicted as it will be a factor of the fraction is 11, the number of digits in the cycle is a 207 factor of 10). The steeper the side of a pyramid, the fewer the sekeds. The field of calculus was further developed later in the 1800s. His solution was to base the division of the pot on the ratio between the size of the lead and the length of the game, but this also gave unsatisfactory results for games with many rounds. After publishing their through gravitational results, they were told that their proof settled a lensing. Simultaneous equations A set of several equations that include the same unknown quantities, such as x, y, and z. As a boy, he showed an aptitude for math and began his studies at the University of Königsberg aged 15. The natural consequence was an ever-widening triangle. 1940 Hellmuth Kneser gives the first constructive variant of the Argand FTA proof that allows for the roots to be found. To find how far the guess of 7 is from the required quantity, 19 is divided by 8 (the "false" answer). He has written around 1,000 books on a range of subjects, including internationally acclaimed titles such as The Oceans Atlas, Do You Think You're Clever? Pure mathematics Topics in mathematics that are studied for their own sake rather than for any practical application. Its genesis was largely due to French mathematician Évariste Galois, who developed it in order to understand why only some polynomial equations could be solved algebraically. The golden spiral can be drawn by splitting a golden rectangle (a rectangle with side lengths in the golden ratio) into successively smaller squares and golden rectangles, and inscribing quarter circles inside the squares. Appointed keeper of the Jardin du Roi, the royal botanical gardens in Paris, in 1739, Buffon enriched its collections and doubled its size. Langlands' marriage of harmonic analysis and number theory could lead to a wealth of new tools, just as the 19th-century unification of electricity and magnetism into electromagnetism provided a new understanding of the physical world. He wrote up his results in Miscellanea Analytica. Gödel's theorem also implied that some as-yet unproven theorems in mathematics, such as the Goldbach conjecture, may never be proved. The UK uses a point for the decimal separator and a comma as a delimiter. I realized that it was necessary... to start again right from the foundations if I wanted to establish anything in the sciences that was stable and likely to last. Transcendental number Any irrational number Any irrati for any further solutions of xm - yn = 1. AFTER 1774 In his Memoir on the Probability of the Causes of Events, Pierre-Simon Laplace introduces the principle of inverse probability. This ascetic brotherhood required its members to live for intellectual pursuits, while following strict rules of diet and clothing. Sissa's concept of wheat on a chessboard is an early example of how quickly numbers can increase with exponential growth. (Numbers from 1 million onward are approximate.) The wheat on this chessboard would total over 18 million trillion grains. 33 The Berlin papyrus was copied and published by German Egyptologist Hans SchackSchackenburg in 1900. Calculus can show how the position of a moving vehicle changes over time, how the brightness of a light source dims as it moves further away, or how the position of a person's eyes alters as they follow a moving object. Centuries before Pythagoras, the Egyptians had used a triangle with sides of 3, 4, and 5 units as a building tool to ensure corners were square. 1240-50. Emmy Noether Significant works Noether began her work on abstract algebra shortly before World War I with her exploration of invariant theory, which explained how some algebraic expressions stay the same while other quantities change. Eric Temple Bell Scottish mathematician Galois groups and fields Groups are just one kind of abstract algebraic structure among many. For example, in French, 80 is expressed as quatre-vingt (4 × 20); Welsh and Irish also express some numbers • Euclid's Elements • Eratosthenes' sieve • Zu Chongzhi • Calculus • Euler's number • Buffon's needle experiment 95 IN CONTEXT KEY FIGURE Eratosthenes (c. In 1579, French mathematician François Viète used 393 regular polygons each with 216 sides to calculate π to 10 decimal places. In spherical geometry, almost all the axioms look different from the postulates set out in Euclid's Elements. He used only powers of 2 and 3, solving the equations 3n - 2m = 1 and 2m - 3n = 1. This blend is exactly that exhibited by the energy levels of the nuclei of heavy atoms, according to quantum theory. Such sets of whole-number solutions to the equation as Pythagorean triples. In the 1500s, the solution of cubic and quartic equations occupied Italian mathematicians such as Gerolamo Cardano, while Marin Mersenne devised a method of finding prime numbers, and Rafael Bombelli laid down rules for using imaginary numbers. 1500 BCE were often encrypted to protect recipes for pottery glazes and other such commercially valuable information. A loxodrome starts at the North or South Pole, and spirals around the globe, crossing each meridian at the same angle. In the 1600s, such problems attracted the attention of French mathematicians Blaise Pascal and Pierre de Fermat. AFTER 16th century Mathematicians in Italy create jealously guarded methods to solve cubic equations in the fastest time. two types of mind... the mathematical, and... the intuitive. 565 Logic The study of reasoning, that is, how conclusions can be deduced correctly from given starting information (premises) by following valid rules. 96 In addition to calculating Earth's circumference and the distances from Earth to the Moon and Sun, the Greek polymath Eratosthenes devised a method for finding prime numbers. During this period, he also worked on the Anglo-French Survey, using trigonometry to calculate the distance between the Paris Observatory and London's Royal Greenwich Observatory. In 1844, Eugène Catalan claimed that there is only one solution to the equation xm - yn = 1, where x, y, m, and n are the distance between the Paris Observatory. In 1844, Eugène Catalan claimed that there is only one solution to the equation xm - yn = 1, where x, y, m, and n are the distance between the Paris Observatory. natural numbers (positive integers) and m and n are greater than 1. Frege's work was followed by Charles Sanders Peirce and another German logician, Ernst Schröder, who introduced quantification into Boole's algebra and produced substantial works using Boole's algebra and boole's algebra and produced substantial works using Boole's algebra and prod numbers,  $31/3 \times 31/3 \times 31/3 = 1000/27 = 295$  37.037... the largest for three numbers. In mathematics, the art of asking questions is more valuable than solving problems. The group, which acts in secrecy and whose members must resign at age 50, still exists, although Bourbaki now publishes infrequently. Pi is not merely the ubiquitous factor in high school geometry problems; it is stitched across the whole tapestry of mathematics. Fermat knew that this equation had an infinity of integer solutions for x, y, and z, such as 3, 4, and 5 (9 + 16 = 25) and 5, 12, and 13 (25 + 144 = 169), known as "Pythagorean triples." He then wondered if other triples could be found to the power of 3, 4, or any integer beyond 2. To meet the era's practical needs, mathematicians devised tables of relevant calculations, and John Napier developed a means of calculating with logarithms 177 in the 1600s. Arches in the shape of an inverted catenary are often used in architecture and construction due to their strength. This way of seeing numbers as points on a line finally led to the acceptance of negative numbers, and by the end of the 1800s, they had been formally defined within mathematics, separate from notions of quantities. Arithmetic of symmetries A group is an abstract object—it consists of a set of elements and an operation that combines them, subject to some axioms. Many of the symbols and abbreviations are still in use today. Similarly, if you multiply all the numbers by the same quantity, you still have a magic square. Alan Turing test In 1950, Turing developed a test of a machine's ability to exhibit intelligent behavior equivalent to, or indistinguishable from, that of a human. Such numbers, divisible only by 1 and themselves, had intrigued mathematicians for centuries. Space scientists constantly use n in their calculations. In diagrams, vectors are often represented by bold arrows. New developments in trigonometry gathered pace in the 1600s. His weak constitution could not resist the cold winter. British mathematician e anticipated the computer age by more than a century with two ideas for mechanical calculators and "thinking row is always one less than the sum of the numbers in the given row. 200-c. Where the circles met, he called that point C, and he could draw two more lines AC and BC, calling on his first postulate. Each of these six transformations fits the triangle onto itself—it looks exactly the same, except that the vertices are permuted (rearranged). As a teenager, his father took him to Marin Mersenne's mathematical salon in Paris. By the 3rd and 4th centuries, a place value system had long been in use, and by the 7th century—the time of Brahmagupta—the use of a circular symbol as a placeholder was already well established there. It follows that AB = BC = CA, meaning that he had drawn an equilateral triangle on AB. This allows bacteria to spread very quickly. 1484 In France, Nicolas Chuquet writes an article about calculation using geometric series. Three volumes were published in 1572, the year he died; the last two incomplete volumes were published in 1572, the year he died; the last two incomplete volumes were published in 1572, the year he died; the last two incomplete volumes were published in 1572, the year he died; the last two incomplete volumes were published in 1572, the year he died; the last two incomplete volumes were published in 1572, the year he died; the last two incomplete volumes were published in 1572, the year he died; the last two incomplete volumes were published in 1572, the year he died; the last two incomplete volumes were published in 1572, the year he died; the last two incomplete volumes were published in 1572, the year he died; the last two incomplete volumes were published in 1572, the year he died; the last two incomplete volumes were published in 1572, the year he died; the last two incomplete volumes were published in 1572, the year he died; the last two incomplete volumes were published in 1572, the year he died; the last two incomplete volumes were published in 1572, the year he died; the last two incomplete volumes were published in 1572, the year he died; the last two incomplete volumes were published in 1572, the year he died; the last two incomplete volumes were published in 1572. probably used the sequence e = 1 + 1 + 1/2 + 1/6 + 1/24 + 1/120 + 1/720, going up to 20 terms. Fractals have applications in medical research, such as understanding the behavior of viruses and the development of tumors. In 1634, Gilles Personne de Roberval used this method to work out that the area beneath a cycloid (the arc traced by the rim of a rolling wheel) is three times the area of the circle used to generate the cycloid. AFTER 1928 British physicist Frank Benford wrote is impossible. So 7 (the original "false" value) is multiplied by 2 + 1/4 + 1/8 (the scaling factor) to give the quantity 16 + 1/2 + 1/8 (or 165/8). In 1938, American physicist Frank Benford wrote a paper on this phenomenon; mathematicians later referred to it as Benford's law. In Italy, however, it is known as Tartaglia's triangle after mathematician Niccolò Tartaglia, who wrote about it in the 1400s. Dodecahedron A 3-D polyhedron made up of 12 pentagonal (5-sided) faces. In his works, he pioneered the move away from the Arabic algebra of procedures toward what is known as symbolic algebra. One of these tables poses the question: "If a pyramid is 250 cubits high and the side of its base is 360 cubits long, what is its seked?" The word seked means slope, so the problem is purely trigonometrical. Bridgeman Images: Private Collection / Archives Charmet (cla). Gottfried Leibniz makes the same discovery in Germany three years later. I have had my results for a long time, but I do not yet know how to arrive at them. He could finally afford to marry when he was awarded a full professorship by Göttingen in 1862. AFTER 1740 Leonhard Euler applies the ideas of calculus to synthesize calculus, complex algebra, and trigonometry. While other countries had kept academics at home, French mathematicians had joined their countrymen in the trenches and a generation of teachers had been killed. One formulation of quantum physics, called matrix mechanics, makes extensive use of matrix algebra, and particle physicists and cosmologists use transformation matrices and groups and groups at home. theory to study the symmetries of the Universe. In binary, the number 111 means 1 × 22 + 1 × 21 + 1 × 20, that is 4 + 2 + 1, or 7 in our decimal number system. The highest power of the equation determines how many solutions it has: in this case, there are three solutions. In the late 1500s, Italian mathematician Rafael Bombelli broke new ground when he laid down the rules for using imaginary and complex numbers in his book Algebra. More precision In the 5th century, astronomer and mathematician Zu Chongzhi, who was renowned for his meticulous calculations, set about obtaining an even more accurate value for n. When a signaler pressed a letter on the keyboard, a corresponding letter lit up on the lampboard. Seven years later, her doctoral thesis was examined by mathematician John E. Omar Khayyam A passion for geometric forms is evident in Islamic architecture, seen here in the tile patterns, curved arches, and domes of the Masjid-i Kabud, the "Blue Mosque," in Tabriz, Iran. Lovelace died in 1852 of uterine cancer; in line with her wishes, she was buried next to her father. The base-60 system had obvious drawbacks. These symbols slowly replaced the letters "p" for plus and "m" for minus as they were taken up by scholars, first in Italy, then in England. Between every algebraic number of transcendentals. We appear to inhabit a 3-D world, but to make any sense of its shape we need to take ourselves outside this, into four dimensions. There was no practical value in identifying such numbers, but their quirkiness and the beauty of their patterns fascinated Pythagoras and his brotherhood. Galois groups of quartic equations and simpler polynomials are solvable, but those of higher degree polynomials are not. 515 The Enigma machine was used in German espionage between 1923 and 1945. How the planets move By 1667, Newton had already developed early versions of his three laws of motion and knew about the force needed to enable a body to move in a circular path. 553 See also: Calculus • Newton's laws of motion • Non-Euclidean geometries ALEXANDER GROTHENDIECK 1928-2014 Considered by many to be the greatest pure mathematician of the second half of the 20th century, Grothendieck was unorthodox in every respect. AFTER 1942-46 The Manhattan Project, a US-led body for developing nuclear weapons, makes extensive use of Monte Carlo methods (computational processes that model risk by generating random variables). He found there was more than others. Gottfried Leibniz CARL GAUSS Born in Brunswick, Germany, in 1777, Carl Gauss showed his mathematical talents early: aged only three, he corrected an error in his father's payroll calculations, and by the age of five he was taking care of his father's accounts. In the latter part of the 18th century, France became the European center of mathematical enquiry, with Joseph-Louis Lagrange in particular emerging as a significant figure. A line running parallel to y = 2x would pass through the y axis at a point other than the origin, such as at (0,2). ÉMILE BOREL Born in 1871 in Saint-Affrique, France, Émile Borel was a mathematics prodigy and graduated top of his class from the École Normale Supérieure in 1893. In February 1650, Descartes caught pneumonia and died. The Riemann zeta function, for example, is a function of complex numbers that provides information about primes. Binary code Turing did not initially envisage that his machine would use only binary data. Matrices with different dimensions cannot be added. Although numbers such as since ancient Greek times and from even earlier Babylonian clay tablets, in 825 CE, al-Khwarizm was the first to make the distinction between rational numbers, which have an indefinite string of decimals with no recurring pattern. Beautifully complex, the Mandelbrot set displays self-similarity at all scales: magnification reveals smaller replicas of the Mandelbrot set itself. 1733 Abraham de Moivre discovers "normal distribution": the way that values for most data cluster at a central point and taper off at the extremes. Since A5 has a finite number of elements, it is a finite group. Jacob Bernoulli The false probability 287 A coin toss is an example of the law of large numbers. The shape is used in the universal symbol for recycling and also 400 suggested in the mathematical symbol for infinity ( $\infty$ ), echoing the eternity image in the ancient Roman mosaic. He identified certain basic facts and progressed from two throws of a dice was 1/36, that is, 1/6 as likely as getting an ace with one die in one roll. Key works Mensuram orae ad terram (On the Measurement of the Earth) Geography) See also: Mersenne primes • The Riemann hypothesis • The prime number theorem • Finite simple groups 99 IN CONTEXT KEY FIGURE Apollonius of Perga (c. To make these odds the same, he argued that a pair of dice should be rolled six times for each roll of the single die. The river flowed around an island and then forked. For example, multiplying each term in the first parentheses by each term in the first parentheses results in (a + b)2 = a2 + 2ab - a2 + a2b - ab2. See also: The Rhind papyrus • Euclid's Elements • Calculating pi HERO OF ALEXANDRIA C. Getty Images: f8 Imaging / Hulton Archive (crb). C. The law was named by French mathematician Siméon Poisson in 1835, but its origin is credited to Swiss mathematician Jacob Bernoulli. Alfred North Whitehead British mathematician The importance of 10 Simon Stevin, a Flemish engineer and mathematician in the late 16th and early 17th century, used many calculations in his work. Pythagoras was reputedly an excellent lyre player. He proposed his configuration, known as the "von Neumann architecture," in 1945, and today, a similar process is used in almost every computing device. Arthur Cayley JAMES JOSEPH SYLVESTER Born in 1814, James Joseph Sylvester began his studies at University College London, but left when he was accused by another student of wielding a knife. As the 18th century progressed, calculus proved increasingly useful as a tool for describing and understanding the physical world. The Babylonian tablet YBC 7289 (c. 1891 Édouard Lucas coins the name Fibonacci sequence in Théorie des Nombres (Number Theory). Tangent (1) A line which grazes the outside of a curve, just touching it at one point. Probability theory While ancient and medieval law graded probability in the assessment of judicial evidence, there was no theory on which to base it. He wrote up his findings as a short paper, then included it in the 1738 edition of his Doctrine of Chances. This is shown in the graph below. In 1967, the young Canadian-American mathematician Robert Langlands suggested a set of profound links between two major and seemingly unconnected areas of mathematics—number theory and harmonic analysis. Mapping Establishing a relationship between members of one mathematical set and another. Read it carefully and communicate it to such others as are worthy of it. Riemann's new framework enabled mathematicians to explore shapes in four dimensions or higher, including seemingly "impossible" shapes. A and B can be combined to make the statement "my dog is hairy OR my dog is brown," which is also true, and also has a value of 1. The genesis of such tools, the elliptic functions, began in England with 17thcentury mathematicians John Wallis and Isaac Newton. Pebbles were placed on these lines to count out values. Over the decades, sets of such numbers acquired their own universal symbols that could be used in formulae. M. There he came across the Hindu-Arabic symbols used to represent the numbers 1 to 9. Proposition 16 In any triangle, if one of the sides is extended, the angle between the triangle and the extended side is greater than any of the angle and the extended side is greater than any of the angle between the triangle. It is expressed as the symbol i. One suggests that the idea dawned on him as he watched a fly moving over the ceiling of his bedroom. See also: The logic of mathematics • The Turing machine GRACE HOPPER 1906-92 Born Grace Murray in New York City, Hopper was a pioneering computer programmer. 560 Coordinates Combinations of numbers that describe the position of a point, line, or shape on a graph or a geographical position on a map. Because so much of Clarke's work was secret, the full extent of her accomplishments is still unknown. A Roman mosaic dating from c. Where those problems involve areas or paths of curves such as the flight of a ball or a rocket. Taught initially by his father, a Protestant minister who had some mathematical training and was also a friend of the Bernoulli family, Euler developed a passion for mathematics. 200 BCE Eratosthenes devises a method for calculating prime numbers. The Romans upgraded the Greek counting table into a device that greatly simplified calculations. 67 68 The perfect symmetry of the five Platonic solids was probably known to scholars long before the Greek philosopher Plato popularized these functions) in 1824. For the Königsberg graph, 303 A has degree 5 and B, C, and D each have degree 3. Her actions helped to establish a Royal Commission on health in the army in 1856. However, in Renaissance Italy, scholar Gerolamo Cardano produced in-depth analyses of outcomes involving dice. 80 To construct an equilateral triangle, for Proposition 1, Euclid drew a line and centered a circle on its endpoints, here A and B. Associative law This states that if you add, for example, 1 + 2 + 3, the numbers can be added in any order. 195 Over the centuries, scholars have needed to extend the concept of the number in order to solve different problems. DANIEL GORENSTEIN Born in Boston, Massachusetts, in 1923, Daniel Gorenstein had taught himself calculus by the age of 12 and later attended Harvard University. Suddenly Wiles's approach was brought into question. AFTER 1920 Karl Pearson, a British statistician, expresses regret about describing the Gaussian curve as the "normal curve" because it gives the impression that all other probability distributions were "abnormal." 1922 In the US, the New York Stock Exchange introduces the use of normal distribution to model the risks of investments. Alamy Stock Photo: Jessica Moore (tr). Other Renaissance artists—such as Raphael and Michelangelo—also used the ratio. See also: Pythagoras • Diophantine equations • Catalan's conjecture • 23 problems for the 20th century • Finite simple groups 530 IN CONTEXT KEY FIGURE Grigori Perelman (1966-) FIELDS Geometry, topology BEFORE 1904 Henri Poincaré states his conjecture on the equivalence of shapes in 4-D space. 398 A Möbius strip can be made from a simple length of paper. For each set of data, a chi-squared value 438 can be found from the sum of all the differences between observed and expected values. French mathematician and philosopher Nicole Oresme studied the velocity of an accelerating object against time, and he realized that the area under a graph depicting this relationship was equivalent to the distance traveled by the object. Women were welcome in the brotherhood, and formed a significant part of its 600 members. It shows that the ancient Egyptians used quadratic equations. The three key operations of sets, as well as basic mathematical functions. Around 1628, Frenchman Marin Mersenne challenged his fellow mathematicians, including de Roberval, René Descartes, and Pierre de Fermat, to find both the area under the arch of a cycloid and a tangent to a point on the curve. Fourier's career was disrupted by two arrests—one for criticizing the French Revolution, the other for supporting it—but in 1798, he accompanied Napoleon's forces into Egypt as a diplomat. Mathematical education was now taken more seriously. As quarter-master general from 1604, Stevin was responsible for several innovative military and engineering ideas that were adopted across Europe. The first prime number—and also the only even prime—is 2. See also: Calculus JEAN LE ROND D'ALEMBERT 1717-83 The illegitimate son of a celebrated Paris hostess, d'Alembert was brought up by a glazier's wife. He began to study mathematics seriously, starting by reading a book on calculus. Welsh mathematics seriously, starting by reading a book on calculus. volume of a sphere has been understood for millennia. The equation x2 + 1 can equal zero when x is i or -i. Famously, he measured the size of seeds produced by sweet pea plants grown from seven sets of seeds. 780-c. Where they meet is the origin (O). In the late 18th and early 19th century, Pierre-Simon Laplace applied probability theory to practical and scientific problems, setting out his methods in his Théorie Analytique des Probabilités (Analytic Theory of Probabilities) in 1812. He also produced the least-squares method for estimating a quantity based on consideration of measurement errors, and gave his name to three forms of elliptic integrals—the Legendre transform, transformation, and polynomials. He calculated Earth's circumference as a distance close to the current accepted figure. Pythagoras's reputation for ruthlessness is also highlighted in a story about a member of the brotherhood who was executed for publicly disclosing that the Pythagoreans had discovered a new regular polyhedron. One application of networks is to address the so-called "traveling salesperson problem." This involves finding the shortest route for a salesperson to travel from their home to a series of cities and back again. In developing a nonlinear model of the atmosphere, Lorenz stumbled across the area of chaos theory that would later be dubbed the butterfly effect. Both Fibonacci's sequence and the golden ratio appear to exist widely in nature. Boolean logic is also at the heart of how internet search engines work. In 1963, American mathematician John G. They claimed that every elliptic curve (an algebraic structure) could be associated with a unique modular form, one of a class of highly symmetrical structures belonging to number theory. Without the economy of modern notation, Khayyam expressed his equations in words, describing x3 as "cubes", x2 as "squares," x as "lengths," and numbers as "amounts." For example, he described x3 + 200x = 20x2 + 2,000 as a problem of finding a cube that "with two hundred times its side" is equal to "twenty squares of its side and two thousand." For a simpler equation, such as x3 + 36x = 144, Khayyam's method was to draw a geometric diagram. It was at this time, too, that he is thought to have figured out the relationship between geometry and algebra that is the basis of the coordinate system. But it also highlights where mathematics could do better things like the glaring omission of women from the history of mathematics, he used practical approaches to establish the ratio of the volumes of a cylinder, sphere, and cone with the same maximum radius and height to be 3:2:1. For example, the reciprocal of 3 is 1/3. The study of trigonometry purely as an aid to astronomy persisted well into the 1500s, when new developments in Europe began to gain momentum. The first perfect number is 6, as its divisors 1, 2, and 3 add up to 6. This led mathematicians to analyze the properties of right-angled 104 triangles: all right-angled triangles contain two shorter sides (which may or may not be of equal length) and a diagonal, or hypotenuse, which is longer than either of the others; all triangles contain three angles; and right-angled triangles have one angle of 90°. Since ancient times, mathematicians had discussed using "indivisibles"— elements about it become meaningless; the only meaningful statements about it are imprecise. Dutch physicist and mathematician Christiaan Huygens wrote a treatise translated as "On reasoning in games of chance," which was the first book on probability theory. H. The idea of telling a machine what it should do-programming-came from a French weaver, Joseph Marie Jacquard. Shannon demonstrates Theseus, his electromechanical "mouse," which used a "brain" of telephone relays to find its way around a maze. Descartes also made a contribution with his use of x, y, and z for the unknowns in equations, and a, b, and c for known figures. DIOPHANTUS Little is known about the life of the Greek mathematician and philosopher Diophantus, but he was probably born in Alexandria, Egypt, in C. Kepler looked at ways of calculating the areas and volumes of curved shapes. AFTER 1690 Swiss mathematician Jacob Bernoulli draws on Huygens's imperfect solution to the tautochrone problem to solve the brachistochrone problem—finding a curve of curved shapes. the fastest descent. This explores triangles and other shapes on the surface of a sphere rather than on a plane. During the Golden Age of Islam (8th-14th century), mathematicians attempted to prove the PP. To generate his logarithms, Napier imagined two particles traveling along two parallel lines. In 1822, Fourier was made the secretary 342 IN CONTEXT KEY FIGURE Pierre-Simon Laplace (1749-1827) FIELD Mathematical philosophy BEFORE 1665 Calculus is developed by Isaac Newton to analyze and describe the motion of falling bodies and other complex mechanical systems. The search continues Prime numbers attracted the attention of mathematicians from the 1600s onward, when figures such as Pierre de Fermat, Marin Mersenne, Leonhard Euler, and Carl Friedrich Gauss probed further into their properties. His motto, "Another roof, another proof," referred to his habit of staying at the homes of fellow mathematicians in order to "collaborate" for a while. His analysis of the various forms of arguments marked the beginning of logic as a subject for study in its own right. One problem facing Babbage was what to do with numbers carried over into the next column when adding up columns of digits. Computers are not easily programmed to create truly random sequences of numbers, so companies use physical phenomena to generate them. Almost immediately, he began to teach infinitesimal calculus at the same institution, where he was appointed a full professor in 1889. More than 2,000 years before, Euclid had proved that there are infinitely many primes, but it was only at the end of the 1700s that Legendre stated his conjecture—a formula to describe the distribution of primes. The coordinates within each 226 octant follow one of eight sequences of values for x, y, and z, ranging from all negative values, with six possible negative and positive values, with six possible negative and positive values for x, y, and z, ranging from all negative values for 1831 Premier Mémoire (First Memoir) The equilateral triangle has six symmetries. Newton died in 1727. The theoretical chance of winning the game is W, and Bernoulli suspected that the fraction of games (f) that resulted in a win would converge on W as the number of games increased. Ancient roots Algorithms have ancient origins. In 2005–06, here the fraction of games increased and the fraction of games (f) that resulted in a win would converge on W as the number of games increased. was made a Gatsby Teacher Fellow for creating the popular mathematics website Risps. In 1615, he devised a way of working out the maximum volumes of solids with curved shapes, such as barrels. Representation theory, as this field is known, is applied in number theory and analysis, and in physics. In modern notation, these would be: ax2 = bx; ax2 = c; ax2 + bx = c; ax2 + c = bx; ax2 = bx + c; and b2 = c. In this way, zero entered the world as a form of punctuation. The system was employed for trading and tax calculations: amounts received were represented by red rods, and debts by black rods. In his innovative 1888 book Natural Inheritance, Galton showed how two sets of data can be compared to show if there is a significant relationship between them. His huge output—much of it never published—included revolutionary advances in algebraic topology, number theory, and category theory. Bernoulli worked on calculus with his younger brother Johann BERNHARD RIEMANN The son of a pastor, Bernhard Riemann was born in Germany in 1826. Ideal In abstract algebra, a mathematical ring that is a component of a larger ring. Graph theory uses matrices to encode how a set of vertices (points) is connected by edges (lines). worked out uniquely from the value of other numbers, using a particular rule. AFTER 1975 Benoit Mandelbrot uses computer graphics to create more complex fractals (shapes that self-repeat). 1934 British mathematician Henry Whitehead stirs interest in Poincaré's conjecture by publishing an erroneous proof. )—there is an infinite Counting infinitie: To help identify where a number is located, Cantor drew a distinction between two kinds of numbers: cardinals, which are the counting numbers 1, 2, 3... that denote the size of a set; and ordinals, such as 1st, 2nd, or 3rd, which list order. 1872 German mathematician Felix Klein defines geometry in terms of group theory. Hardy, English mathematician A million monkeys banging on a million typewriters Robert Wilensky, American computer scientist She changed the face of algebra Hermann Weyl, German mathematician A blueprint for the digital age Robert Gallagher, American engineer A small positive vibration can change the entire cosmos Amit Ray, Indian author 578 A grand unifying theory of mathematician Endless variety and unlimited complication Roger Penrose, British mathematician 579 ACKNOWLEDGMENTS Dorling Kindersley would like to thank Gadi Farfour, Meenal Goel, Debjyoti Mukherjee, Sonali Rawat, and Garima Agarwal for design assistance; and Gilian Reid, Amy Knight, Jacqueline Street-Elkayam, and Anita Yadav for production assistance. 1202 Fibonacci identifies what becomes known as the Fibonacci sequence of numbers. He stated that 2n-1 was valid when n = 2, 3, 5, 7, 13, 17, 19, 31, 67, 127, and 257. Origin The point at which the x and y axes of a graph intersect. Although apparent, it is not mentioned in his preliminary assumptions. Don Zagier American mathematician JACQUES HADAMARD Born in Versailles, France, in 1865, Jacques-Salomon Hadamard became interested in mathematics thanks to an inspiring teacher. These are found by generating random numbers and testing their primality with Pierre de Fermat's "little theorem": if a number (p) is prime, when another number (n) is raised to the power of p, and n is subtracted from the result, the answer is a multiple of p. 299 Using normal distribution From the mid-1700s, the bell curve cropped up as a model for all kinds of data. Alamy Stock Photo: Hemis (tl). Two of Mersenne's n values (67 and 257) were proved incorrect, but in 1947, three new primes were found: n = 61, 89 and 107 (M61, M89, M107), and in 2018, the Great Internet Mersenne Prime Search uncovered the 51st known Mersenne prime. Alternatively, draw a line about a third of the 399 strip, then turn the scissors 90° and cut along its length: the result is one twisted loop that is twice as long. 131 IN CONTEXT KEY FIGURE Brahmagupta (c. He arrived at these denominators by using the factorial for each integer. Someone spying could know the public key, n and a, but would have no idea whether M is 2, 12, or 1,002 (all divisible by 10 with a remainder of 2). Alamy Stock Photo: INTERFOTO (tr). See also: Mersenne primes • The law of large numbers • The Riemann hypothesis • The prime number theorem 307 IN CONTEXT KEY FIGURE Leonhard Euler (1707-83) FIELD Number theory BEFORE 1714 Roger Cotes, the English mathematician who proofread Newton's Principia, creates an early formula similar to Euler's, but using image inary numbers and a complex logarithm (a type of logarithm used when the base is a complex number). The unique identity element is 0, and the inverse of any integer n is -n as n + -n = 0 = -n + n. Pythagoras may have been killed when his school was set on fire, or shortly afterward. The kite and dart must be exactly the same shape as the ones shown (above); the area of the kite to that of the dart is expressed by the golden ratio. He is best known for the fractal—Von Koch's "snowflake" curve—he described in a 1906 paper. The tangent is used to calculate the rate of change at a specific point on a curve. As a woman she was ineligible to study at the École Polytechnique, but she obtained lecture notes and corresponded with the mathematician Joseph-Louis Lagrange. He also compiled a star catalogue, which may be the one used by Ptolemy in Almagest. Only in the early 1800s, when the influential German mathematician Carl Gauss favored using x2, did superscript notation begin to stick. In 1615, he published his results in Nova stereometria doliorum vinariorum (New solid geometry of wine barrels). It can be shown by connecting six points on a circle, a proof valid for other conics, too. 1500 Indian astronomer Nilakantha Somayaji uses an infinite series (the sum of terms of an infinite series, too. 1500 Indian astronomer Nilakantha Somayaji uses an infinite series (the sum of terms of an infinite series). It can be shown by connecting six points on a circle, a proof valid for other conics, too. 1500 Indian astronomer Nilakantha Somayaji uses an infinite series (the sum of terms of an infinite series). It can be shown by connecting six points on a circle, a proof valid for other conics, too. 1500 Indian astronomer Nilakantha Somayaji uses an infinite series (the sum of terms of an infinite series). It can be shown by connecting six points on a circle, a proof valid for other conics, too. 1500 Indian astronomer Nilakantha Somayaji uses an infinite series (the sum of terms of an infinite series). Early research The FTA can be stated in a number of ways, but its most common formulation is that every polynomial with complex root. Pascal's triangle can be any size, ranging from just a few rows in depth to any number. 21 IN CONTEXT KEY CIVILIZATION Babylonians FIELD Arithmetic BEFORE 40,000 years ago Stone Age people in Europe and Africa count using tally marks on wood or bone. 312 Key work 1736 An Introduction to the Doctrine of Fluxions, and a Defence of the Mathematicians Against the Objections of the Analyst See also: Probability • The law of large numbers • Normal distribution • Laplace's demon • The Poisson distribution • The birth of modern statistics • The Turing machine • Cryptography 313 IN CONTEXT KEY FIGURE Joseph-Louis Lagrange (1736-1813) FIELD Algebra BEFORE 628 Brahmagupta publishes a formula for solving many guadratic equations. His famous conjecture concerns the 2-dimensional surface of a 3-dimensional sphere. Adding more numbers creates a sequence of movements across the plane. Such situations demand a many-valued (fuzzy) reasoning system. Leonardo also traveled to Egypt, Syria, Greece, Sicily, and Provence, exploring different number systems. To find the unknown angle (θ) in a right-angled triangle, the sine formula is used when the lengths of the opposite (opposite θ) and the hypotenuse are known; the cosine formula is used when the lengths of the adjacent and hypotenuse are known; and the tangent formula is used when the lengths of the opposite and adjacent are known; and the tangent formula is used when the lengths of the adjacent are known; and the tangent formula is used when the lengths of the adjacent are known; and the tangent formula is used when the lengths of the adjacent are known; and the tangent formula is used when the lengths of the adjacent are known; and the tangent formula is used when the lengths of the adjacent are known; and the tangent formula is used when the lengths of the adjacent are known; and the tangent formula is used when the lengths of the adjacent are known; and the tangent formula is used when the lengths of the adjacent are known; and the tangent formula is used when the lengths of the adjacent are known; and the tangent formula is used when the lengths of the adjacent are known; and the tangent formula is used when the lengths of the adjacent are known; and the tangent formula is used when the lengths of the adjacent are known; and the tangent formula is used when the lengths of the adjacent are known; and the tangent formula is used when the lengths of the adjacent are known; and the tangent formula is used when the lengths adjacent are known; and the tangent formula is used when the lengths adjacent are known; and the tangent formula is used when the lengths adjacent are known; adjacen Company produces a patent for a conveyor belt based on the Möbius strip. 76 Euclid's work covers two areas: elementary geometry and general mathematics, creating bestselling textbook series for secondary-level students, both in the UK and worldwide. AFTER 1799 Carl Gauss publishes the first proof of the fundamental theorem of algebra. Thales used geometry to calculate the height of pyramids and the distance of ships from the shore. ISBN: 9781465494207 A WORLD OF IDEAS: SEE ALL THERE IS TO KNOW www.dk.com 585 Cogepжание HOW TO USE THIS EBOOK INTRODUCTION ANCIENT AND CLASSICAL PERIODS 6000 BCE-500 CE Numerals take their places • Positional numbers The square as the highest power • Quadratic equations The square as the highest power • Quadratic equations The square as the highest power • Quadratic equations and daemons • Pythagoras A real number that is not rational • Irrational numbers The guickest runner can never overtake the slowest • Zeno's paradoxes of motion Their combinations give rise to endless complexities • The Platonic solids Demonstrative knowledge must rest on necessary basic truths • Syllogistic logic The whole is greater than the part • Euclid's Elements Counting without numbers • The abacus Exploring pi is like exploring pi is like exploring the Universe • Calculating pi We separate the numbers as if by some sieve • Eratosthenes' sieve A geometrical tour de force • Conic sections The art of measuring triangles • Trigonometry Numbers can be less than nothing • Negative numbers The very flower of arithmetic • Diophantine equations An incomparable star in the firmament of wisdom • Hypatia The closest approximation of pi for a millennium • Zu Chongzhi THE MIDDLE AGES 500-1500 A fortune subtracted from zero is a debt • Zero Algebra is a scientific art • Algebra is a scientific art • Algebra freeing algebra from the constraints of geometry • The binomial theorem Fourteen forms with all their branches and cases • Cubic equations The ubiquitous music of the spheres • The Fibonacci sequence The golden ratio Like a large diamond • Mersenne primes 586 8 11 19 22 31 39 43 48 59 63 67 71 75 83 88 96 100 104 113 120 124 126 128 132 139 150 156 163 171 176 179 187 Sailing on a rhumb • Rhumb lines • The equals sign and other symbology Plus of minus makes minus • Imaginary and complex numbers The art of tenths • Decimals Transforming multiplication into addition • Logarithms Nature uses as little as possible of anything • The problem of maxima The fly on the ceiling • Coordinates A device of marvelous invention • The area under a cycloid Three dimensions made by two • Projective geometry Symmetry is what we see at a glance • Pascal's triangle Chance is bridled and governed by law • Probability The sum of the distance equals the altitude • Viviani's triangle theorem The swing of a pendulum • Huygens's tautochrone curve With calculus I can predict the future • Calculus The perfection of the science of numbers THE ENLIGHTENMENT 1680-1800 To every action there is an equal and opposite reaction • Newton's laws of motion Empirical and expected results are the same • The law of large numbers One of those strange numbers that are creatures of their own • Euler's number Random variation makes a pattern • Normal distribution The seven bridges of Königsberg • Graph theory Every even integer is the sum of two primes • The Goldbach conjecture The most beautiful equation • Euler's identity No theory is perfect • Bayes' theorem Simply a question of algebra • The algebra theorem of algebra • The fundamental theorem of algebra • The fundamental theorem of algebra • The complex numbers are coordinates on a plane Nature is the most fertile source of mathematical discoveries • Fourier analysis The imp that knows the positions of every particle in the Universe • Laplace's demon What are the chances? 212 BCE) FIELD Number theory BEFORE c. Our current mathematical body of knowledge has been built up by a haphazard and diverse group of people across time and cultures. In 1686, English astronomer Edmond Halley persuaded Newton to write up his new physics and its applications to planetary motion. (2) In logarithms, a fixed base (usually 10 or Euler's number e) is used; the logarithm of any given number x is the power to which that base must be raised to produce x. Many problems in number theory are easy to pose, but extremely difficult to prove. Take, for example, a chef who needs to forecast the number of baked potatoes that will be ordered in her café. He also began to work on matrices with fellow British mathematician Arthur Cayley. He died in 1517, in Sansepolcro, Tuscany. He knew the surface area of the sphere was four times that of a circle of the same radius, but could not find a square that would give the surface area. Henceforth, space by itself, and time by itself, and time by itself shall fade to mere shadows, and only some union of the two will preserve independent reality. When multiplying fractions, for example, the numerators would be multiplied separately from the denominators, and the resulting fraction would then be reduced. 1959 American writer Erik Frank Russell publishes "Now Inhale," a short story about an alien allowed to play a version of the Tower of Hanoi before his execution. English mathematician Thomas Harriot and French mathematician François Viète, who each made important contributions to developments in algebra, used letters to produce consistent symbolic notation. See also: Conic sections • Algebra • The binomial theorem • Calculus GUILLAUME DE L'HÔPITAL 1661-1704 Born in Paris, l'Hôpital was interested in math from a young age and was elected to the French Academy of Sciences in 1693. The story of mathematics is one of exploration of these different fields, and the discovery of new ones. Pascal combined the numbers step by step to represent the rounds played. His monumental work Éléments of Mathematics, 1960), occupies a key place in university libraries and countless students of mathematics have learned the tools of their trade from his work. The question was working memory system whether a long calculation that could not be checked that allowed it to process by humans, followed by a simple verdict of "yes, the large amounts of data. 1070 Treatise on Demonstration of Problems of Algebra 1077 Commentaries on the difficult postulates of Euclid's book A parabola (pink) for the equation  $x^2 = 6y$  intersects the circle (blue) (x-2)2 + y2 = 4. Cryptography is the development of means of secret communication. 6th century BCE In Greece, the relationship between the side lengths of a rightangled triangle is discovered, and is later attributed to Pythagoras. According to one story, Hippasus drowned when his fellow Pythagoreans threw him over the side of a boat in disgust. Others contributed to the development of the growing field of probability theory. Furthermore, expanding mathematics by simply adding more axioms will lead to further "incompleteness." This meant that the efforts of Russell, Hilbert, Frege, and Peano to develop complete logical frameworks for mathematics were destined to have logical gaps, however watertight they tried to make them. Science Photo Library: New York Public Library (bl). He calculated that a positive number subtracted from zero becomes a negative number subtracted from zero become mathematician, is credited with using the Greek letter phi (φ) for the golden ratio. In 1982, American mathematician Richard Hamilton attempted to prove the conjecture using Ricci flow, a mathematician Richard Hamilton attempted to prove the conjecture using Ricci flow, a mathematician Richard Hamilton attempted to prove the conjecture using Ricci flow, a mathematician Richard Hamilton attempted to prove the conjecture using Ricci flow, a mathematician Richard Hamilton attempted to prove the conjecture using Ricci flow, a mathematician Richard Hamilton attempted to prove the conjecture using Ricci flow, a mathematician Richard Hamilton attempted to prove the conjecture using Ricci flow, a mathematician Richard Hamilton attempted to prove the conjecture using Ricci flow, a mathematician Richard Hamilton attempted to prove the conjecture using Ricci flow, a mathematician Richard Hamilton attempted to prove the conjecture using Ricci flow, a mathematician Richard Hamilton attempted to prove the conjecture using Ricci flow, a mathematician Richard Hamilton attempted to prove the conjecture using Ricci flow, a mathematician Richard Hamilton attempted to prove the conjecture using Ricci flow, a mathematician Richard Hamilton attempted to prove the conjecture using Ricci flow, a mathematician Richard Hamilton attempted to prove the conjecture using Ricci flow, a mathematician Richard Hamilton attempted to prove the conjecture using Ricci flow, a mathematician Richard Hamilton attempted to prove the conjecture using Ricci flow, a mathematician Ricci f three, five, or eight petals—numbers that belong to the Fibonacci sequence. "Mathematics" is not simply, as many people think, "stuff to do with numbers." That would exclude a huge range of mathematical topics, including much of the geometry and topology covered in this book. Then he split his machine into two parts, the "Mill" and the "Store," which made it possible to separate the addition and carryover processes. By their nature, dynamical states are "nondeterministic": two systems that are nearly identical. Some scholars have suggested that "O" was used because it was the shape left when a counter was removed. As most guadratic equations could 196 be solved with an algebraic formula, the search was on for a similar formula that worked for cubic equations. Importantly for mathematical rules. Huygens identified the cycloid, a curve that was steep at the top and shallow at the bottom. He taught mathematics at both universities and wrote the first English book on algebra in 1543. See also: The Fibonacci sequence • Decimals 30 IN CONTEXT KEY CIVILIZATIONS Egyptians (c. The machine would then work through the entire calculation automatically. In 1543, Recorde's The Grounde of Artes introduced the symbols for addition (+) and subtraction (-) to mathematics in England. The Hindu-Arabic system used today is based on the ten numerals 0 to 9: when 10 is reached, 1 is written again, but with a 0 after it. 230 A modified form of polar coordinates that gives an aircraft's destination in terms of angle and distance can be used as an alternative to GPS. Kepler's interest in this field began in 1613, when he married his second wife. In 2016, Peter Trueb, a Swiss particle physicist, used the y-cruncher software to calculate  $\pi$  to 22.4 trillion digits. Eventually, the study of areas and volumes was formalized into what became known as integration, while calculating rates of change was termed differentiation. 204 The decimal system makes it easier to divide and multiply fractions, especially by 10. RENÉ DESCARTES The son of a minor noble, René Descartes was born in Touraine, France, in 1596. By the 9th century, Brahmagupta's zero (highlighted in pink) was widely used in India, from where it spread via the Arab world to Europe. 3rd century CE Greek mathematician Diophantus uses symbols to represent unknown quantities. Fundamental to understanding this is the Riemann hypothesis—an unsolved conjecture—which, if true, could reveal a huge amount more about prime numbers. 100-170 cE) determined  $\pi$  to be 3:8:30 (base-60), that is, 3 + 8/60 + 30/3,600 = 3.1416, which is just 0.007 percent greater than the closest known value of  $\pi$ . 212 The earlier table to the power of 2 (see Generating logarithms) can be thought of as a simple base-2, or log2 table. If you were to break a stick of length h into three, there would always be a point in the triangle from which the pieces form the perpendiculars p, q, and r. AI is therefore entirely the product of a pre-programmed set of rules. 1955 In Japan, Yutaka Taniyama and Goro Shimura propose that every elliptic curve has a modular form. Key works 1202 Liber Abaci (The Book of Calculation) 1220 Practica Geometriae (Practical Geometry) 1225 Liber Quadratorum (The Book of Squares) Generations of bees An example of the Fibonacci sequence cropping up in nature concerns bees in a beehive. The most significant was that of French monk Marin Mersenne in 1644). In modern notation, this translates into two simultaneous equations:  $x^2 + y^2 = 100$  and x = (1/2 + 1/4)y = 3/4 y. A butterfly flaps its wings in the Amazonian jungle, and subsequently a storm ravages half of Europe. 408 Overwhelmed by depression, Cantor was institutionalized for much of his later life. 386 A linear transformation in 2 dimensions maps lines through the origin to other lines through the origin, and parallel lines. For example, the chances of throwing a six with a die can be estimated by throwing the die a given number of times and dividing the amount of sixes thrown by the total number of throws. The Greek Anthology, a compilation of mathematical games and verses published around 500 CE, contains one number of throws. tombstone. The square of a real number cannot be negative—any real numbers. Pythagoreans revered the pentagon, and their symbol was the pentagram, a five-pointed star with a pentagon at its center. When solving the equation x2 + 10x = 39, they gave the solution as 3. Using different bases The Hindu-Arabic numeration that is employed today is a base-10 (decimal) system. Galois theory relates the solvability of a polynomial equation (whose roots are elements of a field) to a group—specifically, to the permutation group that encodes possible rearrangements of its roots. French mathematics was stuck with antiquated textbooks and teachers. Appel and Haken had used the computer to carry out a proof by The IBM System/370 exhaustion—all possibilities were meticulously computer c. In this role, he turned Göttingen into one of the mathematical hubs of the world and taught a number of young mathematicians who later made their own mark. AFTER 2001 The Taniyama-Shimura conjecture is established. To do that, a codebreaker must figure out the prime factors of n, but today's RSA algorithms use values for n with 600 digits or more. Exponential function A mathematical function where, as a quantity gets larger, its rate of increase also gets faster. Key works 388 1850 On a New Class of Theorems 1852 On the principle of the calculus of forms 1876 Treatise on elliptic functions See also: Algebra • Coordinates • Probability • Graph theory • Cryptography 389 IN CONTEXT KEY FIGURE George Boole (1815-64) FIELD Logic BEFORE 350 BCE Aristotle's philosophy discusses syllogisms.

In 1995, he was appointed professor at St. Petersburg University, first as chair of software engineering and later as chair of algebra and number system. The year of Hippasus's death is not known but is likely to have been in the 5th century BCE. If the answer was No, the algorithm M\* never halts, then the machine fear and disbelief of irrational numbers, numbers that cannot be expressed as fractions of two integers, the most famous example being n. Distances between points were not relevant: the only thing that counted was the connections between points. Descartes and Fermat both made use of algebraic notation, the x and y system that François Viète had introduced at the end of the 1500s. The comte also translated a number of scientific works. Gottlob Frege Logical gaps In 1931, Kurt Gödel, an Austrian mathematician and philosopher, published his incompleteness theorem (following on from his completeness theorem (at a parabola, but in which the two extensions of the curve approach two imaginary straight lines at angles to each other without ever touching or crossing the lines. A year later, Albert Einstein produced his theory of special relativity, which proved that the speed of light is the same throughout the Universe. He was born in c.262 BCE in Perga, a center for the worship of the goddess Artemis, in southern Anatolia (now part of Turkey). To encrypt and decrypt an Enigma message, both machines needed to be set up in the correct way. This was still the case in 1545 when Italian polymath Gerolamo Cardano published his Ars Magna (The Great Art), in which he explained how to solve linear, quadratic, and cubic equations. 1650 BCE The Rhind papyrus, written by Middle Kingdom Egyptian scribes as a mathematics guide, includes estimates of the value of π. The top line is for powers, with the coefficients for each power listed in the column below. 2019 In Japan, Emma Haruka Iwao uses a cloud computing service to calculate π to more than 31 trillion decimal places. An atheist, freethinker, and socialist, he became one of the greatest statisticians of the 1900s, but he was also a champion of the discredited science of eugenics. Proposition 9 To bisect a given rectilineal angle. They were painstakingly copied by scribes well versed in arithmetic, geometry, and mensuration (the study of measurements) and are likely to have been used for training of other scribes. In the 6th century BCE, Pythagoras created his quadratic equation for calculating the start of the game, the probability of eight rolls of the die without success is  $(5/6)8 \approx 0.233$ , and the probability of throwing at least one ace is (1-0.233), or 0.7677. He failed to address the obvious next step: although he had established that every real number solution, he had not considered equations built from complex number solution, he had not considered equations built from complex number solution. particularly interested in measurement. Leibniz Around the time that Newton was developing his calculus, German mathematician Gottfried Leibniz was working on his own version, based on the consideration of infinitesimal changes in the two coordinates defining a point on a curve. Journeys of discovery Pythagoras was well-traveled, and the ideas he absorbed from other countries undoubtedly fueled his mathematical inspiration. Even though the system is deterministic, every measurement of that system is an approximation. Cohen graduated and later received his doctorate, in 1958, from the University of 554 Chicago before moving to the Massachusetts Institute of Technology (MIT), Princeton University, and finally Stanford University, where he became professor emeritus in 2004. This was proved in 1761 by Johann Heinrich Lambert— earlier estimations of π had been 3 or 22/7. See also: The Platonic solids • Euclid's Elements • The complex plane • NonEuclidean geometries • Topology 507 IN CONTEXT KEY FIGURES Kenneth Appel (1932-2013), Wolfgang Haken (1928-) FIELD Topology BEFORE 1852 South African law student Francis Guthrie asserts that four colors are needed to color a map so that adjacent areas are not the same color. The triangle is divided into three smaller triangles by drawing a line from the point to each corner of the main triangle. Lie groups are continuous, not discrete—for example, they model the infinite number of rotational symmetries, such as those associated with a circle, rather than the finite number of transformations of a polygon. For example, if statement A, "my dog is brown" is true, it also has a value of 1. This is only 1/4 of 100, so the areas must be quadrupled to match the Berlin papyrus equation. In 2013, a computer tested every even number up to 4 × 1018 without finding one. By treating the integers as complex analysis (a study of functions with complex numbers) to investigate the integers. Where one side of the pentagram crosses another, it divides each side into two parts, the ratio of which is  $\phi$ . By the 1840s, mathematicians such as Auguste Bravais were looking at the level of error that could be accepted for this line, and tried to pin down the significance of the midpoint or "median" of a set of data. Group theory is used to study symmetries in chemistry and physics, for example, and can be used in public key cryptography, which secures much of today's digital communication. Ramanujan was not the first to make note of this number's unique properties; French mathematician Bernard Frénicle de Bessy had also written about them in the 1600s. Two of these numbers, e and i, were introduced by Euler himself. AFTER 400 BCE Theodorus of Cyrene proves the irrationality of the square roots of the nonsquare numbers. Perhaps because of its very concrete nature, for many ancient civilizations, geometry was the cornerstone of mathematics, and provided a means of problem-solving and proof in other fields. 1854 George Boole introduces the algebra that will form the basis for computing. Also at the turn of the 20th century, German mathematician David Hilbert invented the idea of a space that took the Euclidean spaces of two and three dimensions and generalized them to infinite dimensions. AFTER 1854 Bernhard Riemann describes the kind of surface that has hyperbolic geometry. One of the inputs needed for this formula is a series of complex numbers defined by what is now called the Riemann zeta function,  $\zeta(s)$ . For example, if r is 2, then the circle crosses the x axis at (2,0) and (-2,0), and it crosses the y axis at (0,2) and (0,-2). Cartographers have long known that any map, however complicated, can be colored in with just four colors, so that no two nations or regions sharing a border are the same color. His mathematical accomplishments included describing a method for computing the square roots and cubic roots of numbers. Composed of polygons Only five regular polyhedra are possible—each one created either from identical equilateral triangles, squares, or regular pentagons, as Euclid explained in Book XIII of his Elements. In a few cases, where the bodies start in very specific places, the motion is periodic—it repeats the same paths over and 485 over again. Just a single coordinate number is needed to describe an exact point on the line—as when measuring a distance with a ruler from zero to a unit of length. He then went on to add five axioms, or common notions; if A = B and B = C, then A = C; if A = B and C = D, A + C = B + D; if A coincides with a ruler from zero to a unit of length. He then went on to add five axioms, or common notions; if A = B and B = C, then A = C; if A = B and C = D, A + C = B + D; if A coincides with a ruler from zero to a unit of length. B, then A and B are equal; and the whole of A is greater than part of A. 427 In 2000, 21st-century mathematics was heralded in a similar spirit to that of the 20th century, when the Clay Mathematics announced seven Millennium Prize Problems, offering a \$1 million prize for any of their solutions. In around 370 BCE, the Greek mathematician Eudoxus of Cnidus proposed a method of calculating the area of a shape by filling it with identical polygons of known area, and then making the polygons of known area, and then making the polygons infinitely smaller. If one or both of A and B are false, then A AND B is false. He later updated Newtonian mechanics into a system that enabled further advances to be made in electromagnetism and quantum mechanics. It was also a period of considerable progress in mathematics. French mathematician François Viète was key to this development. Euler explored e further in his 1748 Introduction). An order-two magic square does not exist because it would only work if all the numbers were identical. 1650 BCE) FIELD Arithmetic BEFORE c. This book presents many of the "big ideas" in mathematics, from the earliest discovered them, and what makes them significant. In 1829, Galois enrolled at the Ecole Préparatoire, only to be expelled in 1830 for his politics. Multiplication was especially long-winded and much more difficult than addition. When events are certain, probability = 1 (or 100 percent). Born in Germany to anarchist parents, at the age of 10 he fled the Nazi regime as a refugee to France, where he spent most of his life. It was one of three symbols used by the Mayans for arithmetic; the other two were a dot representing 1 and a bar for 5. Alan Turing 467 Clerks at work in Hut 8, Bletchley Park, UK, during World War II. He began to receive plaudits in the early 1900s, but lived out his old age in poverty. Parallel Of a line, going in exactly the same direction as another line. The challenger is presented with three poles, one of which holds three disks in order of size, with the largest disk on the bottom. Key work 1939 "Report on the Applications of Probability to Cryptography" 468 The halting problem Turing approached this problem as a thought experiment. This marble plaque on the rue de Vaugirard, Paris, is one of 16 original meter markers installed in 1791, after the French Académie des Sciences defined the meter for the first time. He studied logic and theology at the University of Edinburgh and followed his father into the ministry, spending much of his life leading a Presbyterian chapel in Tunbridge Wells, Kent. Many mathematicians argued that it could not. Alamy Stock Photo: Ian Robinson (clb). Apex The vertex that is furthest from the base in a 3-D shape. In 1952, Turing was convicted of homosexuality, then a crime in the UK. Laplace later sided with the Bourbons (the French royal family) and was restored. He also argued that Earth could be rotating in space, 540 rather than the Churchapproved view that the celestial bodies circled around Earth. By alternating quantities of vertical rods, this system could indicate ones, tens, hundreds, thousands, and more powers of 10, much as the decimal system does not vary -often symbolized by a letter such as a, b, or c. The five Platonic solids are the tetrahedron, cube, octahedron, and icosahedron, and icosahedron, and icosahedron, and icosahedron. Commutative law The law that states that 1 + 2 = 2 + 1, for example, and that the order in which the numbers are set down doesn't matter. From the middle of the first millennium BCE, the first pure mathematicians began to appear in Greece, and slightly later in India and China, building on the legacy of the practical pioneers, astronomers, and explorers of earlier civilizations. In theory these can be fitted into any shape and added up. Scipione del Ferro, a mathematics professor at Bologna University, took a major step forward when he discovered an algebraic method for solving some cubic equations, but the quest for a comprehensive formula continued. Pascal gives an example with two opponents playing a sequence of games, each with an equal chance of winning, where the first to win three games wins the stake. Today, negative numbers are used in many areas, ranging from banking and temperature scales to the charge on subatomic particles. There is an ancient and innate sense in people that numbers ought not to misbehave. 2018 The largest known prime to date is found to be 282,589,933 - 1. and Do Not Open, and contributed to major books such as Science and Science Year by Year. 71 In the Square of Opposition, S is a subject, such as "sugar," and P a predicate, such as "sweet." A and O are contradictory, as are E and I (if one is true, the other is false, and vice versa). In contrast to the pluralists, who believed that the Universe could be divided into its constituent atoms, Eleatics believed in the indivisibility of all things. The prime numbers—those positive whole numbers that have only two factors, themselves and 1—have long fascinated mathematicians. Riemann only calculated three. The relationship with n can be used in a number of probability problems. Proposition 12 To a given infinite straight line, from a given point which is not on it, to draw a perpendicular straight line. Another significant development was the formalization of a number system of base-10, and Simon Stevin's introduction of the decimal point in 1585. Just a month later, he fell ill and his health deteriorated until he died of tuberculosis in 1866. She claims that the crocheting process helps develop geometrical intuition. As the Industrial Revolution spread and 1848's "Year of Revolution" saw nationalism surge across old empires, there was a renewed drive to understand the workings of the Universe A mathematical expression or operation that is the opposite of another one and undoes it. Gerolamo Cardano encountered negative roots while working on cubic equations in the 1500s. To avoid prison, Turing agreed to hormone treatment to reduce his libido. Hungarian mathematician Paul Erdős wrote and cowrote around 1,500 academic papers in his lifetime. start and finish (if they were different locations). Today, Fourier analysis plays a key role in many applications including digital file compression, analyzing MRI scans, speech recognition software, and determining the composition of planetary atmospheres. Scalene triangle where none of the sides and none of the angles are the same size. Proposition 13 If a straight line set up on a straight line makes angles, it will make either two right angles or angles equal to two right angles. He was right; the study of numbers, such as complex analysis developed, fueling the proofs of Hadamard and Poussin. 575 QUOTATIONS The following primary quotations are attributed to people who are not the key figure for the relevant topic. Taking this idea to the limit, Archimedes imagined a polygon with sides of infinitesimally smaller length. Three axes (x, y, z) are set at right angles to each other. supposed discoveries of the golden ratio in the human body were a result of imprecise measurements. It led to protracted bitterness between the two rivals and across much of the mathematical community. The Scientific Revolution was well under way, inspiring a new, rational approach not only to the sciences, but to all aspects of culture and society. It was only in the late 1800s that Georg Cantor was able to explain it with mathematical rigor. It made him want to study mathematics at the University of Oxford, and then to get his PhD at Cambridge. He was awarded the Nobel Prize in Literature in 1950, and in 1955 he and Albert Einstein released a joint manifesto calling for a ban or nuclear weapons. 1823 French mathematician Augustin-Louis Cauchy formalizes the fundamental theorem of calculus. Alamy Stock Photo: Interfoto (tr). But applications for mathematical discoveries have usually been found, and advances in science and technology have driven innovations in mathematical thinking. Descartes was aware of Fermat's ideas, no doubt using them to improve his own. partition suggests that the optimal number of splits for 10 is between 3 and 4. Ragwort flowers have 34 or 55. Primes tend to decrease in frequency as numbers get larger. 1643 Pierre de Fermat pioneers number theory. Using projection, this is true for an ellipse, too. Infinite complexity is suggested by the self-similarities of a Romanesco cauliflower. Realizing that the problem related to the geometry to show that it was impossible to devise such a route. This involved moving from arguments expressed in words to a symbolic logic where arguments could be expressed using abstract symbols. The Bourbaki group poses for a photo at the first Bourbaki congress in July 1935. Cayley showed that the rules of matrix algebra are different from those in standard algebra. The paradox of Achilles and the tortoise maintains that a fast object, such as Achilles, will never catch up with a slow one, such as a tortoise. Key works 1630-33 Le Monde (The World) 1637 La Géométrie (Geometry) 1644 Principia philosophia (Principles of Philosophy) Finding a new method There are two accounts of how Descartes came to develop the coordinate system. 172 Bacteria dividing is an example of exponential growth; when a single cell divides, it creates two cells that divide to make four, and so on. Key work 1927 Collected papers of Srinivasa Ramanujan See also: Cubic equations • Catalan's conjecture • The prime number theorem 452 IN CONTEXT KEY FIGURE Émile Borel (1871-1956) FIELD Probability BEFORE 45 BCE The Roman philosopher Cicero argues that a random combination of atoms forming Earth is highly improbable. For example, the set of integers forms a group when addition is the operation. This is true, but Gauss argued that the point needed to be proved. The binomial theorem is a rule for working out binomial coefficients in complex cases. The 2 in 120, for example, has a place value of 20, but in 210 it stands for 200. A square is a special kind of rhombus, with all angles 90 degrees. Rather than increasing in multiples of 10, each column represents a power of 2. ABRAHAM DE MOIVRE Born in 1667, Abraham de Moivre was raised as a Protestant in Catholic France, and lived there until 1685, when Louis XIV expelled the Huguenots. 1030) FIELD Number theory BEFORE c. 250 CE In Arithmetica, Diophantus lays down ideas about algebra later taken up by al-Karaji. Two years later, the Riemann hypothesis was also one of the Clay Institute's Millennium Prize problems. 304 The city of Königsberg had seven bridges linking two parts of the city to its two islands. The symbolism that Diophantus introduced for the first time... provided a short and readily comprehensible means of expressing an equation. However, he could not accept the use of negative numbers in algebra, believing them to be meaningless. However, even using the best strategy and moving one disk per second, they would take 585 billion years to complete the game. The idea of "decimal fractions"— which have powers of 10 as the denominator—had been used five centuries before Stevin, in the Middle East, but it was Stevin who made decimals commonplace in Europe, both for recording and calculating with parts of a whole. The first five lines of it are shown here. The circle's radius can be conceived as the hypotenuse of a right-angled triangle with short sides x . 1677 Gottfried Leibniz suggests a form of symbolic notation for logic, anticipating the development of mathematical logic. Lagrange's investigation into permutations (and symmetries) formed the basis of the even more abstract and general group theory advanced by French mathematician Evariste Galois, who used it to prove why it is impossible to resolve equations. conjecture by South Korean astrophysicist Sun Hong Rhie. Reason is immortal, all else is mortal. 2000 BCE), Babylonians (c. Instead, physicists used a technique invented by Swiss mathematician Daniel Bernoulli in 1738, which used probability theory to model the movement of independent units within a space. the concept of negative numbers was recognized in Indian mathematics, it was not generally accepted until the 1600s. A line from G, the point of intersection, to H on the x axis, gives the value for x (3.14) in the cubic equation x3 + 36x = 144. Bridgeman Images: Pictures from History (cra). Leibniz was a celebrated philosopher as well as a mathematician. The amazing thing is that chaotic systems don't always stay chaotic. He later published work in the 396 Cambridge Mathematical Journal, but still could not afford to study for a degree. In essence, it was a battery connected to 26 lightbulbs, or lamps—one for each letter of the alphabet. It is also now known that Platonic solids are indeed found in nature—in certain crystals, viruses, gases, and the clustering of galaxies. In his spare time, Fermat continued his mathematical investigations, circulating his ideas in letters to friends, such as Blaise Pascal. Standard aspect ratios for television and computer monitor screens, such as the 16:9 display, also come close to  $\phi$ , as do modern bank cards, which are almost perfect golden rectangles. There are also three derangements starting with 3 and three starting with 4, making nine in total. There is no end with pi. Polynomial A mathematical expression made up of two or more terms added together. He became one of the great scholars of his day, making important discoveries in math and astronomy. The sum of the vertices (V) minus the number of edges (E) plus the number of faces (F) always equals 2, that is, V - E + F = 2. 1320 Al-Farisi was born in Tabriz, Persia (now Iran). This enabled him to produce a general theory. Linear equations result in straight lines when they are plotted as graphs. Binary notation Writing numbers using the binary system, in which the only numerals used are 0 and 1. Galois groups turn up in number theory and are a generalization of the groups that Evariste Galois used in order to study roots of polynomials. Benjamin Franklin Talking about a magic square that he discovered 46 Once you have one magic square, you can add the same quantity to every number in the square and still end up with a magic square. The rise of computer networks has allowed people to communicate easily over great distances without ever meeting. Aged 16, he won a gold medal at the International Mathematician James Gregory develops the arctangent method for computing n. 1998 In the US, sociologist Duncan J. When he failed to show a profit, the mines were also closed. Working out probabilities Most real-world events, however, are more complicated than the toss of a coin. They introduced the decimal point, so that the system could also express fractions of whole numbers. The line y = 2x would follow a steeper path along a line including the coordinates (0,0); (1,2); (2,4), for instance. There, he chose elliptic curves as the area of study for his doctoral thesis—a subject that seemed to have little to do with his interest in Fermat. He continued to do so when he found the university courses too basic. One example of a group obeying all four axioms is the set of integers {..., -3, -2, -1, 0, 1, 2, 3, ...} with the operation of addition. Erdős's coauthors had an Erdős number of 2, and so on. Socrates' execution in 399 BCE deeply affected Plato and he left Greece to travel. American engineer Claude Shannon used Boole's Mathematical Analysis of Logic to establish the basis of modern digital computer circuits. Getty Images: Keystone Features / Stringer / Hulton Archive (bc). At one point, Turing led the work of Hut 8, which deciphered communiqués between Adolf Hitler and his forces. Coloring in all of the numbers divisible by a particular number creates a fractal pattern, while coloring all of the even numbers creates a pattern of triangles identified by Polish mathematician Wacław Sierpinski in 1915. In such systems, the value of a number is indicated both by its symbol and its position. If the parabola touches the x axis only once, this means that there are coincident roots (the solutions are equal to each other). A successful academic career followed, including a spell at several research institutes in the US, where he solved a major geometry problem called the Soul conjecture. To a topologist, however, two shapes are identical—or invariant, in topologist, however, two shapes are identical—or invariant, interval (the sould conjecture) is the use of the sould conjecture invariant, interval (the sould conjecture) is the use of the sould conjecture invariant, interval (the sould conjecture) is the use of the sould conjecture invariant, interval (the sould conjecture) is the use of the use of the sould conjecture invariant (the sould conjecture) is the use of continuous stretching, twisting, or bending, but with no cutting, piercing, or sticking together. Practical solutions Fibonacci's work in the 12th century, renewing an interest in the search for π in Europe. The number 1 can be written as 20, because in the search for π in Europe. is equivalent to 1 multiplied by 2 zero times, leaving 1 unaffected. AFTER 1888 Charles Hinton invents the tesseract, which is an extension of the cube into four spatial dimensions. Siméon Poisson is credited to the true discoverer. 262-190 BCE) FIELD Geometry BEFORE c. [Conic sections are] the necessary key with which to attain the knowledge of the most important laws of nature. Among his claimed inventions is the Archimedes' screw, a revolving screw-shaped blade in a cylinder, which pushes water up a gradient. See also: Fractals ALBERT EINSTEIN 1879-1955 Einstein was an outstandingly gifted physicist and mathematicians. This later became known as "Zu's ratio." Zu's calculations of n were not bettered until European mathematicians set about the task during the Renaissance, almost a millennium later. 1619 Johannes Kepler conducts the first documented study of tessellations. See also: Pythagoras • Irrational numbers • The Platonic solids • Euclid's Elements • Calculating pi • The Fibonacci sequence • Logarithms • The Penrose tile 186 IN CONTEXT KEY FIGURES Hudalrichus Regius (early 1500s), Marin Mersenne (1588-1648) FIELD Number theory BEFORE c. This scale was used until the 1500s, when it was replaced by the even-tempered scale, in which the notes between the two octaves are more evenly spaced. Jennifer Ouellette American science writer 270 Isaac Newton's Opticks, a treatise about the reflections and refractions of light, published in 1704, contains the first details of his work in the area of calculus. "The engine can arrange and combine its numerical quantities exactly as if studied medicine at Cambridge University and was later ordained a priest, he retained the interest in arithmetic he first developed as a schoolboy in Kent, England. While such equations, solving them exactly (algebraically) was not achieved until the 1700s. JACOB BERNOULLI Born in Basel, Switzerland, in 1655, Jacob Bernoulli studied theology, but developed an interest in mathematics. Not all parabolas cut the x axis in two places. Taxicab numbers have little practical use, but they still inspire scholars as curiosities. Some scholars believe that the history of mathematics is above all a story of discovery rather than invention. By showing this, Euler provided the first theorem in graph theory. His work was widely read and came to the attention of the Holy Roman Emperor, Frederick II. 300 CE Parts of the Indian Bakshali text reveal many circular placeholder zeros. His keen interest in astronomy, and a desire to find simpler ways to perform the calculations that it required, led to his invention of logarithms. The furthest thing from my mind has been those efforts which try to establish an artificial similarity [between logic and algebra]. KARL PEARSON Karl Pearson was born in London in 1857. Similarly, in differential equations, the term that has been differentiated most times in a given equation determines its degree or order. According to the philosopher Iamblichus, who wrote a biography of Pythagorean sect called the Mathematici, which fervently believed that all numbers were rational. 1975-76 Iterative algorithms allow computer calculations of π to millions of digits. Practical considerations also prompted progress: commerce required more sophisticated means of accounting, and international trade drove advances in navigation, which demanded a deeper understanding of trigonometry. When he tried to calculate the surface area of a 3-D sphere using quadrature, a process which involves constructing a square of an area equal to a circle, he failed. Lines connecting the corresponding vertices of the triangles (X to x; Y to y, and Z to z) will always meet at P. 341 Fourier's career was disrupted by two arrests—one for criticizing the French Revolution, the other for supporting it—but in 1798, he accompanied Napoleon's forces into Egypt as a diplomat. Non-Euclidean geometries were further developed by Bernhard Riemann, who identified and defined different types of geometry in multiple dimensions. Congruent Having the ground. For a four-way partition,  $21/2 \times 21/2 \times 21$ numbers in the matrix. See also: Algebra • Coordinates • Calculus NICCOLO FONTANA TARTAGLIA 1499-1557 As a child, Tartaglia was attacked by French soldiers invading Venice. Alamy Stock Photo: Stefano Ravera (ca). Angles and symmetry The Pythagoreans were masters of geometry and knew that the sum of the three angles of a triangle (180°) is equal to the sum of two right angles (90° + 90°), a fact which two centuries later was described by Euclid as the triangle postulate. I hope this book helps inspire everyone to get involved. See also: Calculus • Non-Euclidean geometries • The logic of mathematics NIELS VON KOCH 1870-1924 Born in Stockholm, Sweden, Koch studied at the universities of Stockholm and Uppsala, later becoming professor of mathematics at Stockholm. He was able to calculate the area of crescent-shaped figures contained within intersecting circles (lunes). It might then seem most likely that the next toss will produce a tail. Of Hilbert's 23 problems, 10 are considered resolved, seven have been partially solved, two have been classed as too vague to ever be definitively solved, three remain unsolved, and one (also unsolved) is really a physics problem. The Mandelbrot set In the late 1970s, Benoit Mandelbrot used the term "fractal" for the first time. He began by defining it as the result of subtracting a number from itself— for example, 3 - 3 = 0. 428 IN CONTEXT KEY FIGURE David Hilbert (1862-1943) FIELDS Logic, geometry BEFORE 1859 Bernhard Riemann proposes the Riemann hypothesis, a famous problem that will later be Number 8 on Hilbert's list and remains unresolved today. More complex ciphers and keys make brute force more timeconsuming—and, before computers, effectively unworkable for messages long enough to hold large amounts of information. 241 When six arbitrary points are drawn on a circle and connected as shown (Ab, aB; Ac, Ca; Cb, cB), a straight line can be drawn through the points where lines of the same color cross. JONNY GRIFFITHS After studying mathematics and education at Cambridge University the Open University, and the University of East Anglia, Jonny Griffiths taught math at Paston Sixth Form College in Norfolk, UK, for over 20 years. The book also illustrated other methods of multiplication. In 1654, de Méré consulted his friend Pascal about this problem, and about the further question of how a stake should be divided between the players when a game was interrupted before completion. In 1742, Russian mathematician Christian Goldbach wrote to Leonhard Euler, the leading mathematician of the time. Without symbolic algebra, it 147 would be difficult to imagine how modern mathematics would have ever developed. Their circle area calculation—with the circle 41 diameter as twice the radius (2r)—can be expressed as (8/9 × 2r)2, which, simplified, is 256/81 r2, giving an equivalent for pi of 256/81. 300 BCE Euclid produces a formula to find sets of primitive Pythagorean triples. 1900 The 10th problem on David Hilbert's list of unsolved research problems is the quest to find an algorithm to solve all Diophantine equations After teaching for 20 years at the University of Latvia, she moved to Cornell University in the United States in 1996, where a chance encounter opened up a new area of interest. 495 Key works 1967 Letter to André Weil 1976 On the Functional Equations Satisfied by Eisenstein Series 2004 Beyond Endoscopy See also: Fourier analysis • Elliptic functions • Group theory • The prime number theorem • Emmy Noether and abstract algebra • Proving Fermat's last theorem 496 IN CONTEXT KEY FIGURE Paul Erdős (1913-96) FIELD Number theory BEFORE 1929 Hungarian author Frigyes Karinthy postulates the concept of six degrees of separation in his short story. Láncszemek (Chains). In later centuries, trigonometry became invaluable in navigation as well as astronomy. He was a pioneer of the scientific method that maintained that hypotheses should be tested by experiment and not just assumed to be true. binomials. Classical mechanics was not up to the task. Andrew Wiles Fixing an error The next day, the world's press was full of the story, transforming Wiles into the world's most famous mathematician. For their commercial arithmetic, the ancient Chinese used small bamboo rods, laid out on a large board. He is currently investigating ethical issues in AI. Computer programs are simply algorithms; the inputs from a keyboard, microphone, or touchscreen are processed by these algorithms into outputs, such as text on a device's screen. This calculation brought negative numbers into the same number system as positive numbers. that was becoming increasingly abstract during the 1800s. 2000 BCE The Berlin papyrus records a quadratic equation solved in ancient Egypt. See also: Wheat on a chessboard • The problem of maxima • Euler's number • The prime number theorem 216 IN CONTEXT KEY FIGURE Johannes Kepler (1571-1630) FIELD Geometry BEFORE c. The overlaps show what the sets have in common. He was able to convert base-10 numbers to binary and back again, and could also calculate using binary numbers. 1657 Pierre de Fermat writes his last theorem (about a Diophantine equation) in his copy of Arithmetica. Pressing the same key a second time always lit a different lamp (never the same letter as the key) because the connections between battery and 516 lampboard were altered by three rotors that clicked around with every key press. It states that π(x) is approximately equal to x ÷ ln(x) as x gets larger and tends to infinity. Kepler used infinitesimally small strips to measure the distance covered in an orbit. Given a large enough ciphertext, this system worked on any substitution cipher, no matter how elaborate the encryption. Science Source (b). Falling ill in 1843, Jacobi returned to Berlin, where he was supported by a pension from the king of Prussia. By the 1960s, physicists 373 began to use group theory to classify subatomic particles. In mathematics, polar coordinates, which define points on a plane using two numbers, are the closest rivals to Descartes' system. An influential teacher, he included among his pupils the young Russian émigré and pioneering mathematician Sofya Kovalevskaya. Pierre-Simon Laplace Network theory Arcs on a graph may be "weighted" (given degrees of significance) by assigning numerical values to them—for example, to represent the different lengths of roads on a map. To make the biggest advancement in mathematics during the 21st century, we need to include all people. 520 There are similarities between simple groups and symmetry in geometry. Mathematics is often about the identification of number patterns, and one of the most remarkable number patterns, and one of the most remarkable number patterns. When American mathematician and Fields Medal winner Steve Smale came up with his own list of 18 questions in 1998, it included Hilbert's eighth and 16th problems. Blank spaces represent zero. Player versus banker The Pascal-Fermat letters were sent via Pierre de Carcavi, a mutual friend. The catenary The Gateway Arch in St. Louis, Missouri is a flattened catenary arch, designed by Finnish- Sometimes defined as the shape a hanging chain takes if it is only supported at its ends, a catenary is a curve with the formula y = 1/2 × (ex + e-x). His papers suggest that the machine, if built, could have been close to what we now call a computer. Through a process called fuzzy inference, the algorithm applies the rule to each egg based on its fuzzy set membership. When the values successively assigned to the same variable indefinitely approach a fixed value, so as to end by differing from it as little as desired, this fixed value in their reliability. Scottish mathematician Colin Maclaurin published his Treatise on Fluxions in 1742, promoting and furthering Newton's methods, and attempting to make them more rigorous. A square, rectangle, and rhombus 567 are types of parallelogram. One of al-Karaji's successors was the 12th-century scholar Ibn Yahya alMaghribi al-Samaw'al ALAN TURING Born in London in 1912, Alan Turing was described as a genius by his teachers. Hamilton sold the rights to the game for £25. In 1799, he became Minister of the Interior under Napoleon Bonaparte, but was dismissed after only six weeks for being too analytical and ineffectual. In 1735, Swiss mathematician Leonhard Euler noted a further property of Platonic solids, later shown to be true for all polyhedra. An imaginary number, when squared, produces a negative result, defying the usual rules that any number (positive or negative) results in a positive number (positive or negative) results in a positive number (positive or negative) result, defying the usual rules that any number (positive or negative) results in a positive number (positive) results in a positive number (positive) results in a positive) result (positive) result (pos DK LONDON Senior Art Editor Gillian Andrews Senior Editors Camilla Hallinan, Laura Sandford Us Editor Jenny Wilson Illustrations James Graham Jacket Design Development Manager Sophia MTT Producer, Pre-Production Andy Hilliard Producer Rachel Ng Managing Editor Gareth Jones Senior Managing Art Editor Lee Griffiths Associate Publishing Director Liz Wheeler Art Editor Pooja Pipil Art Editor Midushmita Bose Assistant Art Editor Nobina Chakravorty Senior Editor Anita Kakar Editor Addithyan Mohan Senior Jacket Designer Suhita Dharamjit Jackets Editorial Coordinator Priyanka Sharma Senior Dtp Designers Vijay Khandwal, Anita Yadav Picture Research Manager Balwant Singh Production Manager Balwant Singh Production Manager Pankaj Sharma 583 Senior Managing Editor Rohan Sinha Managing Art Editor Sudakshina Basu original styling by STUDIO 8 TOUCAN BOOKS Editorial Director Ellen Dupont Senior Editor Sudakshina Basu original styling by STUDIO 8 TOUCAN BOOKS Editorial Director Ellen Dupont Senior Editor Sudakshina Basu original styling by STUDIO 8 TOUCAN BOOKS Editorial Director Ellen Dupont Senior Editor Sudakshina Basu original styling by STUDIO 8 TOUCAN BOOKS Editorial Director Ellen Dupont Senior Editor Sudakshina Basu original styling by STUDIO 8 TOUCAN BOOKS Editorial Director Ellen Dupont Senior Editor Sudakshina Basu original styling by STUDIO 8 TOUCAN BOOKS Editorial Director Ellen Dupont Senior Editor Sudakshina Basu Fleischer, Isobel Rodel, Gage Rull Additional Text Marcus Weeks Editorial Advisors Tom Le Bas, Robert Snedden Indexer Marie Lorrimer Proofreader Richard Beatty DIGITAL PRODUCTION Digital Programme Manager Rachana Kishore Production Manager Nain Singh Rawat Production Coordinator Manish Bhatt First American Edition, 2019 Published in the United DK, a Divison of Penguin Random House LLC All rights reserved. 200 CE. The frequency of Penguin Random House LLC All rights reserved. 200 CE. The frequency of the note produced by the hammer of weight 6 was double that of the hammer weighing 12, which corresponds with the ratio of their weights. ArceJaeger American author Fierce rivalry The idea of complex numbers first emerged early in Bombelli's lifetime as Italian mathematicians sought to find solutions to cubic equations as efficiently as possible, without relying on the geometrical methods devised by Persian polymath Omar Khayyam in the 12th century. Diophantus only used positive numbers, but mathematicians now look for negative solutions as well. Alamy Stock Photo: Aflo Co. Ltd. Lotfi Zadeh Creating fuzzy sets A basic computer program that mimics the simple human task of soft-boiling an egg might apply a single rule: boil the egg for five minutes. 287-c. It is likely that these equations first arose from the need to subdivide land for inheritance purposes, or to solve problems involving addition and multiplication. As the Greeks developed the use of mathematics in astronomy, they began to use an "O" to represent zero, although it is not clear why. If there are 10 nodes, each with four connections to other nodes, then the average distance between two nodes chosen at random will be ln10/ln4 ≈ 1.66. Birth of a new geometry In 1750, Leonhard Euler revealed that he had been working on a formula for polyhedra—three-dimensional figures with four or more planes, such as a cube or pyramid—that involved their vertices, edges, and faces rather than lines and angles. Their trade and taxation required a sophisticated numerical system, and their building and engineering works relied on both a means of measurement and some knowledge of geometry and algebra. The principles of the Turing machine are still used in modern computers and look set to continue until quantum computing changes how information is processed. Alamy Stock Photo: i creative (br). Heath Historian and mathematical structure that involves the multiplication of vectors by each other and by scalars. Trigonometry Originally, the study of the way the ratios between different sides of a right-angled triangle change, and later extended to all triangles. A computer graphic shows a fractal pattern derived from the Mandelbrot set. For example, in the expressions ax2 and 3x, a and 3 are coefficients. 980-c. Galton was inspired by his cousin Charles Darwin's work on evolution, and his aim was to show how likely it was that factors such as height, physiognomy, and even intelligence and criminal tendencies might be passed from one generation to the next. Rational number A number that can be expressed as a fraction of one whole number over another. 106 In the medieval period, astrolabes applied trigonometric principles to measure the position of celestial bodies. The bell curve is a visual illustration of normal distribution. However, four vertical rods indicated 405 (4 × 100, or 102) + 5 × 1—the absence of horizontal rods meant there were no tens in the number. 470 A Turing Bombe, used to decipher coded messages, has been rebuilt at the museum at Bletchley Park, the British code-breaking center during World War II. Originally developed as a mechanical means of doing the "donkey work" of calculation to provide tables for mathematicians, astronomers and so on, the actual construction of computers required new mathematical thinking. She worked for the National Advisory Committee for Aeronautics (NACA) from 1953 as part of a group of African-American women mathematicians known as the West Area Computers, who later inspired the film Hidden Figures (2016). Bessel's functions are very beautiful functions, in spite of their having practical applications. The number pi (π) and Euler's number e are both transcendental numbers. The point where the line crossed the x axis (where y = 0) gives the value for x in the cubic equation. For example, for the set of 1, 2, 3, where there are six permutations, 6 ÷ e = 2.207... or 2, to the nearest whole number. This 381 sequence (1, 2, 5, 14, 42...) counts, among other things, the ways that polygons can be divided into triangles. The term "topology"—derived from the Greek topos, meaning "a place"—was introduced to the mathematical world by German mathematician Johann Listing in 1847 in his treatise Vorstudien zur Topologie (Introductory Studies in Topology), although he had used the word in correspondence at least 10 years earlier. Kepler taught at the Protestant seminary in Graz, Austria. And even in this small niche of mathematics, there are many magic square developments that there was simply not enough room to include. for manifolds of the fourth, fifth, and higher dimensions than it was for 3-manifolds. 2005 Microsoft researcher Georges Gonthier proves the four-color theorem with general purpose theorem-proving software. The image of any point (x, y) is found by multiplying the matrix by the column vector representing the point (x, y). She set out many ideas about what was to become computing, earning herself a reputation as the first computer programmer. The black keys are in groups of two and three. If the butterfly had not made its small contribution to the initial conditions, then the tornado or other weather event would not have occurred at all, or would have struck some place other than Texas. Like other scholars of his day, he studied geometry problems from the ancient world and applied algebraic methods to try to solve them. Analytic geometry See algebraic methods to try to solve them. Analytic geometry See algebraic methods to try to solve them. out the quadratic equation x2 = 2 by drawing rectangles and trimming them down into squares. In the mid-1950s, Japanese mathematicians Yutaka Taniyama and Goro Shimura had made the bold step of linking two apparently unrelated branches of mathematics. Funded by his estranged father, he studied law and medicine, then turned to mathematics. Algebras are geometric facts which are proved by propositions. An unproved statement is called a conjecture. If the answer was No (A does not halt), then M\* would halt. The Fibonacci sequence turns out to be the key to understanding how nature designs. This simple "on or off" concept has proved vital in computing, for example, where every number can be represented by a series of switchlike on/off actions. Euler's graph shows that it is impossible to construct a route that visits each bridge only once. It includes techniques for solving particular kinds of equations. Although it is almost impossible to visualize such a space, mathematicians can use these tools to describe lines moving in four, five, or as many spatial dimensions as they desire. Scientists tried looking for a particle with that combination of symmetries, and found the Omega minus particle. problem, a version of the conjecture. 2004 Distributed computing is used to prove that the first 10 trillion "nontrivial zeros" lie on the critical line. The numbers (s) for which  $\zeta(s) = 0.1370$  French philosopher Nicole Oresme represents qualities and quantities as lines defined by coordinates. Whole number Any of the negative and positive counting numbers. Mandelbrot mapped each value of c on the complex plane, coloring the connected sets in different colors. As a decimal, this is about 0.6 percent greater than the true value of pi. A quantum bit, or "qubit," uses superposition to be both a 1 and 0 at the same time, which boosts computing power enormously. On average, the letters that reached the target needed six intermediaries. The average number of events per unit of space or time (lambda, or λ) is key. While the Mayans could calculate up to hundreds of millions, their geographical isolation meant that their mathematics never spread to other cultures. See also: Newton's laws of motion • Non-Euclidean geometries • Topology • Minkowski space L. 1079 CE Persian polymath Omar Khayyam uses intersecting conics to solve algebraic equations. It may have been chosen because it can be divided by many other numbers—1, 2, 3, 4, 5, 6, 10, 12, 15, 20, and 30. Al-Karaji used a basic form of this principle, whereby he would show an algebraic statement to be true for the simplest case (say n = 1), then use that fact to show that it must also be true for n = 2 and so on, with the inevitable conclusion that the statement must hold true for n = 2 and so on, with the inevitable conclusion that the statement must hold true for n = 2 and so on, with the inevitable conclusion that the statement must hold true for all possible values of n. Chapters seven and eight deal with solutions to linear equations, including the rule of "double false position," whereby two test (or "false") values for the solution are used in repeated steps to yield the actual solution. Ptolemy, like the mathematicians before him, used the Babylonian system of numbers known as the sexagesimal system, based on the number 60. His work was theoretical—he investigated the structure of polynomial equations to find the circumstances under which a formula could be found for solving them. The circle equation In analytic geometry, all circles centered on the origin can be defined as r = , known as the circle equation. 1591 François Viète introduces symbolic algebra, in which letters are used to abbreviate terms in equations. Getty Images: Mondadori Portfolio / Hulton Fine Art Collection (tr). This method is used in problem 50 to find the area of the square with the resulting side length. Trigonometry is one of the most useful of all the mathematical disciplines, enabling people to navigate the world, to understand electricity, and to measure the height of mountains. Because going forward, the body of mathematics will continue to grow. He died in Castres in 1665. However, binary computer systems are not always well suited for dealing with real-world inputs that are ambiguous or unclear. In rock music, connections to members of the heavy metal group Black Sabbath number." To filter out 479 the truly well-connected, there is the Erdős-Bacon-Sabbath numbers). Probability theory is also used in many other fields, such as psychology, economics, engineering, and sports. By the 1920s, the idea of a probabilistic Universe was solidified with the development of quantum physics, which has uncertainty at its heart. Of the first 20 positive whole numbers, eight are prime—2, 3, 5, 7, 11, 13, 17, and 19. His 13-volume Arithmetica was well-received—the Alexandrian mathematician Hypatia wrote about the first six volumes—but fell into relative obscurity until the 1500s, when interest in his ideas was revived. See also: Trigonometry • Zero IBN AL-HAYTHAM C. He also imagined rotating these vertical slices about a horizontal axis, and worked out the surface area and volume of the disks swept out by this rotation. Starting out with an initial statement that was either true or false, Boole could then construct further statements and use the AND, OR, and NOT operations in order to determine whether or not these further statements were true. The most ambitious and influential exponent of this was French mathematician Henri Poincaré, who used complex topology to throw new light on the "shape" of the Universe itself. The method finds the area of a square whose side length is 8/9 of the diameter, then multiplies this by the height. Practical applications for elliptic functions in both mathematics and physics. I was... struck by the remarkable fact that in geometry all bodies gliding along the cycloid... descend from any point in precisely the same time. The shortest route between two truths in the real domain passes through the complex domain. Many consider Archimedes to be a pioneer of calculus, which was not developed until the 1600s. Theorems that are true for groups are therefore also true for such matrices, and advances in group theory can be applied to matrices. This allowed mathematicians to 145 work on increasingly complex algebraic expressions in a more uniform way, and reinforced algebraic expressions in a more uniform way. The problem was solved by the introduction of the rhumb line by Portuguese mathematician Pedro Nunes in his Treatise on the Sphere (1537). 167 The scales of a pine cone, viewed from above, can be seen to run in two sets of spirals. In 1918, French mathematician Gaston Julia, a former student of Poincaré, explored the concept of self-similarity when he began to map the complex plane (the coordinate system based on complex numbers) under a process called iteration—entering a value into a function. They discovered the basic principles of addition and subtraction—for example, that two things (whether pebbles, berries, or the) when added to another two invariably resulted in four things. The symbols were then repeated to create other numbers. However, after he refused to issue funds to William Herbert, the future Earl of Pembroke, for his army, the mint was closed. Mathematicians at the time did not use letters or symbols, so he wrote his solution in words. but it was similar to the modern formula shown above. AFTER 17th century In Germany, Johannes Kepler notices that the ratio of successive terms in the sequence converges. With decimal fractions, multiplying and dividing by powers of 10 is straightforward—as in the example of 32.567, the decimal separator can be simply moved left or right. Four of these—the dichotomy paradox, Achilles and the tortoise, the arrow paradox, and the stadium paradox—address motion. Both abaci have two decks—the lower deck counting to five, and the upper deck counting the fives. His explorations of number theory included amicable numbers and factorization. While many European currencies were decimalized in the 1800s, it was not until 1971 that decimal currency was introduced in the UK. Problems with cubes The ancient Greeks, who used geometry to work out complex problems, puzzled over cubes. For example, the set of natural numbers to make the full set of real numbers. However, without the PP, many elementary theorems in geometry could not be proved. Proposition 8 If two sides of one triangle are equal in length to two sides of one triangle are equal in length to two sides of one triangle are equal. squares are probably the earliest example of "recreational mathematics." Their exact origin is unknown, but the first known reference, in the Chinese legend of Lo Shu (Scroll of the river Lo), dates from 650 BCE. Books XI to XIII examine three-dimensional solid geometry. At the age of around 40, Zeno 65 traveled to Athens, where he met Socrates. Julia and Fatou mapped these different values on a complex plane, noting which ones converged and which ones diverged. This was impossible without precise calculations of time, so it required an accurate clock to cope with the rolling motion of the waves, which caused wide variations in pendulum swing, leading to time discrepancies. He wrote on perspective and applied mathematics to practical projects, such as designing a spiral staircase and a new form of pump. 1801 Carl Friedrich Gauss uses a matrix of six simultaneous equations to compute the orbit of the asteroid Pallas. continuously changing quantities. A and E are contrary (both cannot be false); I and O are subcontrary: both can be true but both cannot be false); I and O are subcontrary: both can be true but both can be false); I and O are subcontrary: both can be true but b as a "3sphere." A "2-sphere" is equivalent to a "normal" sphere (such as a ball) in a 3-D space. 1895 Henri Poincaré publishes his paper Analysis situs, in which graph theory is generalized to create a new area of mathematics known as topology (the study of properties of geometrical figures that are not affected by continuous deformation). This is the difference between calculation and computation—and the basis of the modern computer. Cardinal numbers Numbers that denote a quantity, such as 1, 2, 3 (in contrast to ordinal numbers). A graph of the quadratic function y = ax2 + bx + c creates a U-shaped curve called a parabola. These depict relations of inclusion (AND) and exclusion (NOT) between sets. Therefore, any mathematical model based on those uncertain measurements will very possibly develop in a different way from the real thing. Self-similarity is even suggested in the poem "Auguries of Innocence" by the 19th-century British poet William Blake, which begins with the line "To see a world in a grain of sand." The work of the Japanese artist Katsushika Hokusai, with its swirling repeated motifs, is often cited as an example of fractal use in art, as is the architecture of Catalan artist Antoni Gaudí. One area of interest was in the relationship between mathematics, proportion, and beauty. Early abaci 83 The word "abacus" may hint at its origins. Tension rose as he piled his results one on top of the other, with only one end in view. Between the numbers 1,000,000 and 1,020, there are only three prime is 1,000,003. In 1933, he moved to the United States to take up a post at the Institute of Advanced Learning, Princeton, New Jersey, and became a US citizen in 1937. 1320-82 Born in Normandy, France, probably to a peasant family, Oresme studied at the College of Navarre, where pupils from poor backgrounds were subsidized by the royal estate. 1827-28 Niels Abel independently derives and publishes the same findings as Gauss. OMAR KHAYYAM Born in Nishapur, Persia (now Iran), in 1048, Omar Khayyam was educated in philosophy and the sciences. The three-body problem Nevertheless, Newton found a flaw with this deterministic view of the Universe. Proposition 2 To place at a given point (as an extremity) a straight line. He extended his mix of geometry and algebra to solve cubic equations using circles, hyperbolas, and ellipses, but never explained how he constructed them, simply saying he "used instruments." Khayyam was among the first to realize that a cubic equation could have more than one root, and therefore more than one root, and therefore more than one solution. of scholars based at Merton College, Oxford, who developed the mean speed theorem, which Oresme later proved. Number system Any system of writing down and expressing numbers. In 421 1896, Jacques Hadamard in France and Charles-Jean de la Vallée Poussin in Belgium both proved the theorem, guite independently. In the 16th century, when Welsh doctor and mathematician Robert Recorde began his work, there was little consensus on the notation used in arithmetic. 559 Circumference The distance all the way around the outside edge of a circle. According to the law, as you make more observations of an event occurring, the measured probability (or chance) of that outcome gets even closer to the 286 theoretical chance as calculated before any observations began. Like the Weierstrass function, this "Cantor set" was considered unsettling by the mathematical establishment, who branded these new shapes "pathological"— meaning "lacking usual properties." In 1904, Swedish mathematical establishment, who branded these new shapes "pathological"— meaning "lacking usual properties." In 1904, Swedish mathematical establishment, who branded these new shapes "pathological"— meaning "lacking usual properties." In 1904, Swedish mathematical establishment, who branded these new shapes "pathological"— meaning "lacking usual properties." In 1904, Swedish mathematical establishment, who branded these new shapes "pathological"— meaning "lacking usual properties." In 1904, Swedish mathematical establishment, who branded these new shapes "pathological"— meaning "lacking usual properties." In 1904, Swedish mathematical establishment, who branded these new shapes "pathological"— meaning "lacking usual properties." In 1904, Swedish mathematical establishment, who branded these new shapes "pathological"— meaning "lacking usual properties." In 1904, Swedish mathematical establishment, who branded these new shapes "pathological"— meaning "lacking usual properties." In 1904, Swedish mathematical establishment, who branded these new shapes "pathological"— meaning "lacking usual properties." In 1904, Swedish mathematical establishment, who branded these new shapes "pathological"— meaning "lacking usual properties." In 1904, Swedish mathematical establishment, who branded these new shapes "pathological"— meaning "lacking usual properties." In 1904, Swedish mathematical establishment, who branded these new shapes "pathological"— meaning "lacking usual properties." In 1904, Swedish mathematical establishment, who branded these new shapes "pathological"— meaning "lacking usual properties." In 1904, Swedish mathematical establishment, who branded these new shapes "pathological"— meaning "lacking usual properties." In 1904, known as the Koch curve or "Koch snowflake," which repeated a triangular motif at an ever smaller size. They include Numbers: How Counting Changed the World; Everything is Mathematical, a book series with Marcus du Sautoy; and Help Your Kids with Science with Carol Vorderman. I have to do as painters do, stand back and view it from a distance, but not too great a distance. When rods of different colors were added together, they canceled each other out—like income erasing a debt. The individual terms in the series define the different ways that a stretched string, for 563 example, or air in a tube, can vibrate to produce sound. See also: Group theory • The Riemann hypothesis • The Turing machine • Information theory • Proving Fermat's last theorem 519 IN CONTEXT KEY FIGURE Daniel Gorenstein (1923-92) FIELD Number theory a simple group. Khayyam did not use coordinates and axes (which were invented about 600 years later). Hipparchus is credited with inventing the device. 1746 The first attempt at a proof of the fundamental theorem of algebra (FTA) is made by Jean d'Alembert. P-functions are used in classical and quantum mechanics. 157 Using modern conventions, cubic equations can be expressed simply, such as x3 + bx = c. The works of al-Khwarizmi and his successors were key to establishing algebra as a discipline in its own right. He is especially remembered for the elegant equation known as "Euler's identity," which highlights the connection between fundamental mathematical constants such as e and  $\pi$ . Returning to London, he studied law and was admitted to the bar in 1850. 1050 In China, Jia Xian, describes the triangle later known as Yang Hui's triangle. The fourth axiom is closure, meaning that a group should have no elements outside the group as a result of performing the operations. Parabolas A function is a group of terms that defines a relationship between variables (often x and y). American in space (1961), John Glenn to be the first American to orbit Earth (1962), Apollo 11 to land on the Moon (1969), and the Space Shuttle program to launch (1981). 134 Brahmagupta's use of the decimal number system and the algorithms he devised spread throughout the world and informed the work of later mathematicians. As ideas from China spread along trade routes such as the Silk Road, other cultures became interested in magic squares. Rüdiger Thiele German mathematician DAVID HILBERT Born in Prussia in 1862 to German mathematics at the University of Königsberg in 1880 and later taught there before becoming professor of mathematics at the University of Göttingen in 1895. Mostly, Poincaré argued, the three bodies do not retrace their paths, and their movement is called aperiodic. Shortly after his release in 1832, he became involved in a duel-it is unclear whether this was over a love affair or politics. In 1656, Dutch physicist and mathematician Christiaan Huygens created the pendulum clock, a clock with a swinging weight that was constant. 350 BCE On Interpretation 335-323 BCE Nichomachean Ethics 335-323 BCE Politics See also: Pythagoras • Zeno's paradoxes of motion • Euclid's Elements • Boolean algebra • The logic of mathematics 74 IN CONTEXT KEY FIGURE Euclid (c. In a base-10 23 system, a two-digit number, such as 22, indicates (2 × 101) + 2; the value of the 2 on the left is ten times that of the 2 on the right. The high-precision mechanics required to build the machine were beyond what any engineer could achieve at the time. Perlman has taught at MIT and the universities of Washington and Harvard, and continues to work on computer network and security protocols. They were housed in a great library, the House of Wisdom, which became a center for research and the data from each section of the tape (either by the head or tape moving). As well as his ideas on 253 probability, Fermat pioneered differential calculus, but is best remembered for his contribution to number theory and Fermat's last theorem. The second, more ambitious, idea was the Analytical Engine. 1979 Manfred Kochen of IBM and Ithiel de Sola Pool at MIT publish a mathematical analysis of social networks. William Rowan Hamilton WILLIAM ROWAN HAMILTON Born in Dublin in 1805, Hamilton became interested in mathematics from the age of eight after meeting Zerah Colburn, a touring American mathematical child prodigy. AFTER 1922 British mathematical child prodigy and the treatise on the theory of Bessel functions. This output is then defuzzified to give a crisp logical output that can be used by the control system. 1893 Charles Sanders Peirce uses truth tables to show outcomes of Boolean algebra. Freqe declared that he was "thunderstruck," and he never found an adequate solution to the paradox. The multiples of 11 up to 100 have already been removed, and so on for all successive numbers. He found that while a vibrating string produces a note with frequency f, halving the length of the string produces a note an octave higher, with frequency 2f. In the late 1600s, Swiss mathematician Jacob Bernoulli 's brother Johann, who first called the number e. Although early Greek mathematics was largely geometry-based, Diophantus developed new algebraic methods in the 3rd century CE, and was the first to use symbols for unknown quantities. R. 1735 Leonhard Euler devises a formula that links the faces, vertices, and edges of polyhedra. In an arithmetic sequence, each number differs from the one preceding it by a constant quantity, such as 1, 2, 3, 4, 5, 6... (going up by 1), or 3, 6, 9, 12... (going up by 3). This seemed impossible, because a negative number multiplied by itself produces a positive result. The quadratic function can predict projectile motion—the height of the object over time. The first two propositions, called the premises, logically entail the third proposition, the conclusion. Although there are two primes between 30 and 40, and three between 40 and 50, the accuracy of the prime number theorem increases at higher number of permutations in a set is equal to the factorial of the highest integer, in this case 3! (short for 3 × 2 × 1). Science Photo Library: Royal Astronomical Society (cra). Mary Everest Boole British mathematician and wife of George Boole Logic is the bedrock of mathematics. After crossing the Mediterranean to Egypt, he was taught by Euclidean scholars in the great cultural city of Alexandria. All the other points on the circle can be seen as one corner of a right-angled triangle moving around in a circle. The machine's algorithm might either instruct the head or the tape to move—to go left, right, or stay still. However, in the early stages, heads and tails are likely to be more unbalanced. Topology is, in simple terms, the study of shapes without measurements. 287 BCE in Syracuse, Sicily, the Greek polymath Archimedes excelled as a mathematician and engineer, and is also remembered for his "eureka" moment, when he realized that the volume of that object. 426 The computer age In the first half of the 1900s, applied mathematics was largely concerned with theoretical physics, especially the implications of Einstein's theories of relativity, but the latter part of the century was increasingly dominated by advances in computer sciences. The Caesar cipher and key are used (in reverse) to decipher the message. Linear equation An equation that contains no variable multiplied by itself (for example, no x2 or x3). He used the term "plus of minus" to describe a positive imaginary unit and "minus" to describe a negative imaginary unit. It is likely that she helped her father, 124 Theon, a respected Alexandrian scholar, to produce his definitive edition of Euclid's Elements, and his Almagest and Handy Tables of Ptolemy. In the Babylonian sexagesimal system, a single symbol was used alone and repeated up to nine times to represent symbols for 1 to 9. Euler was convinced that Goldbach was right, but he could not prove it. 850)— whose theories, particularly on algebra, influenced later European mathematicians—was familiar with the rules of Brahmagupta and understood the use of negative numbers for dealing with debts. Black holes are where God divided by zero. Among his achievements, he established the beginnings of a link between algebra and geometry, building on the work of Euclid and trying to complete the lost eighth volume of Apollonius of Perga's Conics. This chart shows how to write the numbers 1 to 10, from the base-10 system, as both binary numbers and binary visuals—which is how a computer would process them—where 1 is "on" and 0 is "off." Binary power revealed In 1617, Scottish mathematician John Napier announced a binary calculator based on a chessboard. Isaac Newton 264 As civilizations developed, accurate measurement became essential. (2) A mathematical expression used in integral calculus, or the result of performing an integration. GEORGES-LOUIS LECLERC, COMTE DE BUFFON Born in Montbard, France, in 1707, Georges-Louis Leclerc was urged by his parents to pursue a career in law, but was more interested in botany, medicine, and mathematics, which he studied at the University of Angers, France. In Boolean algebra, OR behaves like + (aside from 1 + 1 = 1) and AND behaves like × (see Logic gates). For example, a 1.76 oz egg would have a membership degree of 0.5 for both sets, while an 2.82 oz (80 g) egg would be "large" with degree nearly 1, and also "small" with degree nearly 0. This revived the link between philosophy and mathematics. In China, 3 was often became common from the 2nd century CE. 189 The rhumb spiral A rhumb line cuts across every meridian (line of longitude) at the same angle. In ancient Babylon, the area of a circle was found by multiplying the square of the circumference by 1/12, implying that the value of π was 3. AFTER 1901 Swedish mathematician Helge von Koch proves that the best possible version of the prime counting function relies on the Riemann hypothesis. He later became dean of Rouen Cathedral. 418 The BlackDog<sup>™</sup> robot is designed to carry loads over rough terrain. In the 18th century, French mathematician Abraham de Moivre made an important step forward in statistics; building on Jacob Bernoulli's discovery of binomial distribution, de Moivre showed that events cluster around the mean (b on graph below). Lagrange had made his name working with Euler, but later made important contributions to polynomials and number theory. One derivation of the Latin square—Sudoku—has

become a popular puzzle. 1000 BCE In China, bamboo rods are first used to denote numbers, including negatives. Leonhard Euler Rational answers Many of the equations that could not be expressed rationally and completely using the Hindu-Arabic decimal —the square root of 2—had been known system. While its dimensions and angles may change, collinearity is preserved; this means that if three points XYZ are on a straight line, with Y between X and Z, then their images xyz are also on a straight line, with Y between X and Z. This method was similar to the elimination system introduced by German mathematician Carl Gauss in the 1800s. which is still used today for solving simultaneous equations. After his employer's death in 1673, he took up the role of librarian to the Duke of Brunswick in Hanover. Non-Euclidean geometry, as described by Euclid in ancient times, is that parallel lines never meet (often expressed as meeting at infinity). Advances in astronomy also demanded sophisticated calculations. 1640 Pierre de Fermat states his "little theorem" (on primality), which is still used as a test when searching for primes to use in public key encryption. Takebe Katahiro Japanese mathematician See also: The Rhind papyrus • Irrational numbers • Calculating pi • Euler's identity Buffon's needle experiment 127 128 INTRODUCTION As the Roman Empire collapsed and Europe entered the Middle Ages, the center of scientific and mathematics in ancient Egypt. The topic gained popular appeal in 1982 with the publication of his book The Fractal Geometry of Nature. On the other hand, even these near-solutions are rare. The binary logic of any computer is clear: given valid inputs, it will provide appropriate outputs. Proving the Riemann hypothesis would solve many other theorems. Halting would signify that the algorithm had arrived at a solution. Thompson proved that, with the exception of trivial groups (for example, 0 + 0 = 0, or 1 × 1 = 1), all simple groups have an even number of elements. The abacus is a counting device and calculator that has been in use since ancient times. It includes differential calculus, which is concerned with rates of change, and integral calculus, which calculates areas and volumes under curves or curved surfaces. Even if he did not invent it, Hipparchus is the first person of whose systematic use of trigonometry we have documentary evidence. Al-Khwarizmi was not working in a total vacuum, as he had the translated works of earlier Greek and Indian mathematicians at his disposal. After graduating in math and physical sciences from Hampton University, Virginia, Jackson taught for a while, then in 1951 started work in the West Area Computing Unit of the National Advisory Committee for Aeronautics (NACA). 407 Adding to that set makes a new set of  $\omega + 1$ . Emma Haruka Iwao Japanese computer scientist 93 The perimeter to height ratio of the Great Pyramid of Giza, in Egypt, is almost exactly π, which might suggest that ancient Egyptian architects were aware of the number. 1946 British statistician R. Although much admired by today's mathematicians, Cantor was something of a pariah among his contemporaries. While = 2, it is also true that - ×- × = 2. So too did the Protestant Reformation that challenged the hegemony of the Catholic Church in the 1500s. 185 The ratio of beauty based on white, Western models. Fibonacci sequence is particularly interesting. Lorenz had been late to provide information about his proposed talk, so Merilees had improvised, basing his choice of words on what he knew of Lorenz's work and an earlier comment that "one flap of a seagull's wings" could be enough to change the weather forecast. of truth values for a proposition—not just completely true or completely false, the two values of Boolean logic. Therefore, some Europeans eat cheese." As well as being a widely used tool for sorting data in everyday life, in contexts ranging from school classrooms to boardrooms, Venn diagrams are an integral part of set theory, due to their distinctive ability to express relationships. 1665 Isaac Newton develops the general binomial series from the binomial theorem, forming part of the numbers, around 30 percent of the numbers have a leading digit of 1, around 17 percent have a leading digit of 2, and less than 5 percent 473 have a leading digit of 9. Yet the debate about whether to use the new system rather than Roman numerals and traditional counting methods lasted for several hundred years, before its adoption paved the way for modern mathematical advances. As a result, the 2.82 oz egg would be assigned a boiling time of nearly 6 minutes. Every square matrix has an associated value called its determinant, which encodes many of the matrix's properties and can be computed by arithmetic operations on the matrix's elements. A fraction was shown as a number with a dot above it. Sent to a London prison in 1557 for failure to pay the fine, Recorde died there in 1558. Mesopotamian clay tablets from c. Charting the development of maths around the world from Babylon to Bletchley Park, this book explores big questions like these and explains how the answers help us understand everything from patterns in nature to artificial intelligence. Written in clear English, The Maths Book is packed with short, pithy explanations that cut through the jargon, step-by-step diagrams that untangle knotty theories, memorable quotes, and witty illustrations that play with our ideas about numbers. This diverse and inclusive account of mathematics offers insights into the maths behind world economies and espionage. To create a Platonic solid, a minimum of three identical polygons must meet at a vertex, so the simplest is a tetrahedron— a pyramid made up of four equilateral triangles. 250 BCE Archimedes uses the law of exponents, which states that multiplying exponents can be achieved by adding the powers. The failure of the Riemann hypothesis would create havoc in the distribution of prime numbers. Square matrices Because of their symmetry, square matrices have particular properties. Key works 1897 Commentary on Numbers 1900 "The Problems of Mathematics" (Paris lecture) 1932-35 Collected Works 1934-39 Foundations • Euler's number • The Goldbach conjecture • The Riemann hypothesis • Transfinite numbers 432 IN CONTEXT KEY FIGURE Francis Galton (1822-1911) FIELD Number theory BEFORE 1774 Pierre-Simon Laplace shows the expected pattern of distribution around the norm. (Norwegian cartographer Casper Wessel is now known to have used similar constructions in 1799.) Argand's essay was republished in a mathematics journal in 1813, and in the next year, he used the complex plane to produce the first rigorous proof of the fundamental theorem of algebra. The individual lines within the attractor are highly unstable in their trajectories; those that start in the same area often move far apart at a later point, and lines with very different starting points may end up tracking each other closely for long periods. Like the ancient Chinese, Brahmagupta looked at numbers in financial terms, as "fortunes" (positive) and "debts" (negative), and stated the following rules for multiplying with positive and negative quantities: The product of two fortunes is a fortune. By contrast, numbers such as 4 (22), 9 (32), and 16 (42) are square numbers, with square roots that are also whole numbers and therefore rational. There are three derangements starting with 2: 2, 1, 4, 3; 2, 3, 4, 1; and 2, 4, 1, 3. Unlike most mathematicians, who like to work collaboratively, Wiles decided to pursue this goal on his own, telling no one except his wife. From its early beginnings, when the Babylonians and ancient Egyptians pondered the lengths of shadows cast by a stick in the ground, through architecture and astronomy to modern applications, trigonometry has formed a part of the language of mathematicians, who are driven by curiosity, believe there is still more to discover. The second axiom is the inverse axiom The letters a, b, and c represent known numbers, while x represents the unknown number. Today, the largest known Mersenne prime is 282,589,933 -1. EUCLID Details of Euclid's date and place of birth are unknown and knowledge of his life is scant. It was not until the 1500s, however, that mathematicians began to abbreviate equations by using letters to stand for known and unknown variables. Analysis The branch of mathematics that studies limits and handles infinitely large and small quantities, especially to solve problems in calculus. As well as the sections described by Apollonius, this can be a single point, where the plane cuts across the apex (top vertex), or straight lines cutting through the apex at an angle. In 1843, in a flash of inspiration, he realized that the "third dimensional number, but needed a four-dimensional one (a quaternion). The smaller the segments used, the closer the shape is to a rectangle. The development of arctangent series by Scottish astronomer mathematician James Gregory in 1671, and independently by Gottfried Leibniz in 1674, provided a new approach for finding π. The book was illustrated by Pacioli's friend and colleague Leonardo da Vinci, a leading artist and polymath of the Renaissance. These problems remained unresolved until 1629, when Italian mathematician and Jesuit priest Bonaventura Cavalieri found a method for calculating the areas and volumes of curved shapes by slicing them into parallel pieces (Cavalieri's 235 principle), although he did not publish his results until six years later. 1690 Gottfried Leibniz creates classification circles. In his Principia, Newton used mathematics to show that the consequences of gravity were consistent with what had been observed experimentally. 700 CE The Indian poet and mathematician Virahanka writes about the sequence. To work out lengths and areas, equations involving variables (unknown quantities) and squared terms were 139 devised. Versions with eight disks are often used to test developmental skills of older children. Achieving fame in such contests became essential for any scholar who wanted to gain a post as a mathematics professor at a prestigious university. It is from this time onward that his theorem and other discoveries were probably set down, although no records remain. Fermat's last theorem, for example, remained a conjecture (unproven claim) for 357 years. However, it was not published until 1862. 163 A problem with rabbits One of the problems Fibonacci raised in Liber Abaci concerned the growth of rabbit populations. 1738 Leonhard Euler proves that 8 and 9 are the only consecutive square or cube numbers. Placing digits after the number 22 will create hundreds, thousands, and larger powers of 10. He died around 194 BCE. 1697 Gottfried Leibniz tries, unsuccessfully, to use algebra to formalize logic. Alamy Stock Photo: The History Collection (tr); Jochen Tack (cla). The logical answer was to add a third number line, j, which ran at 90 degrees to both the real and imaginary number lines, but no one could figure out how such a number added, multiplied, and so on. Integers added in any order will give the same result (6 + 7 = 13 and 7 + 6 = 13), so the set of integers with the operation of addition is an Abelian group. Each starting point leads to a unique evolution, but all the lines, whatever the start point, fall into the same region of the space. This method of solving problems by honest confession of one's ignorance is called Algebra. What he postulated became known as Euler's polyhedral formula: V + F E = 2, where V is the number of faces, and E the number of races, and E the number of edges. In 1856, he tried to capitalize on his skills by launching the icosian game, in which players search for a path connecting the points of a dodecahedron without returning to the same point twice. His 1784 paper persuaded Congress to 206 introduce a decimal system for money using dollars, dimes, and cents. He used his knowledge of forces and German astronomer Johannes Kepler's laws of planetary motion to deduce how elliptical orbits were related to the laws of gravitational attraction. Some French people eat cheese. 2002 Preda Mihăilescu proves Catalan's conjecture, 158 years after it was formulated in 1844. Langlands began studying the relationship between integers and periodic functions as part of research into patterns in prime numbers. For example, a square sail under pressure from the wind takes the form of a catenary. Brahmagupta (c. Alamy Stock Photo: National Geographic Image Collection (tl). The notes made by the hammers of weights use in the ratio 3:2. Published in Great Britain by Dorling Kindersley Limited A catalog record for this book is available from the Library of Congress. 1718 Abraham de Moivre defines the statistical independence of events in his book The Doctrine of Chances. From East to West In Europe, mathematical study was under the control of the Church, and was confined to a few early translations of some of Euclid's work. 283 Laws of motion Newton began Principia by stating his book The Doctrine of Chances. three laws of motion. It revolutionized ways of calculating algebraic problems, introducing principles that are the foundation of modern algebra. As well as the individual thinkers and their ideas, it is a story of societies and cultures, a continuously developing thread of thought from the ancient civilizations of Mesopotamia and Egypt, through Greece, China, India, and the Islamic empire to Renaissance Europe and into the modern world. See also cardinal numbers. He also presents television and radio programs for Discovery Channel and the BBC, and his 2019 book Humble Pi: A Comedy of Maths Errors topped the Sunday Times best-seller chart. Although a matrix contains many elements, it is treated like one unit. 214 It was through work such as that of Mengoli that the important concept of the exponential function came to light. Brahmagupta A new algebra still form the basis of algebra today. Also in the 1500s, Dutch cartographer and mathematician Gemma Frisius used trigonometry to determine distances, thus enabling accurate maps to be created for the first time. EDWARD LORENZ Born in 1917, in West Hartford, Connecticut, Edward Lorenz studied mathematics at Dartford College and Harvard University, gaining a masters degree at Harvard in 1940. See also: Calculus • The fundamental theorem of algebra FLORENCE NIGHTINGALE 1820-1910 Named after her Italian birthplace, Florence Nightingale was a British social reformer and pioneer of modern nursing, who based much of her work on the use of statistics. 1913 The infinite monkey theorem explains that given infinite time, random input will eventually produce all possible outcomes. PIERRE SIMON LAPLACE Born into an aristocratic family in 1749, Laplace lived through the French Revolution and the Reign of Terror, in which many of his friends were killed. The spread of Islam through parts of India in the 8th century led to Indian mathematicians sharing their knowledge, including the concept of zero, with scholars in the Arab world. Alamy Stock Photo: David Lyons (bl). The Pythagoreans developed methods for finding sets of triples, and The smallest, or most also proved that there are an infinite number of such primitive, of the sets. 1600 François Viète lays the foundations for solutions of Diophantine equations. This results in a total of five pairs of rabbits. Two centuries later, Euclid developed a formula to generate triples: a = m2 - n2, b = 2mn, c = m2 + n2. Kepler also noticed that the closer to the maximum the shape gets, the less the rate at which the volume increases: an observation that contributed to the birth of calculus, opening up the exploration into maxima and minima. They are tested on their speed and accuracy in a knockout system similar to a spelling bee. In France, the leading academy was that of the Abbé Marin Mersenne, a Jesuit priest and mathematician who held weekly meetings at his Paris home. Such ciphers were first developed in the 1500s, but the most famous one was the encryption produced by the Enigma forces in World War II. A natural logarithm (ln)—socalled because it occurs naturally, revealing the time required to reach a certain level of growth—has a special base, later known as e, with an approximate value of 2.71828. Manifold A kind of abstract mathematical space that in any particular small region resembles ordinary 3-D space. This becomes Proposition 31 of Euclid's Elements. 1650 BCE The Egyptian Rhind papyrus includes a method for calculating the slope of a pyramid. 1847 Johann Listing publishes Vorstudien zur Topology). However, from the 12th century onward, during the slope of a pyramid. 1847 Johann Listing publishes Vorstudien zur Topology). recognized the wealth of scientific knowledge Islamic scholars had amassed. In 1220, Leonardo of Pisa (Fibonacci), who popularized Hindu-Arabic numerals in his book Liber Abaci (The Book of Calculation), 1202, computed π to be 864/275 = 3.141, a small improvement on Archimedes's approximation, but not as accurate as the calculations of Ptolemy, Zu Chongzhi, or Aryabhata. In 1890, French mathematician Henri Poincaré showed that there was no generalized, predictable way in which three bodies move around each other. Other philosophers, notably Zeno of Elea, applied logic to the foundations of mathematics, exposing the problems of infinity and change. (Fractions are not integers.) Integral (1) Relating to integers. At around the same time as the ancient Babylonians, Egypt's mathematicians of her time, Noether laid the foundations for the development of the entire field of abstract algebra with her contributions to ring theory. Dirk Struik Dutch mathematician The key coordinates The Cartesian coordinate system is by no means the only one. Someone who worked with a coauthor (but not with Erdős directly) would have an Erdős number of 2, and so on. Platonic solids also display duality: the vertices of one polyhedron correspond to the faces of another. Using his data on gambling, Pearson calculated a table of probability values, called chi-squared (x2), in which 0 shows no significant difference from expected (the "null hypothesis"), whereas larger values show a significant difference from expected (the "null hypothesis") and the simplest length to use for the larger of the two small squares is 4, because the problem deals with quarters. He survived, but with serious facial injuries and a speech impediment, which earned him the nickname "Tartaglia," or stutterer. The numbers in the Fibonacci sequence follow Benford's law, as do the powers of many integers. Among his beliefs was the idea of metempsychosis, which he may have encountered on his travels in Egypt or elsewhere in the Middle East. It might seem that Catalan's conjecture must be false, since simple calculations quickly yield examples of powers that are almost consecutive. Cuneiform, a word derived from the Latin cuneus ("wedge") to describe the shape of the symbols, was inscribed into wet clay, stone, or metal. By working out the value of x for which both these simpler equations are true simultaneously, he could solve the original cubic equation. 352 To set up the machine for a calculation, each number was represented by a column of cogwheels, and each cogwheel was marked with digits from 0 to 9. An object resting on a table pushes down on it, and the table pushes back with an equal force. Transformation The conversion of a given shape or mathematical expression into another related one, using a particular rule. Geometries in which this and other Euclidean postulates are not valid are called non-Euclidean. I shall call [the imaginary unit] 'plus of minus' when added and when subtracted, 'minus of minus.' Rafael Bombelli Beyond positive numbers. In 2006, American mathematicians Dmitry Khavinson An Einstein ring, first and Genevra Neumann proved that there was an discovered in 1998, is the upper limit to the number of roots of a certain class deformation of light from a source into a ring of harmonic polynomials. A further question was whether Ta(n) exists for all values of n; in 1938, Hardy and British mathematician Edward Wright proved that it does (an existence proof), but developing a method of finding Ta(n) in each case has proved that it does (an existence proof), but developing a method of finding Ta(n) in each case has proved elusive. attempts to understand and predict motion, especially of heavenly bodies. A starting point would be to imagine that we inhabit a Universe that is a 3-D surface embedded within four dimensions. Alamy Stock Photo: Science History Images (br). Euler calculated e to 18 decimal places, writing his first work on e, the Meditatio (Meditation), in 1727. Without limiting the rights under the copyright reserved above, no part of this publication may be reproduced, stored in or introduced into a retrieval system, or therwise), without the prior written permission of the copyright owner. Lambert also invented the first practical hygrometer, used to measure the humidity of air. The equilateral triangle construction is a good example of Euclid's method. This reasoning based on the causes of observed events—rather than using direct probabilities, such as the 50 percent chance of a heads coin toss—became known as inverse probability. One method is to change a digit in a recurring sequence. Thales of Miletus, Hippocrates, and Plato (among others) had all begun to move toward the mathematical mindset that Euclid so brilliantly formalized: the world of proof. In 1907, she was the first woman to receive the Order of Merit, Britain's highest civilian honor. This approach made logarithms much easier to use. Since this shark-fin shape (left) and triangle (right) are the same height and the same width at equivalent points along their height, Cavalieri's principle states that have similar area. Mandelbrot had become interested in the work of Julia and Fatou while working at the IT company IBM. The Fibonacci sequence can be defined by a recurrence relation—an equation that defines a number in a sequence in terms of its previous numbers. The group was often at odds with cutting-edge mathematics and physics, and was so focused on pure math that applied mathematics and physics. developments in calculus were formalized in 1823 when AugustinLouis Cauchy formally stated the fundamental theorem of calculus. This idea was developed by the Greek geometer Apollonius of Perga about 1,750 years before Descartes' birth. The puzzle was allegedly first set as a challenge on the back of a cereal box. 1949 Paul Erdős in Hungary and Atle Selberg in Norway both find a proof of the theorem using only number theory. The formula (1 + 1/)n gives closer values n n for e as n increases. He also came up with a famous example to illustrate impossibility: monkeys hitting typewriter keys at random will eventually type the complete works of Shakespeare. Icosahedron A 3-D polyhedron made up of 20 triangular faces. He suggested a notation system for decimal fractions, replicating the advantages of the Indian place-value system for whole numbers. At the age of 22, while still studying at Trinity College, Dublin, he was appointed both professor of astronomy at the university and Royal Astronomer of Ireland. Shannon's solution was to divide information into the smallest possible chunks, or "bits" (binary digits). In the 3rd century, Liu Hui approached the task using the same method as Archimedes—drawing regular polygons with increasing numbers of sides inside 126 and outside a circle. Yet the Difference Engine was designed to handle numbers of up to 50 digits by means of more than 25,000 moving parts. The calculator could multiply, divide, and even find square roots, but was considered a mere curiosity. These signs had first appeared in print in Mercantile Arithmetic (1489), by German mathematician Johannes 191 Widman, but were probably already used by German mathematician Johannes 191 Widman, but were probably already used by German mathematician Johannes 191 Widman, but were probably already used by German mathematician Johannes 191 Widman, but were probably already used by German mathematician Johannes 191 Widman, but were probably already used by German mathematician Johannes 191 Widman, but were probably already used by German mathematician Johannes 191 Widman, but were probably already used by German mathematician Johannes 191 Widman, but were probably already used by German mathematician Johannes 191 Widman, but were probably already used by German mathematician Johannes 191 Widman, but were probably already used by German mathematician Johannes 191 Widman, but were probably already used by German mathematician Johannes 191 Widman, but were probably already used by German mathematician Johannes 191 Widman, but were probably already used by German mathematician Johannes 191 Widman, but were probably already used by German mathematician Johannes 191 Widman, but were probably already used by German mathematician Johannes 191 Widman, but were probably already used by German mathematician Johannes 191 Widman, but were probably already used by German mathematician Johannes 191 Widman, but were probably already used by German mathematician Johannes 191 Widman, but were probably already used by German mathematician Johannes 191 Widman, but were probably already used by German mathematician Johannes 191 Widman, but were probably already used by German mathematician Johannes 191 Widman, but were probably already used by German mathematician Johannes 191 Widman, but were probably already used by German mathematician Johannes 191 Widman, but were probably already used by German math He felt that to talk openly about working on Fermat would stir up excitement in the mathematics community, and perhaps lead to unwanted competition. This means that when you trace your finger around the complete strip, the left and right sides of the paper are reversed. A set with a first item, and ends with a with a first item, and ends with a with a first item. (omega) item, a transfinite ordinal number. As it evolved, mathematics was also seen to comprise several distinct but interconnected fields of study. It would have fitted well with India's number • Normal distribution • The birth of modern statistics 348 IN CONTEXT KEY FIGURE Friedrich Wilhelm Bessel (1784-1846) FIELD Applied geometry BEFORE 1609 Johannes Kepler shows that the orbits of the planets are ellipses. In 1934, two young lecturers at the University of Strasbourg, André Weil and Henri Cartan, took matters into their own hands. The Greeks understood the importance of primes as the building blocks of all positive integers. In 1949, 2,037 digits of n were calculated in 70 hours. Rhie posited that there would be a maximum number of images produced; this turned out to be exactly the upper bound found by Khavinson and Neumann. 290 The mathematical constant that became known as e, or Euler's number -2.718... to an infinite number of decimal places—first appeared in the early 1600s, when logarithms were invented to help simplify complex calculations. Del Ferro tackled equations of the form x3 + cx = d. All the questions posed in Fermat's marginal scribbles were later solved, except for one. More generally, a graph consists of a discrete set of nodes (or vertices) connected by arcs (or edges). Quadrilateral Any flat 2-D shape with four straight sides. He passed his technique on to only two people, Antonio Fior and Annibale della Nave, swearing them to secrecy. The first he called the Difference Engine, a calculating machine that would work automatically, using a combination of brass cogs and rods. In 1744, Euler described a catenoid— shaped like a waisted cylinder and produced by rotating a catenary around an axis. Extending the concept The taxicab story inspired later mathematicians to examine the property that Ramanujan had recognized and to expand its application. The 2 × 2 matrices below are square matrices, meaning that they have the same number of rows as they have columns. Matrices are rectangular arrays (grids) of elements (numbers or algebraic expressions), arranged in rows and columns enclosed by square brackets. Working from a hypothetical solution to the Fermat equation, he constructed a curious elliptic curve that appeared not to be modular. The with a "handshake," in which the devices agree on a way of securing their connection. French mathematician Edouard Lucas is believed to have invented his Tower of Hanoi game in 1883. In 1748, Agnesi became the first woman to write a math textbook, the two-volume Institutions), which covered arithmetic, algebra, trigonometry, and calculus. 2009 Statistical analysis of the Iranian presidential election results shows that they do not conform to Benford's law, suggesting that the election may have been rigged. Apollonius of Perga A new geometry Early Greek mathematicians such as Euclid focused on the line and circle as the purest geometric forms. From the simple concept of a system of numbers, and even the study of the properties of numbers, and even the study of the very concept itself. Infinite series A mathematicians in ancient China realized the importance of π (pi)—the ratio of a circle's circumference to its diameter—in geometric and other calculations. Mathematicians, however, require a definitive proof. Matrix arithmetic In 1850, British mathematicians, however, require a definitive proof. He term "matrix" to describe an array of numbers. A value of 0 indicates that a disk is on the starting pole; 1 shows that it is on the final pole. He died in 1620. Its origins lie in the efforts of 18th-century mathematicians such as Pierre-Simon Laplace to identify observational errors in astronomy. 209 Solving by series In the 1400s, French mathematicians such as Pierre-Simon Laplace to identify observational errors in astronomy. never existed. Humankind has been exploring and discovering mathematics for millennia. It often looks for a linear relationship—that is, a relationship—that is, a relationship that gives a simple line on a graph, with one variable changing in step with the other. and precision. In 1843, French mathematician Antoine Augustin Cournot questioned the possibility of balancing a cone on its tip. When a fraction; for example, 2/3 in modern notation would be 1/2 + 1/6 in Egyptian notation (not 1/3 + 1/3 because the Egyptians did not allow repeats of the same fraction). In modern terms, if a cube with side length 1 has a volume of 13, what is x if x3 = 2? Trigonometry was to develop even further; although geometry was its natural home, it was also increasingly applied to solve algebraic equations. He also developed partial differential equations, explained the variations in the orbits of Earth and other planets, and researched integral calculus. 1591 French mathematicians in different parts of the world in the 1600s also led to such notations being adopted much more swiftly. The next step was to explain division by zero, which was more problematic. Key works 1913 Le Hasard (Chance) 1914 Principles and classiques du calcul des probabilités (Principles and classiques du calcul des probabilités). demon • Transfinite numbers 456 IN CONTEXT KEY FIGURE Emmy Noether (1882-1935) FIELD Algebra BEFORE 1843 German mathematician Ernst Kummer develops the concept of ideal numbers. The second number, the angular coordinate (θ), is the angular coordinate (θ), is the angular coordinate (θ) is the angular coordinate (θ). Geneva in 1768, but appears to have had no formal education in mathematics. He set out arithmetic rules for negative numbers. As a result of the work of German mathematician Georg Cantor, it was agreed that the first infinity was "smaller" than the second. Herman Melville Moby Dick (1851) See also: The area under a cycloid • Pascal's triangle • The law of large numbers 261 IN CONTEXT KEY FIGURES Isaac Newton (1642-1727), Gottfried Leibniz (1646-1716) FIELD Calculus BEFORE 287-212 BCE Archimedes uses the method of exhaustion to calculate areas and volumes, introducing the concept of infinitesimals. He based his workings on the assumption of two cones joined at the same vertex, with the area of their circular bases potentially stretching to infinity. 332 Perhaps the best known of the pioneers of topology is August Möbius, inventor of the Möbius strip, which had the unusual property of being a two-dimensional surface with only one side. Like other French philosophes, such as Voltaire and Jean-Jacques Rousseau, d'Alembert believed in the supremacy of human reason over religion. In 1806, Swiss mathematician Jean-Robert Argand found a particularly elegant solution. See also: 23 problems for the 20th century CHRISTINE DARDEN 1942- Along with Katherine Johnson and Mary Jackson, Darden is one. of the AfricanAmerican women whose work as mathematicians made key contributions to the work of NASA's space programs. Another story suggests that a fellow Pythagorean discovered irrational numbers, but Hippasus was punished for telling the outside world about them. that using the square roots of negative numbers might help solve cubic equations. For example, four ys multiplied together (y × y × y × y) is called "y raised to the power of 4" and written y4. If Pascal's triangle were drawn at a sufficiently large scale, this number would be found there. The key is +1, because each of the letters in plaintext is substituted with the letter +1 along in the alphabet. The only way to crack the code is to figure out what p and q are and then calculate z. 75 CE A native of Alexandria in the Roman province of Egypt, Hero (or Heron) was an engineer, inventor, and mathematician. The first five chapters are mostly about geometry (areas, lengths, and volumes) and arithmetic (ratios, and square and cube roots). See also: Quadratic equations • The binomial theorem • Cubic equations • The Fibonacci sequence • Mersenne primes • Probability • Fractals 250 IN CONTEXT KEY FIGURES Blaise Pascal (1623-62), Pierre de Fermat (1601-65) FIELD Probability BEFORE 1620 Galileo publishes Sopra le Scoperte dei Dadi (On the Outcomes of Dice), calculating the chances of certain totals when throwing dice. 282 Newton's second and third law help explain how scales work. A classical computer bit is either 1 or 0, never anything in between. As a subject, mathematics is surprisingly hard to pin down with one catch-all definition. Meeting regularly and marshaled by Dieudonné, the group 462 produced book after book, led by Éléments de Mathématique. Mihăilescu built on 20th-century advances (by Ke Zhao, J. Numbers can be added together by entering the units of one number, starting from the right, then adjusting the beads as further numbers are entered. (2) In trigonometry, the tangent function, abbreviated as tan, is defined as the ratio of the side length opposite a given angle to the side length adjacent to that angle, in a right-angled triangle. In 1733, the mathematician and naturalist George Leclerc, the Comte de Buffon, raised—and answered—a fascinating question. The possible rotations of a Rubik's Cube form a mathematical group with 43,252,003,274,489,856,000 elements, but solving the cube from any position requires no more than 26 turns of 90°. Perelman's proof... solved a problem that for more than a century was an indigestible seed at the core of topology. Borel's theory is often applied to stock markets, where the level of chaos means that in some cases random selection performs better than selection based on traditional economic theories. He was a pioneer of game theory, based on the "two-person zero-sum game," whereby one side wins what the other loses. As these quantities—actual lengths, areas, and volumes—can only be positive, the idea of a negative number did not make sense to Greek mathematicians. The binary system is a base-2 counting system and employs just two symbols, 0 and 1. It is often but not always used to mean a one-to-one mapping, where each member of the other set, and vice versa. In 1994, she founded the Women and Mathematics Program at the Institute of Advanced Study in Princeton New Jersey. If that central point is (0,0) on an x, y graph, the circle equation emerges, by drawing on Pythagoras's theorem. Further back, there are five members of the previous generation, eight of the one before that, and so on. Can two parallel lines ever meet? See place value system. After graduating from Carnegie Mellon University in 1948 and being awarded a doctorate from Princeton University in 1950, he joined the Massachusetts Institute of Technology (MIT), where he researched partial differential equations and began the work on game theory that won him the Nobel Prize for Economics in 1994. AFTER 1750 Leonhard Euler confirms that the Mersenne number 231 - 1 is prime. For example, he stated that "Plus of minus multiplied by plus of minus makes minus"—meaning a positive × = -n. Values become less frequent the further they are from the mean, so are least frequent at points a and c. Many natural objects exhibit self-similarity, including mountains, rivers, coastlines, clouds, weather systems, blood circulatory systems, and even cauliflowers. See also sequence. For example, 4 and 8 are roots of 64, with 8 being the square root (8 × 8 = 64) and 4 the cube root (4 × 4 × 4 = 64). An early version of the law of large numbers (LLN)—a theorem examining the results of performing the same action (such as throwing a die) a number of times —was part of Swiss mathematician Jacob Bernoulli's Ars Conjectandi (The Art of Conjecturing, 1713). 54 Pythagoras's own brand of philosophy integrated spiritual ideas with mathematics, science, and reasoning. Laplace's theory had a startling philosophy integrated spiritual ideas with mathematics, science and reasoning. military specialists to model the trajectory of projectiles fired by artillery—such as this MIM-104 Patriot surface-to-air missile, commonly used by the US Army. Turing was interested in whether any algorithm put into the machine to halt. Its approximate value is 1.618. Although Bombelli did not receive a college education, he was taught by an engineer-architect and became an engineer himself, specializing in hydraulics. 1718 Abraham de Moivre publishes The Doctrine of Chances. Unfortunately, hundreds of terms are needed to compute n to even a few decimal places using this series. three axes of space, plus the axis of time, so the 45° worldline is really a "hypercone," a 4-dimensional figure. Albert Einstein has an Erdős number of 2; Paul Erdős's number is 0. In fact, there is really just one combination of numbers in a magic square. The whole 32 would be followed by a 0, because 32 is an integer, whereas the 6/100, for example, was expressed as 6 and a 2 inside a circle. First, the complete manipulation of number. In 1915, German algebraist Emmy Noether demonstrated how Lie groups related to conservation laws (such as the conservation of energy). There would be no problem with the two common notations if not for the use of delimiters—symbols that separate groups of three digits in the whole-number section of a very large or sometimes very small number. Although the Pythagorean scale worked well for music, which was written in different keys and extended across several octaves. In 1835, Belgian mathematician Adolphe Quetelet posited that characteristics, such as body mass, within a human population follow a bell curve pattern, in which values around the mean are most frequent. Multiples of 2, 3, and 5. She presents courses for examination boards both in the UK and internationally and writes and presents enrichment activities for students. However, he does not provide the proof. Karinthy, who was a writer, not a mathematician, coined the phrase "six degrees of separation." Mathematicians have since tried to model the average degree of separation." smaller length. The range of problems Many of Hilbert's questions are highly technical, but some are more accessible. It was also of little interest to the readers of the Arte dell' Abbaco, most of whom wanted to learn how to use numbers in practical business calculations in everyday trading. Since they knew Europa's radius, calculating its surface area was easy. The name  $\phi$  comes from the ancient Greek sculptor Phidias (500-432 BCE), who is believed to have been one of the first to recognize the aesthetic possibilities of the golden ratio. If XYZ were a real triangular object, it would appear as the triangle xyz when viewed from P. The equation is fn = f(n-1) + f(n-2), where n is greater than 166 1. A geometry able to include mountains and clouds now exists... Like everything in science this new geometry has very, very deep and long roots. Finally, his proof was fully accepted by the mathematical community in 2006. No previous calculator had ever worked with numbers larger than four digits. Getty Images: Keystone / Stringer / Hulton Archive (tr). This was easy for shapes with straight edges, but curvilinear shapes caused problems. The classes of objects encountered in the real physical world do not have precisely defined criteria of membership. They undoubtedly had a rudimentary sense of number and arithmetic, but the history of mathematics only properly began with the development of numerical systems in early civilizations. 1631 In Practice of the Art of Analysis, published 10 years after his death, British mathematician Thomas Harriott accepts negative numbers in algebraic notation. These fuzzy values resemble probabilities, but are fundamentally quite distinct—they indicate the degree to which a proposition is true, not how likely it is. In 1795, he entered Göttingen University and in 1798, he constructed a regular heptadecagon (a polygon construction since Euclid's geometry some 2,000 years earlier. Enrico Bombieri Italian mathematician 403 A new formula In 1859, Bernhard Riemann constructed a new formula for  $\pi(n)$ , which would give the most accurate estimates possible. By contrast, the ancient Egyptians used separate symbols for ones, tens, hundreds, thousands, and above, and had no place value system. All the multiples of 9 have gone, as they are multiples of 3, and all the multiples of 10 have been removed, because they are the even multiples of 5. René Descartes describing geometry Building blocks Coordinate geometry, which had barely evolved since Euclid had written Elements in ancient Greece some 2,000 years earlier. ISAAC NEWTON Isaac Newton was born on Christmas Day in 1642 in Lincolnshire, England, and was brought up in early childhood by his grandmother. In addition to the Elements, he wrote about perspective, conic sections, spherical geometry, mathematical astronomy, number theory, and the importance of mathematical rigor. relationship between sides of a right-angled triangle centuries before Pythagoras's birth. Calculus, developed independently by Gottfried Leibniz and Isaac Newton in the 1600s, provided an answer to this problem. Terry Pratchett and Neil Gaiman British authors Chaos theory, which looks at the way complex systems are highly sensitive to initial conditions and are thus extremely unpredictable. It took until 1619 for German mathematician and astronomer Johannes Kepler to show that the golden ratio is revealed if a number in the Fibonacci sequence is divided by the one that precedes it. 2000 BCE Ancient Egyptians use a binary system of doubling and halving to carry out multiplication and division. However, the most commonly used network, the internet, is public, so any symmetric key shared over a connection would be available to unintended parties. Odds were a feature of gaming, but Gerolamo Cardano was the first to apply mathematics to the study of probability. 135 Zero as a number Brahmagupta established rules for calculating with zero. Plotting 3-D space For a three-dimensional space, the coordinates require a third number, ordered in the tuple (x, y, z). He predicted that, like the wheat on the second half of the chessboard, the rate of technological development would rapidly grow out of control, following the model of doubling its previous growth with every leap forward. DAINA TAIMINA Born in Latvia in 1954, Daina Taimina began her career in the fields of computer science and the history of mathematics. In the 5th century CE, the philosopher Proclus talked of an element as "a letter of an alphabet," with 77 combinations of letters creating words in the same way that combinations of axioms-statements that are self-evidently true-create propositions. There, he became acquainted with finite groups, which would become his life's work. • The Poisson distribution An indispensable tool in applied mathematics • Bessel functions It will guide the future course of science • The mechanical computer A new kind of functions I have created another world out of nothing • Non-Euclidean geometries • Group theory Just like a pocket map • Quaternions Powers of natural numbers are almost never consecutive • Catalan's conjecture The matrix is everywhere • Matrices An investigation into the laws of thought • Boolean algebra A shape with just one side • The Möbius strip The music of the primes • The Riemann hypothesis Some infinities are bigger than others • Transfinite numbers A diagrammatic representation of reasonings • Venn diagrammatic repre shape do not matter, only connections • Topology Lost in that silent, measured space • The prime number theoremMODERN MATHEMATICS 1900-PRESENT The veil behind which the future lies hidden • 23 problems for the 20th century Statistics is the grammar of science • The birth of modern statistics A freer logic emancipates us • The logic of mathematics The Universe is four-dimensional • Minkowski space Rather a dull number • Taxicab numbers A million monkeys banging on a million typewriters • The infinite monkey theorem She changed the face of algebra • Emmy Noether and abstract algebra • Emmy Noether algebra • Emmy Noether algebra • Emmy Noether algebra • Emmy Noether algebra • Emmy Noet to compute any computable sequence • The Turing machine Small things are more numerous than large things • Benford's law A blueprint for the digital age • Information A small positive vibration can change the entire cosmos • The butterfly effect Logically things can only partly be true • Fuzzy logic A grand unifying theory of mathematics • The Langlands Program Another roof, another proof • Social mathematics • The Penrose tile Endless variety and unlimited complication • Fractals Four colors but no more • The four-color theorem Securing data with a one-way calculation • Fractals Four colors but no more • The four-color theorem Securing data with a one-way calculation • Fractals Four colors but no more • The four-color theorem Securing data with a one-way calculation • Fractals Four colors but no more • The four-color theorem Securing data with a one-way calculation • Fractals Four colors but no more • The four-color theorem Securing data with a one-way calculation • Fractals Four colors but no more • The four-color theorem Securing data with a one-way calculation • Fractals Four colors but no more • The four-color theorem Securing data with a one-way calculation • Fractals Four colors but no more • The four-color theorem Securing data with a one-way calculation • Fractals Four colors but no more • The four-color theorem Securing data with a one-way calculation • Fractals Four colors but no more • The four-color theorem Securing data with a one-way calculation • Fractals Four colors but no 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CLASSICAL PERIODS 6000 BCE-500 CE Numerals take their places • Positional numbers The square as the highest power • Quadratic equations The accurate reckoning for inquiring into all things • The Rhind papyrus The sum is the same in every direction • Magic squares Number is the cause of gods and daemons • Pythagoras A real number that is not rational • Irrational numbers The quickest runner can never overtake the slowest • Zeno's paradoxes of motion Their combinations give rise to endless complexities • The Platonic solids Demonstrative knowledge must rest on necessary basic truths • Syllogistic logic The whole is greater than the part • Euclid's Elements Counting pi is like exploring pi is like exploring the Universe • Calculating pi We separate the numbers as if by some sieve • Eratosthenes' sieve • Eratost numbers The very flower of arithmetic • Diophantine equations An incomparable star in the firmament of wisdom • Hypatia The closest approximation of pi for a millennium • Zu Chongzhi THE MIDDLE AGES 500-1500 A fortune subtracted from zero is a debt • Zero Algebra is a scientific art • Algebra Freeing algebra from the constraints of geometry • The binomial theorem Fourteen forms with all their branches and cases • Cubic equations The ubiquitous music of the spheres • The Fibonacci sequence 4 The power of doubling • Wheat on a chessboard THE RENAISSANCE 1500-1680 The geometry of art and life • The golden ratio Like a large diamond • Mersenne primes Sailing on a rhumb • Rhumb lines A pair of equal-length lines • The equals sign and other symbology Plus of minus times plus of minus makes minus • Imaginary and complex numbers The art of tenths • Decimals Transforming multiplication into addition • Logarithms Nature uses as little as possible of anything • The problem of maxima The fly on the ceiling Coordinates A device of marvelous invention • The area under a cycloid Three dimensions made by two • Projective geometry Symmetry is what we see at a glance • Pascal's triangle theorem The swing of a pendulum • Huygens's tautochrone curve With calculus I can predict the future • Calculus The perfection of the science of numbers • Binary numbers that are creatures of their own • Euler's number Random variation makes a pattern • Normal distribution The seven bridges of Königsberg • Graph theory Every even integer is the sum of two primes • The Goldbach conjecture The most beautiful equation of algebra • Theorem Simply a question of algebraic resolution of equations Let us gather facts • Buffon's needle experiment Algebra 5 THE 19TH CENTURY 1800-1900 Complex numbers are coordinates on a plane • The fundamental theorem of algebra 5 THE 19TH CENTURY 1800-1900 Complex numbers are coordinates on a plane • The fundamental theorem of algebra 5 THE 19TH CENTURY 1800-1900 Complex numbers are coordinates on a plane • The fundamental theorem of algebra 5 THE 19TH CENTURY 1800-1900 Complex numbers are coordinates on a plane • The fundamental theorem of algebra 5 THE 19TH CENTURY 1800-1900 Complex numbers are coordinates on a plane • The fundamental theorem of algebra 5 THE 19TH CENTURY 1800-1900 Complex numbers are coordinates on a plane • The fundamental theorem of algebra 5 THE 19TH CENTURY 1800-1900 Complex numbers are coordinates on a plane • The fundamental theorem of algebra 5 THE 19TH CENTURY 1800-1900 Complex numbers are coordinates on a plane • The fundamental theorem of algebra 5 THE 19TH CENTURY 1800-1900 Complex numbers are coordinates on a plane • The fundamental theorem of algebra 5 THE 19TH CENTURY 1800-1900 Complex numbers are coordinates on a plane • The fundamental theorem of algebra 5 THE 19TH CENTURY 1800-1900 Complex numbers are coordinates on a plane • The fundamental theorem of algebra 5 THE 19TH CENTURY 1800-1900 Complex numbers are coordinates on a plane • The fundamental theorem of algebra 5 THE 19TH CENTURY 1800-1900 Complex numbers are coordinates on a plane • The fundamental theorem of algebra 5 THE 19TH CENTURY 1800-1900 Complex numbers are coordinates on a plane • The fundamental theorem of algebra 5 THE 19TH CENTURY 1800-1900 Complex numbers are coordinates on a plane • The fundamental theorem of algebra 5 THE 19TH CENTURY 1800-1900 Complex numbers are coordinates on a plane • The fundamental theorem of algebra 5 THE 19TH CENTURY 1800-1900 Complex numbers are coordinates on a plane • THE 19TH CENTURY 1800-1900 Complex numbers are coordinates on a plane • THE 19TH CENTURY 1800-1900 Complex numbers are coordinates on a p The imp that knows the positions of every particle in the Universe • Laplace's demon What are the chances? For example, the set of all integers forms a ring when taken together with the operations addition and multiplication, because performing these operations of the set. Inputting n into n gives the number of primes between 1 and n. Generating logarithms Napier was fascinated by numbers and spent much of his time finding ways of making calculations easier. Irrational numbers in the 5th century BCE, as he worked on geometrical problems. 1843 Irish mathematician William Hamilton adds two new imaginary units, creating quaternions, which are plotted in four-dimensional space. 1937 Claude Shannon uses Boolean logic as the basis for computer design in his A Symbolic Analysis of Relay and Switching Circuits. 624-C. There was much debate about who discovered modern calculus first: Newton or Leibniz. This number is hugely significant in mathematics due to its links with natural growth and decay. To determine the powers in the expansion, you take the degree of the binomial as n. Alamy Stock Photo: NASA Image Collection (clb). The length of the year In 1074, the ruling sultan of Persia, Jalal al-Din Malik Shah I, commissioned Omar Khayyam to reform the lunar calendar used since the 7th century, replacing it with a solar calendar. Bayes was made a fellow of the Royal Society in 1742 and died in 1761. In essence, this states that the process of differentiation (working out rates of change of a variable represented by a curve) is the inverse of the process of integration (calculating the area beneath a curve). Multiplying the triple 3, 4, and 5 by 2 produces another Pythagorean triple: 6, 8, and 10 (36 + 64 = 100). He called his machine the Difference Engine because it allowed complex multiplications and subtractions -"differences"—that could be handled by scores of interlocking cogs. AFTER 1798 British economist Thomas Malthus predicts that the human population will grow exponentially while the food supply will increase more slowly, causing a catastrophe. He graduated within a year, and in 1832 he became a professor at the University of Königsberg. See also integer. The first square of the chessboard contains 1 grain, so 1 is the first term of this series. Its properties have been put to use in everyday applications, too. In the early 1920s, Noether's work focused on rings and ideals. However, in 1890, fellow British mathematician Percy Heawood found a hole in Kempe's proof, and Kempe himself acknowledged that he had made a mistake that he could not rectify. Möbius junction, the Möbius junction, the Möbius inversion formula. Surface area The area of a flat or curved surface, or of the outside of a 3-D object. Some polynomials, however, do not have real-number roots. In a 1929 short story Chains, Friques Karinthy suggested that people's average connection-number • Graph theory • The complex plane • Proving Fermat's last theorem 511 IN CONTEXT KEY FIGURES Ron Rivest (1947-), Adi Shamir (1952-), Leonard Adleman (1945-) FIELD Computer science BEFORE 9th century CE Al-Kindi develops frequency analysis. Their distributed to the 3rdcentury BCE Greek mathematician. In basic terms, his areas of interest are Galois groups and functions called automorphic forms. All place value (positional) systems require a way of denoting "there is nothing here." The Babylonians (1894-539 BCE), for example, who at first used context to differentiate between, say, 35 and 305, eventually used a double wedge mark rather like inverted commas to indicate the empty value. In addition to his work on probability, Bernoulli is remembered for discovering the mathematical constant e by calculating the growth of funds that received 288 compound interest continuously in infinitesimal increments. In 1806, he moved to Paris to manage a bookshop, and self-published the work containing the geometrical interpretation of complex numbers for which he is known. The Plimpton tablet In the early 1900s, an examination of triangles, dating back to around 1800 BCE, was discovered on an ancient Babylonian clay tablet. Unfortunately, Tijdeman's upper bound is astronomically 380 large, making such computation practically unfeasible even for modern computers. Instead, al-Khwarizmi followed geometric methods to solve linear or quadratic equations. The Möbius strip's form has inspired artists and architects. 1966 In an episode of the BBC's Doctor Who, the villain, The Celestial Toymaker, forces the Doctor to play a ten-disk version of the game. Even in the age of computers, determining whether a large number is prime remains highly challenging. If a needle is dropped onto a series of parallel lines, all the same width apart, what is the likelihood that the needle will cross one of the lines? There is no possible starting point within the attractor that can lead to a trajectory that escapes from it. 572 Transfinite number Another term for an infinite number. He could not exclude negative solutions to his equations and even used a sign, "m," to denote a negative number. He applied the theorem to a right-angled triangle that has both shorter sides equal to 1. The Chinese military strategist Sun Tzu, who lived around 500 BCE, used such rods to make calculations before battles. 1290 Catalan mystic Ramon Llull devises classification systems using devices such as trees, ladders, and wheels. I cannot judge my work while I am doing it. This work, which encapsulated the essence of their mathematical knowledge, included algorithms that assumed negative guantities were possible—for example, as solutions to problems on profit and loss. This was closer to Catalan's conjecture, but did not allow for the possibility that larger powers or exponents could result in consecutive numbers. He could not, however, accept the value of negative quantities as solutions to equations but referred to them as "false roots" rather than true numbers. Lines, surfaces, and shapes could also be interpreted as a series of defined points, which changed the way people thought about natural phenomena. Zeno collected his arguments in a book, but this did not survive. After the way, Lorenz studied meteorology at the Massachusetts Institute of Technology and began to develop ways to predict the behavior of the atmosphere. Since then, Perelman's work has been closely studied, fuelling new developments in topology, including a more powerful version of his and Hamilton's technique for using Ricci flow to smooth singularities. In so doing, Russell managed to circumvent the paradox completely. 510 Computer proofs When Appel and Haken proved the four-color theorem in 1977, it was the first time that a computer had been used to prove a mathematical theorem. It includes the study of prime number as it is 5 × 5 (52). Fellow architect Leon Battista Alberti used "vanishing points" to create a sense of 3-D perspective and wrote about the use of perspective in art. The use 210 of such tables facilitated complex calculations and advanced the development of trigonometry. D. Numbers at the bottom, and there is evidence of Mayan calculations up to hundreds of millions. 146 Plotting equations had challenged mathematicians since the time of Diophantus of Alexandria. Numbers would later be known as "imaginary" numbers. The markings on the turtle's back form a magic square, with numbers, and i = 325), creates what is called a complex number. What Gödel called the "incompleteness theorem" found that there was a mismatch between mathematical truth and mathematical proof. Fourier analysis can separate out pure tones, which can be represented as sine waves on a graph, from each other. Tartaglia remained unsure whether the problem was solvable in a way that would convince all players of its fairness. The recognition of infinitesimals was a pivotal moment in the development of calculus: previously insoluble puzzles, such as Zeno's paradoxes of motion, could now be solved. Around the same time, Thomas Harriot was experimenting with number systems, including the binary system. Arguably, the first programmable computer— rather than calculator—was created by Konrad Zuse in 1938. 1600 BCE) FIELD Algebra BEFORE c. Poincaré's conjecture, conceived by the French mathematician in 1904, is stated as: "Every simply connected, closed 3-manifold is homeomorphic to the 3sphere." In topology, a field that studies the geometrical properties. structure, and 532 spatial relations of shapes, a sphere (a 3-D object in geometry) is said to be a 2manifold with a 2-D surface existing within a 3-D space—a solid ball, for example. Alamy Stock Photo: Ian Dagnall (tl). The 17th-century French mathematician Girard Desargues was a founder of such geometry. James Hilton British novelist AL-KARAJI Born around 980 CE, Abu Bakr ibn Muhammad ibn al-Husayn al-Karaji most likely got his name from the city of Karaj, near Tehran, but he lived most of his life in Baghdad, at the court of the caliph. 4th century BCE Theaetetus, an Athenian contemporary of Plato, discusses the octahedron and icosahedron. Heat and harmony By the end of the 1600s, physicists including Joseph Sauveur were making great strides in studying the relationships between the waves in stretched strings and the pitch and frequency of sounds that they produced. At its simplest, this was done by cutting tally marks in a bone or stick, a rudimentary but reliable means of recording numbers of things. Good architecture should be a projection of life itself. Getty Images: Etienne DE MALGLAIVE / Gamma-Rapho (tr). Finding a solution Fermat's last theorem remained one of the great unsolved problems in mathematics for more than 300 years, until it was proved by British 525 mathematics for more than 300 years. principles of perspective in De Pictura (On Painting). Many seemingly simple polynomials, however, such as x2 + 1, do not equal zero if x is a real number. Lines of longitude are meridians. The sound guality of a musical instrument results principally from the number and relative intensities of the harmonics present in the sound, or its harmonic content. Without the binomial theorem, many mathematician such as Joseph-Louis Lagrange, Carl Friedrich Gauss, and British mathematician Arthur Cayley, but gained traction when German mathematician Richard Dedekind began to study algebraic structures. Decimals [are] a kind of arithmetic invented by the tenth progression, consisting in characters of cyphers. The infinite number of points, it would never reach its goal. The robot's moves are computed using algebraic topology that can predict and model the surrounding "space." Shaping the future Topology developments still continued during the 1900s. See also: Pythagoras • The Platonic solids • Syllogistic logic • Conic sections • The problem of maxima • Non-Euclidean geometries 82 IN CONTEXT KEY CIVILIZATION Ancient Greeks (c. In the case of events such as volcanic eruptions or droughts, plotting elements such as intensity, duration, and frequency could help to identify trends. The shape that results has some unexpected properties, which have advanced our understanding of complex geometrical figures—a branch of study called topology. Dorling Kindersley: Royal Signals Museum, Blandford Camp, Dorset (bc). See also: Trigonometry KAMAL AL-DIN AL-FARISI C. In 1549, after a period practicing medicine in London, Recorde was made controller of the Bristol mint. At the age of about 17, he left to study at Plato's Academy in Athens, where he excelled. 200 CE includes what may be the earliest representation of a Möbius strip which is thought to represent the eternal nature of time. Coincident In geometry, two or more lines or figures that, when superimposed on each other, share all points and occupy exactly the same space. Usually, the equations must be calculated together to solve the value of the unknowns. Other early records of quadratic equations are found in Babylonian clay tablets, where the diagonal of a square is given to five decimal places. Returning to Athens, in 387 BCE he founded the Academy, inscribing over its entrance the words "Let no one ignorant of geometry, believing that its forms -especially the five regular convex polyhedra-could explain the properties of the Universe. Therefore, out of the four possible combinations of A and B, only one results in a true answer. Along with George Fatou, who undertook similar research independently, Julia found that by taking a complex number, squaring it, adding a constant (a fixed number or a letter standing for a fixed number) to it, and then repeating the process, some initial values would diverge to infinity while others would converge to a finite value. AFTER 1925 Russian mathematician Pavel Aleksandrov establishes the basis for studying the essential properties of topological spaces. Cayley was a pioneer of group theory and matrix algebra, devised the theories of higher singularities and invariants, worked in higherdimensional geometry, and extended the quaternions of William Hamilton to create octonions. He also found that while every starting point in his model rendered unique results, they were all confined within certain limits. He was a fiction created in the 1930s by young French mathematicians who were striving to fill the vacuum left by the 461 devastation of World War I. Hardy's frequent retelling of this story ensured that 1,729 449 would become one of the best-known numbers in mathematics. In Boole's logical algebra, truth and falsity were reduced to binary values: 1 for true and 0 for false. So if the elements 1, 2, and 3 are members of a group, then  $(1 \times 2) \times 3 = 2 \times 3 = 6$ , and  $1 \times (2 \times 3) = 1 \times 6 = 6$ , all giving the same result. A prime example is the symbiotic relationship between mathematics and computers. Mathematics and computers is the symbiotic relationship between mathematics and computers. Minkowski—whose primary interest was in geometry and the geometrical method applied to problems in number theory—explored the notion of multiple dimensions, and suggested spacetime as a possible fourth dimension. 2000 BCE, may have been inspired by the abacus. The first 16 propositions in Book 1 Proposition 1 On a given finite straight line, to construct an equilateral triangle. Stephen Marguardt, an American plastic surgeon, created a "mask" (see above) based on applying the golden ratio to the human face. The first field to emerge, and in many ways the most fundamental, is the study of numbers and guantities, which we now call arithmetic, from the Greek word arithmetic ("number"). In 1557, Recorde went on to recommend a new symbol of his own. Meanwhile, there were other developments in mathematics that were to have farreaching implications for later technological progress. Eratosthenes' method starts with a table of consecutive numbers. At the time, messages could only be transmitted using a continuous, analog signal. A regular dodecahedron is one of the five Platonic solids. For example, a3 was aaa and x4 was xxxx. This was seen at the time as a mathematical abnormality that, unlike the sensible Euclidean shapes, had no real-world relevance. determine how far we've traveled? A great deal has now been discovered about Babylonian mathematics, which extended to multiplication, division, geometry, fractions, square roots, cube roots, equations, and other forms, 26 because—unlike Egyptian papyrus scrolls—the clay tablets have survived well. By the time of Diophantus, around 250 CE, linear and quadratic equations were used to solve problems, but any unknown quantity was still represented geometrically—by a length. 180 Discovering phi Some believe that proportions related to  $\phi$  can be found in ancient Greek architecture—and even earlier in ancient Greek architecture. circles The Venn diagram considers two or three different sets or groups of things, or all planets of the solar system. In syllogistic logic, this means that if A is true, I must be true, but that if I is false, A must be true, but that if I is false, A must be true, but that if I is false as well. for a voyage. The solutions for x are where y = 0 and the curve crosses the x axis. It was revolutionary because it was written in everyday Venetian for merchants and anyone else who wanted to solve problems. These are often used to solve problems in mechanics, physics, astronomy, and engineering, and involve powers of an unknown value, such as x2. Computers are programmed to follow the movements of lava lamps, measure radioactive decay, or listen to white noise made by radio transmissions, turning that input into random numbers to use for encryption. Any ambiguity about their status in mathematics is long gone. She published more than 40 research papers in a 20-year career that was cut short by cancer. Pythagoras 52 Fascinated by numbers Another type of number that excited Pythagoras was the perfect number. Hamilton went on to attempt to prove the theorem himself, but did not succeed. These ideas have since been refined and developed, but they remain at the heart of data analysis. Parallelogram A quadrilateral where each side has the same length as the side opposite to it and the two sides are also parallel. When a plane intersects a cone, it creates a conic section. See also: Euclid's Elements • Eratosthenes' sieve • 23 Problems for the 20th century • Information theory • Cryptography 472 IN CONTEXT KEY FIGURE Frank Benford (1883-1948) FIELD Number theory BEFORE 1881 Canadian astronomer Simon Newcomb notices that the pages most often referred to in logarithm tables are for numbers starting with 1. Like the Egyptians, the Babylonians needed scribes to administer their complex society, and many of the tablets bearing mathematical records are thought to be from training schools for scribes. Rather, it produces a longer, continuous twisted loop. This concept captured the public's imagination following an interview with American actor Kevin Bacon, in which he said he had worked with every actor in Hollywood or with someone who had worked with them. Iteration Performing the same operation again and again to achieve a desired result. An Argand diagram uses the x and y axes to represent real numbers, combining them to plot complex numbers, combining them to plot complex numbers, combining them to plot complex numbers. Russia, and became the most prolific mathematician of 291 all time, contributing greatly to calculus, geometry, among other fields. When we weigh ourselves, our weight (the mass of an object multiplied by gravity) is a force, now measured in newtons. By adding together the areas of the triangles, Archimedes obtained a close approximation of the area he sought. The first says that a force is needed to create motion, and that this force may be from the gravitational attraction between two bodies or an applied force (such as when a snooker cue strikes a ball). 503 BENOIT MANDELBROT Born into a Jewish family in Warsaw in 1924, Benoit Mandelbrot left Poland in 1936 to escape the Nazis. Dreamstime.com: Vladimir Korostyshevskiy (bl). His contributions toward the development of calculus include originating the idea of the number line, introducing the symbol for infinity, and development of calculus include origination increases entropy. On a 12-hour clock, for example, if you count on four hours from 10 o'clock, you get 2 o'clock; 10 + 4 = 2, because the remainder of 14 ÷ 12 is 2. Irrational number that cannot be expressed as one whole number divided by another and is not an imaginary number. The system was adopted and refined by Arab mathematicians in the 9th century. For example, in the binomial (x + y)3 = 1x3 + 3x2y + 3x spirals in each set are 13 (clockwise) and 8 (counterclockwise)—two Fibonacci numbers. See also: The logic of mathematicians to investigate what would later be known as chaos theory. Using tables, the Babylonians could also

calculate volumes, such as the space within a grain store. 141 Al-Khwarizmi introduced some fundamental algebraic operations, which he described as reduction, rejoining, and balancing. After Lorenz, we came to see that... in the long run, things could be unpredictable. During this period his discovery of the work of Pythagoras inspired a love of each state... in the long run, things could be unpredictable. mathematics. Calculations were carried out by manipulating the rods on a counting board. This, for example, gives financial companies the confidence to set prices for insurance and pension products, knowing their chances of having to pay out, and ensures that casinos will always make a profit from their gambling customers—eventually. In particular, Listing was interested in shapes that did not satisfy Euler's formula or defied the conventions of having distinct "outside" and "inside" and "inside to one another. The distance the first particle has traveled is the logarithm of the distance the second particle has yet to go. Kurt Vogel German mathematical historian The quest for solutions 121 Many of the problems now called Diophantine equations were known well before Diophantus's time. No a quadratic equation which, when he completed the square, gave numbers available to mathematicians at the time gave a negative number when multiplied by themselves, but Cardano suggested suspending belief and working with the square root of negative 15 to find the equation's two solutions. Inspired by Galton's work, Pearson set out to develop the mathematical framework for correlation and regression. He authored many books on a variety of subjects, including mathematics. The lowest to highest on the right. Irrational numbers are on the left, the highest on the right. Frational numbers are on the left, the highest on the right. Elements • Trigonometry EUDOXUS OF CNIDUS C. Coordinates make it possible to convert curves and shapes into algebraic equations, which can be shown visually. After 1,000 tosses, the imbalance apparent in those first 10 tosses becomes negligible. There, she built her reputation as an aeronautical engineer, specializing in supersonic flight. In the 1990s, British mathematicians Terrence Sheil-Small and Alan Wilmshurst extended the FTA to harmonic polynomials. After World War II, Mandelbrot gained scholarships to study in France and then the US, before returning to Paris, where he was awarded a doctorate in mathematical sciences from the city's university in 1952. Term In an algebraic expression, one or more numbers or variables, usually separated by a plus (+) or minus (-) sign, or in a sequence, by a comma. It provides us with the rules of reasoning and gives us a basis for deciding on the validity of an argument or proposition. Parabolas prove useful in the real world because of their reflective. Four years later, it took around 13 minutes to compute 3,089 digits. Halfway between the starting point and the end of the line, the second particle is traveling with a quarter of its initial velocity; and so on. John Napier Aid to astronomy Along with the developments in trigonometry, there was a gradual and corresponding shift in the way people viewed the heavens. 198 Jacques Hadamard French mathematician Applying complex numbers, real numbers, read numbers, real numbers, real other increasingly sophisticated mathematical tasks. Johnson graduated in 1937 from West Virginia State College and became one of 552 the first African-Americans to enroll in a graduate program at West Virginia State College and became one of 552 the first African-Americans to enroll in a graduate program at West Virginia State College and became one of 552 the first African-Americans to enroll in a graduate program at West Virginia State College and became one of 552 the first African-Americans to enroll in a graduate program at West Virginia State College and became one of 552 the first African-Americans to enroll in a graduate program at West Virginia State College and became one of 552 the first African-Americans to enroll in a graduate program at West Virginia State College and became one of 552 the first African-Americans to enroll in a graduate program at West Virginia State College and became one of 552 the first African-Americans to enroll in a graduate program at West Virginia State College and became one of 552 the first African-Americans to enroll in a graduate program at West Virginia State College and became one of 552 the first African-Americans to enroll in a graduate program at West Virginia State College and became one of 552 the first African-Americans to enroll in a graduate program at West Virginia State College and became one of 552 the first African-Americans to enroll in a graduate program at West Virginia State College and became one of 552 the first African-Americans to enroll in a graduate program at West Virginia State College and became one of 552 the first African-Americans to enroll in a graduate program at West Virginia State College and became one of 552 the first African-Americans to enroll in a graduate program at West Virginia State College and became one of 552 the first African-Americans to enroll in a graduate program at West Virginia State College and became one of 552 the first African-Americans to enroll in a graduate program at West Virginia State College and became on curvature of spacetime. See also: Topology JOAN CLARKE 1917-96 London-born Clarke achieved a double first in math at the University of Cambridge on the eve of World War II but was denied a full degree because of her gender. 200 BCE Conics See also: Euclid's Elements • Coordinates • The area under a cycloid • Projective geometry • The complex plane • Non-Euclidean geometries • Proving Fermat's last theorem 103 IN CONTEXT KEY FIGURE Hipparchus (c. 334 IN CONTEXT KEY FIGURE Jean-Robert Argand (1768-1822) FIELD Number theory BEFORE 1545 Italian scholar Gerolamo uses negative square roots to solve cubic equations in his book Ars Magna. Andrew Wiles After the theorem Fermat was amazingly far-sighted in his original conjecture, but it is unlikely that the "marvelous proof" he claimed to have discovered existed. An elegant illustration 318 Buffon originally used the needle experiment to estimate n (pi)—the ratio of a circle's circumference to its diameter. When applied to technology, the singularity marks the point at which the cognitive ability of artificial intelligence will surpass that of humans. It is given to us to calculus, to measure, to observe; this is natural philosophy. Desargues died in 1661. 268 The fundamental theorem of calculus. For example, if interest is calculated monthly, the monthly rate is 1/12 of the yearly rate. The annual Abel Prize for mathematics is awarded in his honor. See also: Irrational numbers • Diophantine equations • Zero • Algebra • The binomial theorem • Cubic equations • Zero • Algebra • The binomial theorem • Cubic equations • Imaginary and complex numbers • Diophantine equations • Zero • Algebra • The binomial theorem • Cubic equations • Zero • Algebra • The binomial theorem • Cubic equations • Zero • Algebra • The binomial theorem • Cubic equations • Zero • Algebra • The binomial theorem • Cubic equations • Zero • Algebra • The binomial theorem • Cubic equations • Zero • Algebra • The binomial theorem • Cubic equations • Zero • Algebra • The binomial theorem • Cubic equations • Zero • Algebra • The binomial theorem • Cubic equations • Zero • Algebra • The binomial theorem • Cubic equations • Zero • Algebra • The binomial theorem • Cubic equations • Zero • Algebra • The binomial theorem • Cubic equations • Zero • Algebra • Zero • Ze CIVILIZATION Ancient Egyptians (c. Hadamard's brilliant career was marred 423 by personal loss; two of his sons died in World War I, and another in World War I. It was mathematicians, as much as engineers, who provided the means of building mechanical, and then electronic computing devices, which in turn could be used as tools in the discovery of new mathematical ideas. He is the author of We've Got Your Number, a book on mathematics for children, and film scripts voiced by Tilda Swinton. Their understanding was shaky at best, and it was Pascal's work with the triangle that pulled the strands together. GIRARD DESARGUES Born in 1591, Girard Desargues lived in Lyon all hi life. He turned down the two greatest accolades for a mathematician: the Fields Medal in 2006 and the Clay Mathematics Institute prize (and its \$1 million award) in 2010, saying it belonged as much to Hamilton. The notation of modern calculus Invented by Newton for differentiation. The sound created by vibrating strings has been a topic of research for more than 2,500 years. He also applied the theory of conic sections (circles, ellipses, parabolas, and hyperbolas) to solve optical problems, and explained that the differential calculus (or differentiation), which together with the related field of integral calculus led to the fundamental theorem of calculus (see box, right). This further enabled people to plot a course just by drawing a straight line on the map. Problem 41 asks for the volume of a cylindrical store with a diameter of 9 cubits and a height of 10 cubits. This is the starting point of all probability theory. When the sum of all the numbers above a given row is a prime, it is a Mersenne prime—a prime number that is one less 248 than a power of 2, such as 3 (22 - 1), 7 (23 - 1), and 31 (25 - 1). [Frederick the Great is] always at war; in summer with the Austrians, in winter with mathematicians. He adopted the Hindu-Arabic numeral system, and the use of symbols in algebra, and contributed many original ideas, including the Fibonacci arithmetical sequence. David Percy British mathematician See also: Calculating pi • Trigonometry • Imaginary and complex numbers • Logarithms • Euler's number 309 IN CONTEXT KEY FIGURE Thomas Bayes (1702-61) FIELD Probability BEFORE 1713 Jacob Bernoulli's Ars Conjectandi (The Art of Conjecturing), published after his death, sets out his new mathematical theory of probability. 354 A supporting genius One of the greatest advocates for Babbage's work was his fellow mathematical theory of probability. loom weaves flowers and leaves." As a teenager in 1832, Lovelace had seen one of the Difference Engine models working and had been instantly entranced. When a coin is flipped, there are two possible outcomes: "success" and "failure." This type of test, with two equally likely outcomes, is called a Bernoulli trial. He embarked on his travels at the age of 20, and spent many years away. This all changed in the early 1900s with the rise of "abstract" algebra, which became one of the key fields of mathematician Emmy Noether. 121 Alamy Stock Photo: Art Collection 3 (bl). AFTER 1749 Abraham de Moivre uses Euler's formula to prove his theorem, which links complex numbers and trigonometry. The works of Archimedes are, without exception, works of mathematical exposition. Lagrange's insight into the symmetries that arose from these permutations of symmetries and roots) why a formula for the quintic equation needed a different approach. In the 9th century, the Islamic mathematician alKhwarizmi wrote a treatise on Hindu-Arabic numbers, which described the place value system including zero. In a simple case, such as the movement of a pendulum or the orbit of a planet around a star, this imaginary body oscillates (moves back and forth) around a fixed point on the surface, following a repeating what is called a limit cycle. Set theory strongly indicates that there are many more irrational numbers, even though there are many more irrational numbers. in much the same way that there were rules for adding, subtracting, or multiplying numbers. In the early 1960s, American mathematician Daniel Gorenstein began to pioneer the classification of finite simple groups and issued his complete classification of finite simple groups and issued his complete classification of finite simple groups and issued his complete classification of finite simple groups and issued his complete classification of groups and issued his complete cla significance of differences between expected and observed frequencies. Formulated by Leonhard Euler in 1747, the equation known as Euler's identity, ein + 1 = 0, encompasses the five most important numbers in mathematics: 0 (zero), which is neutral for addition and division; e (2.718..., the number at the heart of exponential , the fundamental imaginary number); and π (3.142..., growth and decay); i (the ratio of a circle's circumference to its diameter, which occurs in many equations in mathematics and physics). 410 BCE Originally a merchant on the Greek island of Chios, Hippocrates later moved to Athens, where he first studied, then practiced mathematics. In a Lorenz attractor, small changes in starting conditions result in huge changes to the paths each line takes, yet the lines still fall within the confines of the same shape, providing order within the confines of the same shape. As all other even numbers are divisible by 2, they cannot be primes, so all other primes must be odd. Big Ideas Simply Explained is actually not a bad definition. Limit The end number that is approached as certain calculations are iterated to infinity. The Lune of Hippocrates, as it was later called, is bounded by the arcs of two circles, the smaller of which has as its diameter a chord spanning a right angle on the larger circle. He died in 1956 in Paris. Computers work in a similar way, using switches and electricity rather than gates and marbles. 2000s Fourier analysis is used to create a number of speech recognition programs for computers and smartphones. These lines were displayed as six-line hexagrams, combined into a total of 64 different patterns. 391 Boole wanted to discover a way to frame logical arguments so that they could be manipulated and solved mathematically. Explaining the numbers Rafael Bombelli assimilated the tussles between the various mathematically. human scale, Borel considered events with a probability of less than 10-6 (or 0.000001) to be impossible. 573 CONTRIBUTORS KARL WARSI, CONSULTANT EDITOR Karl Warsi taught mathematics in UK schools and colleges for many years. Key work C. The way the ratios change is described by trigonometric functions, which are now fundamental to many branches of mathematics. 187 Mersenne's influence Regius's work on primes was continued by others who proposed new hypotheses with 2n - 1. 1202 Leonardo of Pisa's Liber Abaci (The Book of Calculation) brings the Arabic number system to Europe. However, in 1903, Bertrand Russell published The Principles of Mathematics, which revealed a flaw in the logic of one area of mathematics. After exhaustive tests that involved tossing coins and drawing lottery tickets, Pearson came up with the key idea of "standard deviation," which shows how much on average observed values differ from expected. Using this method implies that in sapproximately 3.1605 calculated to four decimal places, which is just 0.6 per cent greater than the most accurate known value of  $\pi$ . Each recipient then sent the letter on to a person they knew to get it closer to its target destination. He had colored a map of the 508 English counties using just four colors and believed that the same could be done with any map, however complex. Two-circle Venn diagrams can represent categorical propositions, such as "All A are B," "No A are B," and "Some A are not B." Three-circle diagrams can also represent syllogisms, in which there are two categorical premises and a categorical conclusion. Together, the two axes create a field of points called a coordinate plane, which extends outward 224 in two dimensions with the origin (0,0) at the center. The Eye of Horus, an Egyptian god, was a symbol of power and protection. But it also traces the history of maths, from ancient ideas such as magic squares and the abacus to modern cryptography, fractals, and the final proof of Fermat's Last Theorem. Nevertheless most AI is "narrow," in that it is tasked with doing one job very well, generally better than a human can, but it cannot learn to do anything else and is unaware of what it does not know. 120 In the 3rd century CE, the Greek mathematician Diophantus, a pioneer of number theory and arithmetic, created a prodigious work called Arithmetica. Ptolemy's contribution Around 300 years later, in the Egyptian city of Alexandria, the gifted GrecoRoman polymath Claudius Ptolemaeus, better known as Ptolemy, wrote a mathematical treatise called the Syntaxis Mathematikos (later renamed the Almagest by Islamic scholars). Zeno introduced the Socratic philosophers to Eleatic ideas. Unlike many of his peers in the 1900s, Poincaré did not confine himself to any one single field of mathematics. The formula for this is A = P (1 + r)t, where A is the number of years. This allows for calculations involving the Chinese base-16 system, which uses 15 units rather than the 9 used in the decimal system. Infinity was a concept that mathematicians had long instinctively mistrusted. Launched in 1989, the World Wide Web facilitated the rapid transmission of knowledge, and computers became a part of everyday life, especially in the field of information technology. Each other number is the 153 sum of its adjacent number in the preceding column and the number above that adjacent number. Hypatia may have intended these editions to serve as textbooks for students, as she offered commentaries providing clarification, and developed some of the concepts further. a teacher before making his mark in statistics. She later became a symbol for feminists. This represents how Stevin would have written the number now expressed as 32.567. Proposition 10 To bisect a given finite straight line. Today, computer aided design (CAD) software makes extensive use of matrices for this purpose. (2) The root of an equation is its solution. Euclid's very first proposition was criticized by later writers. The highest point of the curve (b) represents the mean, which the values of Hindu-Arabic numerals in Europe through his book Liber Abaci (1202). It is a fundamental transcendental number that appears in many branches of mathematics. In this way, Cantor's set theory creates infinities nestled inside each, expanding forever. The potential importance of their conjecture was gradually understood over the next three decades and it became part of an ongoing program to link different mathematical disciplines. JAN DANGERFIELD A lecturer and senior examiner in Further Mathematics, Jan Dangerfield is also a fellow of the UK's Chartered Institute of Educational Assessors and a Fellow of the Royal Statistical Society. One day, he thought he had spotted a hole in the argument. This opening page of Euclid's Elements shows illuminated Latin text with diagrams and comes from the first printed edition, produced in Venice in 1482. Euclid's Elements discussed the Platonic solids described earlier by Plato (such as the tetrahedron), and demonstrated the golden ratio") in their proportions. For example, A AND (B OR NOT C) is true when A and B are both true and C is false, and is false when A is false and both B and C are true. After graduating in 1943, he stayed at Harvard for several years, first to teach 522 mathematics to military personnel during World War II, then to earn his PhD under mathematics to military personnel during World War II, then to earn his PhD under mathematics to military personnel during World War II, then to earn his PhD under mathematics to military personnel during World War II, then to earn his PhD under mathematics to military personnel during World War II, then to earn his PhD under mathematics to military personnel during World War II, then to earn his PhD under mathematics to military personnel during World War II, then to earn his PhD under mathematics to military personnel during World War II, then to earn his PhD under mathematics to military personnel during World War II, then to earn his PhD under mathematics to military personnel during World War II, then to earn his PhD under mathematics to military personnel during World War II, then to earn his PhD under mathematics to military personnel during World War II, then to earn his PhD under mathematics to military personnel during World War II, then to earn his PhD under mathematics to military personnel during World War II, then to earn his PhD under mathematics to military personnel during World War II, then to earn his PhD under mathematics to military personnel during World War II, then to earn his PhD under mathematics to military personnel during World War II, then to earn his PhD under mathematics to military personnel during World War II, then to earn his PhD under mathematics to military personnel during World War II, then to earn his PhD under mathematics to military personnel during World War II, then to earn his PhD under mathematics to military personnel during War II, then to earn his PhD under mathematics to military personnel during War II, then to earn his PhD under mathematics to military personnel during War II, then to earn his PhD under mathematics to military personnel during born Perlman has been described as the "mother of the internet." While a student at the Massachusetts Institute of Technology (MIT), she worked on a program that introduced children as young as three to computer programming. 1827 Carl Friedrich Gauss publishes his Disguisitiones generales circa superficies curvas (General Investigations of Curved Surfaces), defining the "intrinsic curvature" of a space, which can be deduced from within the space. In the 1600s, Japanese mathematician Seki Takakaze had shown how to calculate the determinants of matrices up to size 5 × 5. AFTER 1777 Leonhard Euler introduces the notation i for . 800 BCE The Indian scholar Baudhayana finds solutions to some "Diophantine" equations. One such assumption was that polynomials of odd degree (such as cubics and quintics) Always have a real root. Alamy Stock Photo: Classic Image (tr). Index (plural indices) Another word for an exponent. Trigonometry, like other branches of mathematics, was not the work of any one man, or nation. SUE POPE A mathematics educator, Sue Pope is a long-standing member of the Association of Teachers of Mathematician Arthur Cayley extends the work of Galois to a full theory of abstract groups. In 1750, Swiss mathematician Gabriel Cramer stated a general rule (now called Cramer's rule) for the determinant of a matrix with m rows and n columns, but he failed to give the proof of this rule. Such a machine could carry out any conceivable algorithm. The crucial number in this algorithm is n, which is formed by multiplying two prime numbers: p and q. Complex numbers enable all polynomial equations to be solved completely, but have also proved immensely useful in many other branches of mathematics— even in numbers. 2012 In Australia, the largest 3-D map of the sky suggests that the Universe is fractal up to a point, with clusters of matter within larger clusters, but ultimately matter is distributed evenly. Theoretical probability of any single event is calculated by dividing the number of desired outcomes by the total number of desired outcomes. each other, such as a mug and a doughnut. While the number of simple groups is infinite, the number of types of simple groups is not— at least, not when simple groups is not— at least, not when simple groups is not— at least, not when simple groups of finite size are considered. Any linear motion from point A to B can be expressed as the addition of two complex numbers. His meticulous analysis of observations led him to realize that the orbits of the planets around the Sun are elliptical, not circular, and he described the three key laws of planetary motion. It concerns the prime numbers or 1.455 Borel is best known for his infinite monkey theorem, but his lasting achievement was in laying the foundations for the modern understanding of complex functions—what a variable must be altered by to achieve a particular output. It was here around 1015 that he probably wrote his three key mathematics texts. However, the numbers -1, 0, and 1 with the multiplication operation form a finite group; multiplying any members of the group produces only -1, 0, or 1. See also: The Platonic solids • Trigonometry • Cubic equations • The complex plane GEROLAMO CARDANO 1501-76 A contemporary of Niccolò Tartaglia, Cardano was born in Lombardy and became an outstanding physician, astronomer, and biologist, as well as a renowned mathematical framework for Einstein's theory of Niccolò Tartaglia, Cardano was born in Lombardy and became an outstanding physician, astronomer, and biologist, as well as a renowned mathematical framework for Einstein's theory of Niccolò Tartaglia, Cardano was born in Lombardy and became an outstanding physician, astronomer, and biologist, as well as a renowned mathematical framework for Einstein's theory of Niccolò Tartaglia, Cardano was born in Lombardy and became an outstanding physician, astronomer, and biologist, as well as a renowned mathematical framework for Einstein's theory of Niccolò Tartaglia, Cardano was born in Lombardy and became an outstanding physician, astronomer, and biologist, as well as a renowned mathematical framework for Einstein's theory of Niccolò Tartaglia, Cardano was born in Lombardy and became an outstanding physician, astronomer, and biologist, as well as a renowned mathematical framework for Einstein's theory of Niccolò Tartaglia, Cardano was born in Lombardy and became an outstanding physician, astronomer, and biologist, as well as a renowned mathematical framework for Einstein's theory of Niccolò Tartaglia, Cardano was born in Lombardy and became an outstanding physician, astronomer, and biologist, as well as a renowned mathematical framework for Einstein's theory of Niccolò Tartaglia, Cardano was born in Lombardy and became an outstanding physician, astronomer, and biologist, as well as a renowned mathematical framework for Einstein's theory of Niccolò Tartaglia, Cardano was born in Lombardy and became an outstanding physician, astronomer, and biologist, as well as a renowned mathematical framework for Einstein's theory of Niccolò Tartaglia, Cardano was born in Lombardy and became an outstanding physician, as a renowned mathematica relativity, allowing scientists to develop and expand this theory. Alamy Stock Photo: World History Archive (tr). The AIs must fool human judges into thinking they are human rather than a computer program. The third axiom concerns associativity, which means that the result of operations on elements does not depend on the order in which they are applied. Raymond Louis Wilder American mathematician 419 HENRI POINCARÉ Born in 1854, in Nancy, France, Henri Poincaré showed such early promise that he was described by a teacher as a "monster of mathematics." He graduated in the subject from the University of Paris. A pair of related conjectures can be strong or weak: if the strong conjecture is proved, then the weak conjecture is also proved, but not vice versa. He was already familiar as a teenager with the works of Lagrange, Gauss, and Cauchy, but failed (twice) to enter the prestigious Ecole Polytechnique—possibly due to his mathematical and political impetuousness, though no doubt affected by the suicide of his father. First there was a tape, as long as it needed to be, divided into sections, each section carrying a coded character. Pythagoras then constructed a musical scale, starting with one note and the note an octave above it, filling in the notes between using perfect fifths. Another golden ratio approximated in nature is the golden spiral, which gets wider by a factor of  $\phi$  for every quarter turn it makes. In 1904, Henri Poincaré went even further, producing a theory that would help to lay a topological basis for understanding the shape of the more formal integration to come, Kepler's method was further developed in 1635 by Italian mathematician Bonaventura Cavalieri in Geometry, Advanced in a New Way by the Indivisibilibus continuorum nova quadam ratione promota (Geometry, Advanced in a New Way by the Indivisibilibus continuorum nova quadam ratione promota (Geometry, Advanced in a New Way by the Indivisibilibus continuorum nova quadam ratione promota (Geometry, Advanced in a New Way by the Indivisibilibus continuorum nova quadam ratione promota (Geometry, Advanced in a New Way by the Indivisibilibus continuorum nova quadam ratione promota (Geometry, Advanced in a New Way by the Indivisibilibus continuorum nova quadam ratione promota (Geometry, Advanced in a New Way by the Indivisibilibus continuorum nova quadam ratione promota (Geometry, Advanced in a New Way by the Indivisibilibus continuorum nova quadam ratione promota (Geometry, Advanced in a New Way by the Indivisibilibus continuorum nova quadam ratione promota (Geometry, Advanced in a New Way by the Indivisibilibus continuorum nova quadam ratione promota (Geometry, Advanced in a New Way by the Indivisibilibus continuorum nova quadam ratione promota (Geometry, Advanced in a New Way by the Indivisibilibus continuorum nova quadam ratione promota (Geometry, Advanced in a New Way by the Indivisibilibus continuorum nova quadam ratione promota (Geometry, Advanced in a New Way by the Indivisibilibus continuorum nova quadam ratione promota (Geometry, Advanced in a New Way by the Indivisibilibus continuorum nova quadam ratione promota (Geometry, Advanced in a New Way by the Indivisibilibus continuorum nova quadam ratione promota (Geometry, Advanced in a New Way by the Indivisibilibus continuorum nova quadam ratione promota (Geometry, Advanced in a New Way by the Indivisibilibus continuorum nova quadam ratione promota (Geometry, Advanced in a New Way by the Indivisibilibus continuorum nova quadam ratione promota (Geometry, Advanced in a New Way by the Indivisibilibus continuorum nova quadam ratione promota (Ge Venn diagram A diagram that shows sets of data as overlapping circles. However, many irregular shapes also tessellate, and semiregular tessellations are those involving unknown numbers to the power of 2 but not x3, x4, and so on. Algebraic notation may have taken a long time to catch on, but when a symbol made sense and helped mathematicians work through problems, it became the norm. Throughout the Industrial Revolution, applied mathematics was prioritized over areas of study that were more theoretical. In his Introduction to the Analytic Arts (1591), Viète suggested that mathematicians should use letters to symbolize the variables in an equation: vowels to represent unknown quantities and consonants to represent the known. In the 1980s, Ukrainian mathematician Vladimir Drinfel'd expanded the Program's scope to show that there might be a Langlands-type connection between specific topics within harmonic analysis and others within geometry. See also: Binary numbers • Matrices • The infinite monkey theorem • The four-color theorem 357 IN CONTEXT KEY FIGURE Carl Gustav Jacob Jacobi (1804-51) FIELDS Number theory, geometry BEFORE 1655 John Wallis applies calculus to the length of an elliptic curve; the elliptic integral he derives is defined by an infinite series of terms. In reality, the Vitruvian Man, which illustrated the theories of ancient Roman architect Vitruvius, does not quite align with golden proportions. It is zero if the element in question (blue candies) is absent, and 0.5 (or 1/2, or 50 percent) if half of all candies are blue. Heawood did prove correctly that no more than five colors were needed to color any map. See also: Non-Euclidean geometries • The Riemann hypothesis • Topology 557 GLOSSARY In this glossary, terms defined within another entry are identified with italic type. specific numerical value that will replace the unknown value to make the polynomial equal 0. Da Vinci's painting The Last Supper (1494-98) is said to incorporate the golden ratio. His eventual success led to him receiving the Abel Prize—the highest honor in mathematics —in 2016. Complex powers Mathematicians such as Euler asked themselves if it would be meaningful to raise a number to a complex power—a complex number being a number that combines a real number s. Theorem A significant proven result on a mathematical topic, especially one that is not self-evident. in number theory, including finding Pythagorean triples-groups of three integers that represent the lengths of the sides of rightangled triangles. This was groundbreaking in a scientific world that had up until then been governed by certainty. This was controversial among mathematicians, who were used to solving problems through logic that could be checked by their peers. In India, the mathematician-astronomer Aryabhata included a method for obtaining π in his Aryabhatiyam astronomical treatise of 499 CE: "Add 4 to 100, multiply by 8, and then add 62,000. 1977 Voyager 1 is launched. This fractal is constructed from an equilateral triangle in which the central third of each side is replaced by the base of another equilateral year-old Sophie Germain was confined to her wealthy father's house in Paris and began to study the mathematics books in his library. Input Any variable, which when combined with a function, produces an output. Complex numbers The collection of all positive and negative, rational numbers together make up the real numbers. The need to fill two-dimensional space as efficiently as possible led to the study of tessellations—the fitting together of polygons with no gaps or overlap. After graduating from the University of Christiana (now Oslo) in 1822, he traveled widely in Europe, visiting leading mathematicians. This is also called a tiling. So while the two-dimensional Cartesian geometry developed by Descartes worked across four quadrants—with both positive and negative coordinates—Apollonius effectively worked in just one. After training as a meteorologist, he served with the US Army Air Corps in World War II. Like Fermat's conjecture, Catalan's conjecture is a deceptively simple claim about powers of positive integers that was proved long after its initial statement. At much the same time, German mathematician and astronomer Johannes Kepler was researching the area enclosed by a planetary orbit, which he recognized as elliptical rather than circular. Alamy Stock Photo: The Picture Art Collection (crb). The circle can and y, so  $r^2 = x^2 + y^2$ , which can be rewritten as r = then be plotted on axes using different values of x and y that give the same value of r. In the 1800s, many statisticians studied variation in experimental results. He would then have found an approximate numerical solution using trigonometric tables, which were common in astronomy. The annual Loebner Prize in Artificial Intelligence (AI) was inaugurated in 1990 by American inventor Hugh Loebner and the Cambridge Center for Behavioral Studies, Massachusetts. The natural world is full of fractals, from ferns and sunflowers to ammonites and seashells. An ounce of algebra is worth a ton of verbal argument. The finest type of man gives himself up to discovering the meaning and purpose of life itself... this is the man I call a philosopher. A regular octahedron is one of the five Platonic solids. Walter Gropius German architect From maps to math As Western explorers sailed to new lands, they needed accurate maps depicting the spherical world in two dimensions. Library of Congress, Washington, D.C.: LCUSZ62-10191 (b&w film copy neg.) (bl). The Mill was where the arithmetical operations were held before processing and then applied it as a rule of thumb, whereas the Pythagoreans set about rigorously showing the principle, offering a proof that it is true for all right-angled triangles. The next most common letter is "t," then "a," and so on. The Mill was Babbage's version of a computer's CPU, while the Store acted as its memory. In 1954, he committed suicide. Finding the mean In 1721, Scottish baronet Alexander Cuming gave de Moivre a problem concerning the expected winnings in a game of chance. Plotting x2 + 1 on a graph with an x and y axis creates a neat curve that never passes through the origin, or (0,0) point. Chapter six covers taxes, and includes the ideas of direct, inverse, and compound proportions, most of which did not appear in Europe until around the 1500s. 360 BCE. During a lifetime of mathematical study, von Neumann contributed to virtually every area of the discipline. For to show that example, 2/11 is decimalized as 0.18181818..., denoted as both the 1 and 8 recur. Mathematicians then developed his equations to try to understand how the finite speed of light fit into the coordinate system of space and time. The graphic below shows how matrices are added together by adding the elements in corresponding positions. 375 BCE The Republic c. Calculations also became more complex, developing the concepts of multiplication and division from addition and subtraction—allowing the area of land to be calculated, for example. The next prime is 5, so all other multiples of 5 cannot be prime. This would mean that every aspect of a person's life up until their death has already been predetermined; they have no free will and no agency over their thoughts and deeds. All real numbers can be placed on a number line. To explain the problem, Russell used an analogy known as the barber paradox in which a barber shaves every man in town aside from those who shave themselves, creating two sets of people: those who shave themselves, creating two sets of people: those who shave themselves, creating two sets of people: those who shave themselves and those shaved by the barber. AFTER 1894 Lucas's work on recreational mathematics is posthumously published in four volumes. 445 It was in the 1700s that scientists first began questioning whether threedimensional Euclidean geometry could describe the entire Universe. Key work 361 1829 Fundamenta nova theoria functionum ellipticarum (The foundations of a new theory of elliptic function) See also: Huygens's tautochrone curve • Calculus • Newton's laws of motion • Cryptography • Proving Fermat's last theorem 362 IN CONTEXT KEY FIGURE János Bolyai (1802-60) FIELD Geometry BEFORE 1733 In Italy, mathematician Giovanni Saccheri fails to prove Euclid's parallel postulates. In 1851, aged just 46, Jacobi contracted smallpox and died. Algebraic numbers All the rational numbers and those irrational numbers that can be obtained by calculating the roots of a rational number. A center of mathematics and astronomy since Brahmagupta worked there in the 7th century, it lies on the intersection of a former zero meridian of longitude and the Tropic of Cancer. Each step on a logarithmic scale is a multiple of the preceding step. 1650 BCE The area of a circle is calculated using π as (169)2 ≈ 3.1605 in the Rhind papyrus. He studied mathematics and philosophy at Cambridge University, but was dismissed from an academic post there in 1916 for anti-war activities. With the advent of computers, the secure transfer of data became an issue, and mathematicians devised complex cryptosystems using the factorization of large prime numbers. In 1963, when it became possible to plot this data, the shape created became known as the Lorenz attractor. Leibniz saw links between this binary approach to divination and his work with binary numbers. 1871 Richard Dedekind builds on Kummer's idea to formulate definitions of rings and ideals more generally. Once complex numbers were understood, the next challenge for mathematicians was to create a number that worked the same way in a three-dimensional space. 2006 Grigori Perelman's proof of the Poincaré conjecture is confirmed. Persian polymath Nasir al-Din al-Tusi showed that the PP is equivalent to stating that the sum of angles in any triangle is 180°, but the PP nonetheless remained controversial. The conclusion Fermat reached was that no integer greater than 2 could stand for n. His ideas were not always original, but the rigor with which he and his followers developed them, using axioms and logic to build a system of mathematics, was a fine legacy for those who succeeded him. Boole was not directly involved, but the event spurred him to set down his ideas concerning how logic could be formalized with mathematics, in his 1847 essay Mathematical Analysis of Logic. The second half of the chessboard Recent thinkers have used the chessboard problem as a metaphor for the rate of change in technology over recent years. He lived frugally, but managed to buy a farm for his extended family and leased out plots to generate income. Just as Euclid had done in the 3rd century BCE, scholars including Gottlob Frege and Bertrand Russell sought to discover the logical foundations on which mathematical principles are based. The modern consensus is that Leibniz and Newton developed their ideas on the subject independently. The Babylonian and Assyrian civilizations have perished...yet Babylonian mathematics is still interesting, and the Babylonian scale of 60 is still used in astronomical tables produced by poorly paid and unreliable workers. Each number is the sum of the two adjacent numbers in the row above. The result is > 1 for points within it. Pacioli also devised a method of accounting that is still in use today. Typically, plotting two variables, such as height and age, on a graph creates a messy scatter of data points that cannot be linked by a neat line. The Pythagoreans were convinced that the Universe was based on numbers; they also believed that all numbers could be described as the ratio of two integers. It works for ordinary addition and multiplication, but not for subtraction and division. One of the pioneers of this shift to mathematical logic was British mathematician George Boole, who sought to apply methods from the emerging field of symbolic algebra to logic. In "Note G," Lovelace described possibly the first computer algorithm, "to show an implicit function can be worked out by the engine without human head and hands first." She also theorized that the engine could solve problems by repeating a series of instructions—a process known today as "looping." Lovelace envisaged a program card, or set of cards, that returned repeatedly to its original position to work on the next data card or set. The advantage of logarithms is that when numbers need to be multiplied, the calculation can be simplified by adding their logarithms instead. Sometimes the discovery would come as a game-changing revelation, providing a way into unexplored fields; at other times it was a case of "standing on the shoulders of giants," 17 developing the ideas of previous thinkers, or finding practical applications for them. In 1809, Carl Friedrich Gauss pioneered normal distribution as a useful statistical tool in its own right. Al-Khwarizmi described rational numbers as "inaudible" and irrational numbers as "inaudible" Al-Khwarizmi's work was developed further by Egyptian mathematician Abu Kamil Shuja ibn Aslam (c. Area is measured in square units, such as square inches (in2). Getty Images: Matt Cardy / Stringer / Getty Images News (clb). Wellcome Collection (cla). With the limited computational power available at the time, Julia and Fatou were unable to see the true significance of their discovery, but they had found what would become known as the Julia set. The six degrees of separation theory shows how any two seemingly unconnected people can be connected in no more than six steps by their friends and acquaintances. Hardy went to Putney, London, to visit his protégé Srinivasa Ramanujan, who was unwell. In creating this code, Shannon drew on binary mathematics, the idea that figures can be represented by just 0s and 1s, which had been developed by Gottfried Leibniz. He calculated upper and lower limits for n by using Pythagoras's theorem—that the area of the square o me, everything turns into mathematics. Argand's was the first truly rigorous proof of the FTA. 1809 Carl Friedrich Gauss develops the least squares method of finding the best fit line for a scatter of data. Tile patterns have been a feature of art and construction for millennia, especially in the Islamic world. Each ring describes a set of numbers. A search for new methods Over the centuries, as mathematics developed, problems became longer and more complex, and scholars sought new ways to shorten and simplify them. He graduated in mathematics in 1904 from the 549 University of Amsterdam, where he also taught from 1909 to 1951. Although they did the same work as the male code breakers, Clarke and the other Bletchley women were paid less. The bigger the number, the more pairs of primes can create it, so it seems highly likely that the conjecture is valid and no exception will be found. The principal object of Algebra... is to determine the value of quantities which were before unknown... by considering attentively the conditions given... expressed in known numbers. He showed that even the most powerful computers could not produce accurate long-term weather forecasts. The total volume is the sum of the volumes of the layers. First articulated by James Gregory in his 1668 Geometry), it was then generalized by Isaac Barrow in 1670, and formalized in 1823 by Augustin-Louis Cauchy. In a clash with Nearchus, Zeno is reported to have bitten off the man's ear. 1799-1824 Italian scholar Paolo Ruffini and Norwegian mathematician Niels Henrik Abel show that no algebraic formulas exist for equations involving terms to the power of 5 and higher. Many mathematicians attempted to reconstruct Fermat's claimed proof after his death, or to find their own. Polar coordinates are used to help manipulate complex numbers plotted on a plane, especially for multiplication. See also place value system. However, it would be more than a thousand years before standard algebraic notation was accepted. Ada Lovelace ADA LOVELACE Born Augusta Byron in London in 1815, Ada, Countess of Lovelace, was the only legitimate child of the poet Lord Byron. In 2015, NASA scientists applied this method to compute the time it took the spacecraft Dawn to orbit Ceres, a dwarf planet in the asteroid belt between Mars and Jupiter. If a machine is expected to be infallible, it cannot also be intelligent. AFTER 1960s The New Mathematics movement, which focuses on set theory, becomes popular in American and European schools. In 451 1913, Hardy invited Ramanujan to work with him in Cambridge. 250 BCE On the Measurement of a Circle c. Argand proved that the solution for every equation built from complex numbers could be found among the complex number system with base 2. When her application to enlist in the US Navy was rejected, she joined the Naval Reserve and began her transition to computer science. Signals received by the dish will reflect off the parabola and be directed to one single point—the receiver. Mathematicians realized that this was a fundamental property of natural growth—a repetition of a pattern on many scales, from the macro to the micro. Del Ferro soon had competition from Niccolò Fontana (known as Tartaglia, or "the stutterer"). Linear transformations, reflections, enlargements, stretches, and shears (lines that slide parallel to a fixed line, in proportion to their distance from the fixed line). Many of these ideas were published by l'Hôpital in Infinitesimal Calculus. He worked on fractional exponents and infinite series and was the first to prove the divergence of harmonic series, but his proof was lost and the theory was not proven again until the 1600s. They used them to represent complex as numbers) as vectors plotted in two, three, or more dimensions. How can maths help us predict the future? He demonstrated that, provided the sample — averaged out into a bell-shaped curve, later named the "normal distribution" by German mathematician Carl Gauss Argand's work allowed complex numbers to be represented geometrically. The diagram above shows the results of applying one symmetry after another to e, the identity element (rotation through 0°), and how they are written— $\rho 2\sigma$  (the last equilateral triangle in the diagram) means "rotate through 120 degrees twice and reflect." Axioms of group theory Group theory has four main axioms. This idea was pioneered as long ago as the 1370s, when a French monk called Nicole Oresme used rectangular coordinates and the geometric forms created by his results to understand, for instance, the relationship of elements such as speed and time, or the links between heat intensity and the degree of expansion due to heat. The binomial theorem and a Bach fugue are, in the long run, more important than all the battles of history. By this rule the calculation of the 91 circumference of a circle with a diameter of 20,000 can be approached." This works out as [8(100 + 4) + 62,000] ÷ 20,000 = 62,832 ÷ 20,000 = 3.1416. Fuzzy logic is an offshoot of the fuzzy set theory 489 developed in 1965 by Lotfi Zadeh, an Iranian-American computer scientist. What distinguished the Babylonians from neighbors such as Egypt was their use of a positional (place value) number system. It states that if one body is moving with a uniform speed equal to the mean speed of the first body, and both bodies are moving for the same duration, they will cover the same distance. He even devised a version of the Möbius strip—a surface that has only one side and one edge—a few months before August Möbius. He argued that it is possible but highly unlikely, and made the distinction between a physical certainty—an event that can happen physically, like the balancing cone—and a practical certainty, which is so unlikely that in practical terms it is considered impossible. You would not be able to gather in the string without pulling it beyond the Universe. Some may be familiar, others less so. As the ice cube melts, the heavier blue dye sinks 375 Movements and rotations Complex numbers are two-dimensional: they are made up of a real and an imaginary part, for example, 1 + 2i. He lectured in all areas of mathematics, specializing in calculus, but wrote mostly on the philosophy of the subject, bringing the two disciplines 547 together to almost single-handedly invent modern mathematical logic. The Turing machine consists of a head that reads data from an infinitely long tape. Around this time, Descartes began to formulate philosophical ideas and mathematical theorems. Newton v. Algebra breaks free Al-Karaji's discovery of the binomial theorem helped to open the way for the full development of algebra, by allowing mathematicians to manipulate complicated algebraic expressions. The uranium atom is one example of a heavy atom whose nucleus follows the same statistical behavior as prime numbers, making it extremely difficult to predict. Key works 1968 Finite groups 1986 "Classifying the finite simple groups" See also: The Platonic solids • Algebra • Projective geometry • Group theory • Cryptography • Proving Fermat's last theorem 523 IN CONTEXT KEY FIGURE Andrew Wiles (1953-) FIELD Number theory BEFORE 1637 Pierre de Fermat states that there are no sets of positive whole numbers x, y, and z that satisfy the equation xn + yn = zn, where n is greater than 2. At the age of 19, he wrote to Leonhard Euler, who recognized his talent. Aged just 21, Shannon published in 1937. Today, mathematicians and physicists use coordinates to go much further than that and to imagine a space with any number of dimensions. He then explored the effect of calculating with zero. He died in London in 1754. He later attended a Jesuit college, then went to study law in Poitiers. Among these was philosopher, scientist, and mathematician René Descartes, whose methodical approach to problem-solving set the scene for the modern scientific era. This need arose from the discovery of transcendental numbers, 406 such as n and e, which are irrational, infinitely long, and are not themselves an algebraic root. 1859 By merging two complex planes, Bernhard Riemann develops a 4-D surface to help him analyze complex functions. For many centuries, Europeans used Roman numerals to record numbers and to do calculations. 1300 BCE The Berlin papyrus is produced. The formula for this is A = P(1 + r/n)nt, where n is the number of times the interest is calculated in each year. In 1822, Fourier was made the secretary of the French Academy of Sciences, a post he held until his death in 1830. Apollonius wrote on many subjects, including optics (how light rays travel) and astronomy, as well as geometry. See also: The golden ratio • The problem of maxima • Fractals 500 IN CONTEXT KEY FIGURE Benoit Mandelbrot (1924-2010) FIELDS Geometry, topology BEFORE c. In Liber Abaci (1202), for example, he described how to solve many of the problems encountered in commerce, including calculating profit margins and converting currencies. A theory of types Russell went on to produce his own response to his paradox, developing a "theory") by creating a hierarchy so that "the set of all sets" would be treated differently from its constituent smaller sets. 300 BCE Euclid's Elements contains an axiomatic approach to geometry. Book V addresses the idea of ratio and proportion, inspired by the thinking of the Greek mathematician and astronomer Eudoxus of Cnidus. She has published textbooks for Hodder Education and managed publications for the UK's Association of Teachers of Mathematics. By drawing a line from each endpoint to C, where the circles intersect, he created a triangle with sides AB, AC, and BC of equal length. La Géométrie contains much discussion about curves, which were the subject of renewed interest in the 1600s—partly because treatises by ancient Greek mathematicians had been newly translated, but also because curves featured prominently in fields of scientific exploration such as astronomy and mechanics. The equation describing a particular bellshaped curve called the "witch of Agnesi" is named in her honor, although "witch" was a mistranslation from the Italian word for "curve." See also: Trigonometry • Algebra • Calculus JOHANN LAMBERT 1728-77 Lambert was a Swiss-German polymath, born in Mulhouse (now in France), who taught himself math, philosophy, and Asian languages. 48 Thales of Miletus, one of the Seven Sages of ancient Greece, possibly inspired the younger Pythagoras with his geometrical and scientific ideas. A classic conundrum was how to produce a cube that was twice the volume of another cube. The more closely a face aligns with the mask, the more beautiful it supposedly is. See also: Positional numbers • Diophantus (c. Babbage only managed to part-build the machine, but even this was able to process complex calculations accurately in moments. In binary, as in all modern number systems whatever their base, the principles of place value are always the same. He wrote a book on geography, helped construct a world map, took part in a project to determine the circumference of Earth, developed the astrolabe (an earlier 148 Greek tool for navigation), and compiled a set of astronomical tables. See also: Topology • 23 problems for the 20th century • The logic of mathematics EUPHEMIA LOFTON HAYNES 1890-1980 Born in Washington, DC, Lofton Haynes was the first African-American woman to gain a doctorate in mathematics. JOHN FARNDON A widely published author of popular books on science and nature, John Farndon has been shortlisted five times for the Royal Society's Young People's Science Book Prize, among other awards. At 18, Mary met renowned mathematician George Boole (who, like her, was self-taught) in Ireland. There is also no such thince the second s as subtraction in Boole's algebra. The probability of the needle crossing a line can be calculated by multiplying each side of the formula by p, then dividing each side by  $\pi$  to get  $p \approx 2l \div \pi d$ . There is no royal road to geometry. The binomial theorem simplifies the problem by unlocking the pattern in the coefficients— numbers, such as 2 in 2ab, by which the unknown terms are multiplied. 219 Kepler then used his cylinder method to find the barrel shapes with the maximum volume. The dichotomy paradox shows the absurdity of the pluralist view that motion can be divided. See also: Wheat on a chessboard • Mersenne primes • Binary numbers 413 IN CONTEXT KEY FIGURE Henri Poincaré (1854-1912) FIELD Geometry BEFORE 1736 Leonhard Euler solves the historical topology" as a mathematical subject. The hammers of weights 12 and 9 produced a harmonious sound—a perfect fourth—as their weights were in the ratio 4:3. Brahmagupta went on to describe the effect of subtracting numbers from zero. E. Rijksmuseum, Amsterdam: Gift of J. Also called an index. Every year, the best abacists from across Japan take part in the Soroban Championship. To compare it with the Cartesian system, the polar axis would be the Cartesian system. Cartesian coordinates (1,0). False start In 1879, British mathematician Alfred Kempe claimed a proof for the four-color theorem in the scientific knowledge, and wisdom, but in 415 she was killed by Christian zealots for her "pagan" philosophy. A staunch republican, he was arrested in 1831 and imprisoned for eight months. BLAISE PASCAL Born in Clermont-Ferrand, France, in 1623, Blaise Pascal was a mathematics prodigy. dy/dx Invented by Leibniz for differentiation. For example, 0.1010010001... with an extra zero between each successive pair of 1s, continuing indefinitely, is an irrational number. AFTER 1657 Christiaan Huygens writes a treatise on probability theory and its applications to games of chance. 1630 Pierre de Fermat uses a new technique for finding tangents to curves, locating their maximum and minimum points. 1040 Also known as Alhazen, Ibn al-Haytham was an Arab mathematician and astronomer, born in Basra, now in Irag, who worked at the court of the Fatimid Caliphate in Cairo. The calculation for the power 2 is manageable, but for greater powers, the resulting expression becomes increasingly complicated. He began by imagining a machine that was able to say whether any algorithm (A) would halt (provide an answer and stop running) when given an input to which the answer was either Yes or No. Turing was not concerned with the physical mechanics of such a machine. This proved prescient; the math world rose to the challenge. It dominated human conceptions of space and number for more than 2,000 years and was the standard geometrical textbook until the start of the 1900s. Within a large square there are four rightangled triangles of equal size (sides labeled a, b, and c). While practical notions of negative quantities were used from ancient times, particularly in China, negative quantities were used from ancient times, particularly in China, negative quantities were used from ancient times. 1642-44 In France, Blaise Pascal creates a calculating machine. Problems which can be constructed by means of circles and straight lines only. In 762 CE, Caliph al-Mansur established a capital in Baghdad, which swiftly became a major center of culture, learning, and commerce. By the second millennium CE, the suanpan and its counting methods pread across Asia. Matt performs live comedy with Festival of the Spoken Nerd and once calculated pi live in front of a sold-out Royal Albert Hall. 1960 American mathematicians began to were becomind look at geometric objects differently, considering the global properties of shapes beyond the confines of lines and angles. See also: Algebra • The fundamental theorem of algebra & The fundamental theorem of a girls' school in Wismar, northern Germany, Frege studied mathematics, physics, chemistry, and philosophy at the universities of Jena and Göttingen. Cantor himself was sure this was true, but he could not prove it. A successful route needs land areas (nodes or vertices) to have an even number of bridges (arcs) to enter and exit by. 1711 Abraham de Moivre's De Mensura Sortis (On the Measurement of Chance), describes what is later known as the Poisson distribution. properties. Translation A function that moves an object a certain distance in a direction without affecting its shape, size, or orientation. The curvature of an imaginary body can be a mathematician Nicholas Chuquet used superscripts in 1484 to represent exponents ("to the power of"), but did not record them as such; for example, 6x2 was 6.2. It took more than 150 years for superscripts to become common; René Descartes used recognizable examples in 1637 when writing 3x + 5x3, yet continued to write x2 as xx. Like the complex plane, simple quaternion mathematics, combined with basic trigonometry, offers a way to describe all kinds of movements within threedimensional space. The popular view that scientists proceed inexorably from well-established fact, never being influenced by any unproved conjecture, is quite mistaken. Proposition 7 Given two straight lines constructed on a straight line (from its extremities) and meeting in a point, there cannot be constructed on the same straight line (from its extremities), and on the same side of it, two other straight lines meeting in another point and equal to the former two respectively, namely each to that which starts at the same extremity. cannonballs was critical for his designs, which led him to pioneer the study of ballistics. The soroban is also used for far more complex calculations. 571 Statistics (1) Measurable data collected in an orderly way for any purpose. To explain the theorem slightly differently, for a large 422 number x, the average gap between primes from 1 to x is approximately ln(x). The first line was of infinite length, while the second was of fixed length. Cone A 3-D shape with a circular base and a side that narrows upward toward a point (apex). Gauss acknowledged its validity, but claimed to have discovered it first. (tl). Science Photo Library: Frederic Woirgard / Look At Sciences (bl). In Stevin's new notation, numbers that would previously have been written as the sum of fractions—for example, 32 + 5/10 + 6/100 + 7/1,000—could now be written as a single number. Perpendicular lines (p, q, and r) are drawn from that point to each of the three sides, meeting each side at 90°. According to the story, the inventor of chess, Sissa ben Dahir, was summoned to an audience with his ruler, King Sharim. Statistics investigates whether the pattern of recorded data is significant or random. However, in his Utriusque Arithmetices, published in 1536, a scholar known to us only as Hudalrichus Regius pointed out that 211 - 1 = 2,047. AFTER 1872 Ludwig Boltzmann uses statistical mechanics to show how the thermodynamics of a system always results in an increase in entropy. Science Photo Library: Dr Mitsuo Ohtsuki (cb). While in the US, Wiles made contributions to some of the most elusive problems in his field, including the Taniyama-Shimura conjecture. As alKaraji discovered, the coefficients can be laid out in a grid, with the columns showing the coefficients needed for multiplying out each power. The Lorenz attractor, which revealed the butterfly effect, is a fractal. Vector A mathematical or physical quantity that has both magnitude and direction. Getty Images: Bettmann (tr). It might be expected that in any large set of numbers, those that start with the digit 3 would occur with roughly the same frequency as those that start with any other digit. Boole's binaries In 1854, Boole published his most important work, An investigation into the laws of thought. Alamy Stock Photo: The History Collection (b). He believed that nontrivial zeros have one thing in common: when they are plotted on the complex plane, they all lie on "the critical line," where the real part of the number is 0.5. This belief is called the Riemann hypothesis. Mostly, the bodies do not move in rhythmic, repeating patterns. Gauss used a method called "least squares," which involves adding up the squares of the data; this is still used by statisticians. He solved cubic equations, too, inspired by al-Khwarizmi's use of Euclid's geometrical constructions for working out quadratic equations. He argued that such a curve could only exist if the Taniyama-Shimura conjecture were false, in which case Fermat's last theorem would also be false. Recorde later tried to sue Pembroke for misconduct, but was instead countersued for libel. Over the course of three games, the first player wins twice and the other once. First-century Indian numerals did not use zero. The used as the value of π, until latter is 2.1 percent greater than π. His proof contained an unproved assumption that d'Alembert had avoided—that a polynomial will always have roots. If this were not true, the object would move. Instead of constructing a prototype computer, Turing used a 465 thought experiment in order to solve the Entscheidungsproblem (decision problem) that had been posed by German mathematician David Hilbert in 1928. 2480 BCE Stone carvings record flood levels on the River Nile, measured in cubits—about 201/2 in (52 thought experiment) that had been posed by German mathematician David Hilbert in 1928. cm)—and palms—about 3 in (7.5 cm). Above all, Leibniz was driven by his religious faith. Mathematics, rightly viewed, possesses not only truth, but supreme beauty. However, Johann became jealous of his brother's achievements and their relationship broke down several years before Jacob died in 1705. The solution is x = 3, m = 2, y = 2, and n = 3, since 32 379 23 = 1. However, the flow failed to handle spikelike "singularities"—deformities including "cigars" and infinitely dense "necks." Perelman, who learned much from Hamilton during a two-vear fellowship at Berkeley in the early 1990s, continued to study Ricci flow and its application to the Poincaré conjecture when he returned to Russia. Cartesian coordinates have had an immense impact in mathematics, and in many fields of science and the arts, from engineering and economics to robotics and computer animation. Connie Willis American writer 483 Strange attractor The computing power available to Lorenz in the early 1960s was unable to plot the modeled atmospheric variables in a three-dimensional space, where the values on the x, y, and z axes represented, for example, air temperature, pressure, and humidity (or triplets of other weather data). Antoine Augustin Cournot 454 Infinite monkeys Borel's law, which he called the law of single chance, gave a scale to practical certainty. Simon Stevin's notation used circles to indicate the power of ten of the denominator of the converted fraction. Plato Beyond Euclidean geometry, an area explored by two of Euclid's successors, Theodosius of Bithynia and Menelaus of Alexandria. After two years, it would be 100 × 1.03 × 1.03 = \$106.09, and after 10 years it would be \$100 × 1.0310 = \$134.39. A quaternion can define a vector, or a line in three-dimensional space, and can describe an angle and direction of rotation around that vector. This smaller result for a five-way. In topology, two figures are homeomorphic if they can be distorted or stretched into the same shape. Magic squares have been an enduring source of fascination for mathematicians. Oscillation A regular to-and-fro movement between one position or value to another and back again. A new geometry Although it was building on the findings of earlier mathematicians. The entire sum could be written out following this pattern. Like zero, negative numbers were an abstract concept rather than positive values such as lengths or quantities. He was an excellent geometer and shared his ideas with his mathematical friends. In 1977, they developed the RSA algorithm (named for their initials), an encryption procedure that won them the Turing Award in 2002. Russell's barber paradox contradicted Frege's Basic Laws of Arithmetic concerning the logic of mathematics, which Russell had pointed out in a letter to 442 Frege in 1902. Fractions—so named for the Latin word fractio, meaning "break"—were used from around 1800 BCE in Egypt to express parts of a whole. René Descartes had first made the link between shapes and numbers in the 1600s with coordinate geometry, turning geometry into algebra. In the Chinese rod numeral system, red indicates positive numbers, while black indicates positive numbers. Complex numbers take the form a + bi, where a and b are real and i = . Bill Gates A vital concept 137 Mathematics without zero would mean many of the articles in this book could not have been written: there would be no negative numbers, no coordinate systems, no binary systems (and hence no computers), no decimals, and no calculus, because it would not be possible to describe infinitesimally small quantities. If event A occurs, it alters the probability of event B happening, and vice versa. The solution is to have a supply of fresh primes. Root (1) The root of a number, which is another number that when multiplied gives the original number. This spiral crops up commonly in nature: for example, the seeds of pine cones, sunflowers, and coneflowers tend to grow in golden spirals. He became a teacher and traveled throughout Italy. In so doing he created a new field of mathematics, called analytic geometry, where shapes are described in terms of their coordinates and the relationships between a pair of variables, x and y. 1250) FIELD Number theory BEFORE 200 BCE The number sequence later known as the Fibonacci s meters. The Arithmetica of Diophantus strongly influenced 17th-century mathematicians as the study of modern algebra developed. Standard set theory allows degrees of membership or a continuum. Quadratic equations Al-Khwarizmi did not employ symbols; he wrote his equations in words, supported by diagrams. As in algebra, Boole's use of symbols and connectives allowed for the simplification of logical expressions. If all the triangles face outward, the resulting curve takes on the appearance of a snowflake. mathematical constant pi. 300 BCE In his book Elements, Euclid establishes the geometry of 3-D space. Order See degree. Alamy Stock Photo: Ancient Art and Architecture (tr). The results had to be verified before they could be published—and Wiles's proof covered scores of pages. A symbol after a whole number (the standard notation now is a decimal point) can also separate it from its fractional parts, each representing a tenth of the place value of the preceding figure. Harmonic analysis (in which Langlands specialized) is the mathematical study of waveforms, exploring how they can be broken down to sine waves. When Pythagoras used the same ratios that produced harmoniously sounding hammers, and applied them to vibrating strings, he similarly produced notes in harmony with one another. In the 5th century CE, the philosopher Proclus argued that the PP was a 363 theorem that could be derived from the other postulates and should therefore be struck out. These colleagues were completely astounded when Wiles revealed that he had been working on Fermat while still carrying out his daily tasks of lecturing, writing, and teaching. Some natural structures, such as a honeycomb, tessellate. GOTTFRIED LEIBNIZ Born in Leipzig, Germany, in 1646, Gottfried Leibniz was raised in an academic family. The reclusive Perelman did not enjoy the fame his proof brought him. According to Poincaré, if it could be could shown that the Universe did not contain holes, then you could model it as a "3-sphere." To establish whether it contained holes, you could, in theory, conduct an experiment with string. Alamy Stock Photo: Chronicle (cla). For a long time, the catenary's shape was believed to be the same as that of a parabola. Following on from the work of Gilles de Roberval in finding the area under a cycloid, Leibniz and Newton worked on the problems of calculation of such things as continuous change and acceleration, which had puzzled mathematicians ever since Zeno of Elea had presented his famous paradoxes of motion in ancient Greece. 10-C. 1654 Pierre de Fermat and Blaise Pascal develop probability theory. Renaissance art also influenced mathematicians such as Luca Pacioli, Gerolamo Cardano, and Tartaglia had written about how to work out the chances of dice rolling particular numbers or hands of cards coming out a certain way. Newton's subsequent Principia is said by some to have been influenced by Leibniz's work. (2) In graph theory, a graph is a collection of points, called vertices, and lines, called edges, that can be used to model theoretical and real networks, relations, and processes in a range of scientific and social fields. The use of a specific symbol— a simple circle, rather than a blank space or placeholder—to denote zero is attributed to the brilliant mathematician Brahmagupta, who described the rules of its use in calculation. • The Poisson distribution An indispensable tool in applied mathematics • Bessel functions I have created another world out of nothing • Non-Euclidean geometries Algebraic structures have symmetries • Group theory Just like a pocket map • Quaternions Powers of natural numbers are almost never consecutive • Catalan's conjecture The matrix is everywhere • Matrices An investigation into the laws of thought • Boolean algebra A shape with just one side • The Möbius strip The music of the primes • The Riemann hypothesis Some infinities are bigger than others • Transfinite numbers A diagrammatic representation of reasonings • Venn diagrams The tower will fall and the world will end • The Tower of Hanoi Size and shape do not matter, only connections • Topology Lost in that silent, measured space • The prime number theorem MODERN MATHEMATICS 1900-PRESENT The veil behind which the future lies hidden • 23 problems for the 20th century Statistics is the grammar of science • The birth of modern statistics A freer logic emancipates us • The logic of mathematics The Universe is four-dimensional • Minkowski space Rather a dull number • Taxicab numbers A million monkeys banging on a million typewriters • The infinite monkey theorem She changed the face of algebra • Emmy Noether and abstract algebra Structures are the weapons of the mathematician • The Bourbaki group A single machine 6 Small things are more numerous than large things • Benford's law A blueprint for the digital age • Information theory We are all just six steps away from each other • Six degrees of separation A small positive vibration can change the entire cosmos • The butterfly effect Logically things can only partly be true • Fuzzy logic A grand unifying theory of mathematics • The Langlands Program Another roof, another proof • Social mathematics • The Penrose tile Endless variety and unlimited complication • Fractals Four colors but no more • The four-color theorem Securing data with a one-way calculation • Cryptography Jewels strung on an as-yet invisible thread • Finite simple groups A truly marvelous proof • Proving Fermat's last theorem No other recognition is needed • Proving the Poincaré conjecture DIRECTORY GLOSSARY CONTRIBUTORS QUOTATIONS ACKNOWLEDGMENTS COPYRIGHT 7 How to use this eBook Preferred application settings are recommended: Color theme: White background Font size: At the smallest point size Orientation: Landscape (for screen sizes over 9"/23cm), Portrait (for screen sizes below 9"/23cm) Scrolling view: [OFF] Text alignment: Auto-justification [OFF] (if the eBook reader has this feature) Font style: Publisher default setting [ON] (if the eBook reader has this feature) Images: Double tap on the images to see them in full screen and be able to zoom in on them 8 FOREWORD Summarizing all of mathematics in one book is a daunting and indeed impossible task. No observational problem will not be solved by more data. However, Pascal found that the triangle could be used to find the number of possible combinations when you choose a number of objects from a

particular number of available options. The fifth postulate is more elaborate than the other four, and Euclid himself seems to have been wary of it. 24 The Babylonian sun-god Shamash awards a rod and a coiled rope, ancient measuring devices, to newly trained surveyors, on a clay tablet dating from around 1000 BCE. Certain numbers—such as the constants II, e, or the prime and irrational numbers—hold a special fascination and have become the subject of considerable study. Its equation involves e. Transformations in two dimensions are encoded by 2 × 2 matrices, while 3-D transformations in two dimensions are encoded by 2 × 2 matrices. Taxicab numbers were only one area of Hardy and Ramanujan's work. 300 BCE Euclid proves the fundamental theorem of arithmetic that every integer greater than 1 can be expressed as a product of primes in only one way. These ideas were taken up by André Weil and others, using the pseudonym Nicolas Bourbaki. The Babylonians also based their calendar year on the solar year (365.24 days); the number of days in a year was 360 (6 × 60) with additional days for festivals. 276-c. One approach to proving the conjecture appeared to involve making many calculations: in 1976, Robert Tijdeman found an upper bound (maximum size) for x, y, m, and n. AFTER 1976 Dutch number theorist Robert Tijdeman proves that, if more consecutive powers exist, there are only a finite number of them. A new mathematics building at Oxford—as well as an asteroid—9999 Wiles—have been named after him. This was followed in 1916 by the Sierpinski triangle, or Sierpinski triangle Paris, and at the University of Oxford, where he was appointed Regius Professor of Mathematics in 2018. The velocity of the second particle was proportional to the distance remaining to the end of the line. After gaining a doctorate from Yale University in 1934, she taught for several years before the outbreak of World War II. 293 The exponential function can be used to calculate compound interest. Babbage's work in turn inspired Ada Lovelace to devise the forerunner of modern computer algorithms. In the first six months, the number of pairs has increased in the sequence 1, 1, 2, 3, 5, and 8. The average of the two numbers will also be rational, as will the average of that number and either of the number. Cantor created a new transfinite cardinal number. (8), the first letter of the Hebrew alphabet—to denote a set containing an infinite number of elements. 84 The suanpan shown here is set to the number 917,470,346. In the 1600s, English cleric Thomas Malthus posited that population also increases exponentially if there are no checks on its growth, such as war, famine, or food shortages. Over centuries, different "weak" versions of the conjecture have been proved, but no one to date has proved the strong conjecture, which seems destined to defeat even the brightest minds. Although Bombelli's notation was advanced for his time, the use of algebraic symbols was still in its infancy. AFTER 1972 Hal Varian, an American economist, suggests using Benford's law to detect fraud. Avalon: Frances M. I cannot help thinking that Zu Chongzhi was a genius of Antiquity. The Enigma code Another method of increasing the security of encryption is to use a polyalphabetic cipher, where a letter in plaintext can be substituted for several different letters in ciphertext, thus removing the possibility of frequency analysis. To carbon-date organic material, researchers test a sample—here from an ancient human bone—and use Euler's number to calculate its age from the rate of radioactive decay. The hammers in the forge had individual weights of 6, 8, 9, and 12 units. Mode The value that occurs most often in a set of data. This wheel has rolled over a piece of gum. The Rhind and Moscow papyri are the most complete mathematical documents to survive from the height of the ancient Egyptian civilization. Studying even just a sample of mathematics is a great reminder of how much humans have achieved. Catalan was particularly interested in geometry and combinatorics (counting and arranging), and his name is associated with the Catalan numbers. The number of arcs meeting at a node is called its degree. Shapes with five-fold symmetry, such as pentagons and stars, can also be identified. The sequence of bits changes at each move. His approach involved establishing two related concepts that are now at the heart of statistical analysis: correlation and regression. Clarke publishes a study, based on the Poisson distribution, of patterns of V-1 and V-2 flying bomb impacts on London. Numbers in harmony Music was of great importance to Pythagoras. In addition, Wiles solved the theorem using advanced mathematical tools and ideas invented long after Fermat. When numbers are made up, they tend to have a more equal distribution of leading digits than if they followed Benford's law. MUKUL PATEL Mukul Patel, who studied mathematics at Imperial College, London, writes and collaborates across many disciplines. Astronomer Johannes Kepler is best known for his discovery of the elliptical shape of the planets' orbits and his three laws of planetary motion, but he also made a major contribution to mathematics. It says that every element has a unique inverse element; combining the two yields the identity element. Mean An average found by adding up the values of a set of data and dividing by the number of values. He established trigonometry—described methods for calculating sine tables. Some quantities can be represented using coordinates known as vectors, and exist in a purely mathematical "vector space." Vectors are quantities with two values, which can be plotted as a magnitude (the length of a line) and a direction. He turned to American mathematician Nick Katz to check his reasonings. 1732 Daniel Bernoulli uses what later become known as Bessel functions to study the vibrations of a swinging chain. Surd An expression that includes a root that is an irrational number such as . For example, 19 can be written as the sum of three triangular numbers: 1 + 3 + 15 = 19. This is why attempts to achieve an impossible task are sometimes known as "squaring the circle." 90 Another way mathematicians have attempted to square the circle is to slice it into sections and rearrange them into a rectangular shape. It comes in many forms, but all of them work on the same principles: values of different sizes are represented by "counters" arranged in columns or rows. Topology The branch of mathematics that studies surfaces and objects by examining how their parts are connected rather than according to their exact geometrical shapes. 337 BCE Eudoxus lived in the Greek city of Cnidus (now in Turkey). The publisher would like to thank the following for their kind permission to reproduce their photographs: (Key: a-above; b-below/bottom; c-center; f-far; l-left; r-right; t-top) 25 Getty Images: Universal History Archive / Universal Images Group (crb). Although proving the Riemann hypothesis would validate the zeta function's status as the best predictor of the distribution of primes, it still would not allow prime numbers to be fully predicted. The idea of zero as a number was not accepted for centuries. Huygens's idea was to constrain the pendulum by adding cycloid-shaped "cheeks." In theory, the time of each movement would now be the same from any starting point. The 84 problems in the Rhind papyrus illustrate the mathematical methods in common use in ancient Egypt. Much of his work survives only in fragments, but 100 his most influential, Conics, is relatively intact. This concept inspired Einstein's theory of general 447 relativity in 1915, but by then, Minkowski was dead-killed at 44 years old by a ruptured appendix. The Babylonian positional number system, developed c. This ultimately led to "Euler's identity", eiπ+ 1 = 0, an equation that connects five of the most fundamental mathematical quantities (e, i, π, 0, and 1) in a very simple way. The merchant's rod is submerged to an equal extent when pushed at a diagonal into these two barrels, so he charges the same price for both. Turing and others realized that this simple concept could be used as a "computer." At the time, the term "computer." At the term "computer." At the time, the term "computer." At the term "compute calculations. (Napier later settled on the name "logarithms," derived by combining the Greek words logos, meaning proportion, and arithmos, meaning proportion, and arithmos, meaning number.) Adding the two logarithms, and then converting the answer back to an ordinary number, produces the result of multiplying the original numbers. Ellipses have a long history in mathematics. At the International Mathematical Congress in Paris in 1900, he confidently announced his choice of 23 questions that he believed should occupy mathematicians' thoughts in the decades to come. When it introduced the metric system, France also tried to introduce a decimal system for time; there would be 10 hours in a day, 100 minutes in each hour, and 100 seconds in each minute. Each of the five regular convex polyhedra—3-D shapes with flat faces and straight edges—has its own set of identical polygonal faces, the same number of faces meeting at each vertex, as well as equilateral sides, and same-sized angles. This profound connection means the hypothesis may one day be proved not by a mathematician, but by a physicist. Pascal's triangle made it simple to find probabilities. Isosceles triangle A triangle with two sides the same length and two angles the same length and two angles the same length and two angles the same size. The bottom line represented 0 to 4; the line above counted 5s, and the lines above that 10s, 50s, and so on. Product The result of one number or quantity being multiplied by another. The 15th-century Italian mathematicians had accepted the necessity of negative and complex numbers), collected magic squares. Once mathematicians had accepted the necessity of negative and complex numbers for solving certain equations, the question remained as to whether finding roots of higher-degree polynomials would require the introduction of yet new types of number. Parabola, or to a function based on it, such as a quadratic function, which produces a parabola, or to a function, which produces a parabola, or to a function based on it, such as a quadratic function, which produces a parabola, or to a function based on it, such as a quadratic function based o intermediary proposition used to solve a bigger theorem. Methods of solving equations, even quite complex quadratic equations, had been discovered as early as the ancient Babylonians, but it was medieval mathematicians of the Islamic Golden Age who pioneered the use of symbols to simplify the process, giving us the word "algebra," which is derived from the Arabic al-jabr. A rigorous definition of the laws of arithmetic and geometry had emerged by the 1800s, with the work of George Boole, Gottlob Frege, Georg Cantor, Giuseppe Peano, and, in 1899, David Hilbert's Foundations of Geometry. Then all multiples of 2 are crossed out except 2 itself. AFTER 628 CE The Indian mathematician Brahmagupta provides rules for arithmetic with negative numbers. In Boole's algebra, there are only two possible values for any quantity, either 1 or 0. Within weeks, Wiles and Taylor had plugged the gap in the proof. Mesmerizingly beautiful, such images produced with fractal-generating software make popular screen savers. Statistics is the branch of mathematics that is concerned with analyzing and interpreting large quantities of data. theorem has been proved," could be accepted. Determinism was equated with predictability before Lorenz. He was intrigued when the wine merchant at the wedding feast measured the wine in the barrel by sticking a rod diagonally through a hole in the top and checking 217 how far up the stick the wine went. It was a unifying element in his concept of Harmonia, the joining together of the cosmos and the psyche. Finding the others (the "nontrivial zeros" — all other values for which  $\zeta(s) = 0$ ) is more difficult. This is used to find the area of cyclic quadrilaterals, which are four-sided shapes inscribed within a circle. In using mathematics to explain the movement of the planets and of objects on Earth, Isaac Newton fundamentally changed the way we see the Universe. Leibniz used very different notation from Newton's, and in 1684 published a paper on what would later become known as differential calculus. From this basis, Euclid laid out the principles of mathematical proof from axiomatic truths in his Elements, a treatise that was the foundation of mathematics for the next two millennia. This means that one generations back, it has three "great grandparents"—its grandmother's two parents and its grandfather's mother. In the 1800s, analysis and geometry were the leading fields of mathematics, while algebra was considerably less popular. She was working at Stanford University when she died of breast cancer, aged 40. See also convergence. Graph theory and topology began with Leonhard Euler's attempt to find a solution to a mathematical puzzle—whether it was possible to make a circuit of the seven bridges in Königsberg (now Kaliningrad, Russia) without crossing any bridge twice. Polar coordinates, determined by distance and angles from the center of Earth, are also useful for certain types of calculation. The gates can be connected to lights or buzzers which go on and off depending on the output. One solution to the challenge employs binary numbers (0 and 1). The tablet, bought by American publisher George Plimpton in 1923 and known as Plimpton 322, is etched with numerical information relating to right-angled triangles. Over the following century, mathematicians uncovered the rules for finding determinants of larger and larger arrays. Burkatovski (tr). 1928 In the US, Ralph Hartley, an electronics engineer, sees information as a measurable quantity. Euler did this by solving the equation  $x^2 - y^3 = 1$ . Radius Any straight line from the center of a circle or sphere to its circumference. Riemann did not confine his studies to geometry, however. In 1806, the key contribution of Swiss-born mathematician Jean-Robert Argand was to plot complex numbers and y for imaginary numbers. Al-Karaji's work showed how algebra could instead be based entirely on numbers, liberating it from geometry. Like II (3.1415...),  $\phi$  is an irrational number of decimals in a nonrepeating random pattern. Bolyai claimed to have "created another world out of nothing," but the idea was not well received in its time. If λ = 4 (the average number of potatoes ordered in one day), and the number of potato orders on any one day is B, the probability that B is less than or equal to 6 is 89 percent. Cosine (abbreviation cos) A function in trigonometry similar to a sine, except that it is defined as the ratio of the length of the side of a right-angled triangle adjacent to a given angle to the length of the triangle's hypotenuse. Italian mathematicians of this era would publicly challenge one another to solve cubic equations and other problems in the least possible time. In Logarithmo-technica, published in 1668, he set out a series formula for the natural logarithm ln(1 + x) = x - x2/2 + x3/3 - x4/4 + ... This was an extension of Mengoli's formulation, in which 215 the value of x was 1. Every multiplication by i, the imaginary unit, results in a 90° rotation, and a rotation of any other angle is due to some factor or fraction of i. They even explored the strange phenomenon of irrational numbers. Only Fermat succeeded. See also: The algebraic resolution of equations • Emmy Noether and abstract algebra • Finite simple groups 374 IN CONTEXT KEY FIGURE William Rowan Hamilton (1805-65) FIELD Number systems BEFORE 1572 Italy's Rafael Bombelli creates complex numbers, based on the unit 1, with imaginary numbers, based on the unit i. The simplest equation of this form is y = x2. The aim of the puzzle is simple. Everyone wanted to know how this problem had finally been solved. 542 See also: Calculus • Newton's laws of motion • The algebraic resolution of equations MARIA GAETANA AGNESI 1718-99 Born in Milan, then under Austrian Hapsburg rule, Agnesi was a child prodigy who, as a teenager, lectured friends of her father on a wide range of scientific subjects. 1843 Antoine Augustin Cournot makes a distinction between physical and practical certainty. When the pattern is traced back to 165 earlier generations, the Fibonacci sequence continues, with 13, 21, 34, 55 ancestors, and so on. The shape has a single surface; this can be tested by following the surface of the shape with the eye. Proof Any method of showing beyond doubt that a mathematical statement or result is true. In Athens two centuries later, Plato was entranced by the idea and included it in many of his dialogues. However, Ramanujan suffered from poor health. His father was a professor of moral philosophy, while his mother was the daughter of a professor of law. The complex plane is now a powerful tool that works far beyond the interests of number theory. In 1795, Pierre-Simon Laplace tried an FTA proof using the polynomial's "discriminant," a parameter determined from its coefficients which indicates the nature of its roots, such as real or imaginary. Power of exponents The growth of the values in this series is described as exponential. Pythagoras is credited with coining the term "philosopher," from the Greek philos ("love") and sophos ("wisdom"). He challenged Tartaglia to a cubic duel, but was beaten by Tartaglia's superior methods The unencrypted message is known as the plaintext, while the encrypted version is called the ciphertext. Chaos: when the present determines the future, but the approximately determines the future, but the approximately determines the future. mathematician Joseph-Louis Lagrange, used calculus to provide an equation—the Euler-Lagrange equation—for understanding both fluid (gas and liquid) and solid mechanics. While Euclid's definition of "a point" addresses a point on the plane, a point can also be understood as a point on a sphere. His successes had been overshadowed by his calculus dispute with Newton and were only recognized several years after his death. Many problems in the papyrus deal with working out shares of commodities or land. 850) FIELD Algebra BEFORE 1650 BCE The Egyptian Rhind papyrus includes solutions. Zeno was renowned for his paradoxes, which contributed to the development of mathematical rigor. Approximations to the golden ratio may have been the result of an unconscious tendency rather than any deliberate mathematical intention. With any number of disks, the formula 2n - 1 will give the minimum number of moves (where n is equal to the number of disks). The two branches of calculus, integral and differential, offered a method of analyzing such things as the slope of curves on a graph and the area beneath them as a way of describing and calculating change. In 1880, British mathematician John Venn introduced the idea of the Venn diagram in his paper "On the Diagrammatic and Mechanical Representation of Propositions and Reasonings." The Venn diagram is a way of grouping things in overlapping circles (or other curved shapes) to show the relationship between them. Amicable numbers, where the factors of each one add up to form the other. Real numbers, but not imaginary or complex numbers. 1545 Gerolamo end up to form the other. Cardano publishes techniques using determinants. That's why Hilbert risked offering a list of unsolved problems instead of presenting new methods or results. Standard deviation Although Galton's main interest was human heredity, he created a broad range of data sets. Diophantine equations are a type of polynomial—an equation in which the powers of the variables (unknown quantities) are integers, such as x3 + y4 = z5. In spite of advances in computing, no method exists that guarantees to always find the best solution, because the time this takes grows exponentially as the given number of cities increases. In 1618, he left France for the Netherlands and joined the Dutch States Army as the given number of cities increases. a mercenary. This is, in essence, the logical form known as the syllogism: two premises leading to a conclusion. This implies that the coastline cannot be measured in one dimension. It is thought that Euclid died between the mid-4th century and the mid-3rd century BCE. Originally referred to by the Indian scholar Pingala in around 200 BCE, it was later called the Fibonacci sequence after Leonardo of Pisa), an Italian mathematician known as Fibonacci. AFTER 1980s Japanese electronics companies use fuzzy logic control systems in industrial and domestic appliances. The Japanese adopted the Chinese abacus in the 14th century and developed their own abacus, the soroban, which has one bead worth five above the central bar and four beads each worth one below the bar in each column. See also: Calculus • Newton's laws of motion • Non-Euclidean geometries JULIA BOWMAN ROBINSON 1919-85 Born Julia Bowman in St. Louis, Missouri, Robinson gained her mathematics doctorate at the University of California, Berkeley, in 1948. Robinson proved, along with other mathematicians, such as Yuri Matiyasevich, that such an algorithm could not exist. Now known as irrational numbers, these numbers cannot be expressed as a ratio of integers. One grain of wheat (or rice, in some versions of the story) was to be placed on the bottom left square 171 of the chessboard. 218 Infinitesimals The problem with cylinders is that if they have thickness, their straight sides will not fit into the curve of a barrel, while cylinders without thickness have no volume. For Khayyam, the solution would always have been a positive number. HEATHER DAVIS British author and educator Heather Davis has taught mathematics for 30 years. In 1864, with five daughters to raise and no financial support, Mary returned to London, where she worked as a librarian at Queen's College, a girls' school, and later gained a reputation as an eminent children's teacher. So: (a + b) ÷ a = a ÷ b. 385 Multiplying two matrices together is achieved by multiplying the horizontal support. in the first matrix by the vertical numbers in the second (the centered dot indicates multiplication) and adding the results. When you get back to your starting point, you see the end of the significance of the variations in the data within limits set by the researcher and known as "degrees of freedom." The combination of Galton's correlation and regression, and Pearson's standard deviation in Pascal's triangle are extremely simple. One of the oldest surviving examples of a quadratic equation comes from the ancient Egyptian text known as the Berlin papyrus (c. He developed the "method of exhaustion" to prove statements about areas and volumes by successive approximations. Linear equations involve only one variable, which is expressed only to the power of 1, rather than squared or to any higher power. 1847 George Boole invents a form of algebra in which variables can have one of only two values (true or false), paving the way for symbolic, mathematical logic. In addition to his work on primes, Riemann helped to formulate the rules for applying calculus to complex functions (functions using complex numbers). Alamy Stock Photo: Granger Historical Picture Archive (br). By inventing his "sieve" to eliminate nonprimes—using a number grid and crossing off multiples of 2, 3, 5, and above —Eratosthenes made prime numbers. trading stocks, and plays a vital role in programming artificial intelligence systems. These truth tables were first used by American logician Charles Saunders Peirce in 1893, nearly 30 years after Boole's death. Reciprocal A number or expression that is the inverse of another one, meaning that the result of multiplying them together is 1. These two triangles are in perspective from a viewpoint called the center of perspectivity (P). In 1629, French mathematician Albert Girard showed that a polynomial of degree n will have n roots. The almost mystical qualities of geometry and numbers inspired Pythagoras and his followers to establish a cultlike community, dedicated to studying the mathematical principles they believed were the foundations of the Universe and everything in it. 156 The Babylonians could solve quadratic equations, using curves called conic sections—such as circles ellipses, hyperbolas, or parabolas—formed by the golden ratio appear to play an even greater role. I have shown how to find the sides of the square-square, quatro-cube, cubo-cube... to any length, which has not been [done] before now. Although they were initially used for working out geometric problems, today quadratic equations are important in many aspects of mathematics, and technology. Plato's Academy in Athens was dedicated to the study of philosophy and mathematics, and technology. 2004 American mathematicians Michael Aschbacher and Stephen D. In 1961, Michael Gurevitch, an American postgraduate student, published a landmark study of the nature of social networks. Plane geometry of 2-D figures on a flat surface. Al-Khwarizmi is regarded as the "father of algebra" for his systematic rules for solving linear and quadratic equations. Fillipo Brunelleschi had rediscovered the principles of linear 239 perspective known to the ancient Greeks and Romans, and explored them in his architectural plans, sculptures, and paintings. 1927 Austrian mathematician Emil Artin extends the reciprocity theorem to groups. Imagine you are an explorer traveling around the Universe from a set point, and unraveling a ball of string as you go. He and successors Niccolò Tartaglia, Lodovico Ferrari, and Gerolamo Cardano all worked on algebraic formulae to solve cubic equations. If he took his present problemsolving method and added its strengths to an earlier approach of his, then one might fix the other, allowing him to solve the problem. Square matrices whose elements are complex numbers, and whose determinants are not zero, form an algebraic structure called a group. It is another term for integer. His experiment was an 343 exploration of determinism, a philosophical concept that says that all future events are determined by causes in the past. If the she proved that in these commutative rings, every ideal is the unique product of prime ideals. The idea of differential calculus is that the rate at which a variable changes at a point is equal to the gradient of a tangent at that point. 1971 American mathematician Stephen Cook poses the P versus NP problem, which tries to understand why some mathematical problems can quickly be verified but would take billions of years to prove, despite computers' immense calculating power. His advisers calculated that the final square would need 9.2 million trillion grains, and the total number of grains on the chessboard would be 18,446,744,073,709,551,615 (264 - 1). The board consisted of a triangular array of pegs through which beads dropped from top to bottom, where they collected in a series of vertical tubes. This was particularly true of ancient Greece, where geometry and mathematician, and Otto Hölder, a German, prove that all finite groups can be built from finite simple groups. If they now play a fourth game and the first player wins, then he will take the 64 pistoles; if the other wins it, they will have each won two games and are equally likely to win the final game. Without them, breakthroughs such as analytical calculus (which divides quantities into infinitesimally small amounts) and advances in space-time and non-Euclidean geometries could not have happened. 300 BCE Euclid makes the first known written reference to the golden ratio in his Elements. Apollonius's studies inspired many of the advances in geometry seen in the Islamic world during the Middle Ages. 260 Seeking the right curve The key lay in finding a curved path for the pendulum to follow (known as a tautochrone curve), whereby the time the pendulum takes to return to its lowest point. This trend was fostered by the influential work of Carl Friedrich Gauss, regarded by many in the field as the greatest of all mathematicians. It would even print out the results. The game clearly favors the one who makes the throws, rather than the "banker." Choice means probability, and probability, and probability, and probability, and probability, and probability means mathematicians can get to work. geometry, calculus, and foundations—are worthy of study for their own sake, and the popular image of academic mathematics is that of an almost incomprehensible abstraction. He did not like having to use geometry as well as algebra to find a solution, and hoped that his geometrical efforts would one day be replaced by arithmetic. It can be colored in with a crayon in one continuous movement without taking the crayon away from the paper. Key works 1653 Traité du triangle arithmétique (Treatise on the Arithmetical Triangle) 1654 Potestatum Numericarum Summa (Sums of Powers) Binomial calculations As Pascal realized, the answer lay in binomials—expressions with two terms, such as x + y. The whole area is the area of the square, which is  $1 \times 1 = 1$ , so the probability of landing in the shaded area is approximately  $0.78 \div 1 = 0.78$ . Henderson outlines the connection between symmetrical Venn diagrams and prime numbers. Perhaps the most important event in the history of science... [is] the invention of the decimal system... Henri Lebesgue French mathematician Terminating and recurring decimals Fractions are converted to decimals by dividing the numerator by the denominator. The pure tone of a single pitch is produced by a smooth repetitive oscillation called a sine wave (see graph). Its height was five cubits, and a line of thirty cubits measured its circumference." In c. Natural spiral shapes, such as the nautilus shell, have a resemblance to the golden spiral, but do not strictly fit the proportions. He recorded highly accurate observations of celestial 108 objects from Raqqah, Syria. For example, the length of orbits at different altitudes above a planet's surface can be worked out by using the basic principle that if the diameter of a circle is known, its circumference can be calculated by multiplying by n. In 1661, he launched what may have been the world's first public transportation service in Paris, with linked five-person coaches. He also transformed the study of topology by associating it with algebraic structures, in his fixed-point theorem. René Descartes on de Roberval's method for finding the area under a cycloid New spins on the problem 236 The first description of a cycloid was published by Charles de Bovelles in 1503. It is also possible to work out the distance traveled during one rotation of Earth by a person standing at a point on its surface using  $\pi$ , providing the latitude of the person's position is known. A black hole occurs when spacetime warps so much that its curvature becomes infinite at the hole's center. Because the derivative is ex, the slope (a measure of direction and steepness) of the tangent line will always be the same as the y value. During World War I, Borel worked for the War Office and later became minister of the navy. It was here that Leonhard Euler, the next, and arguably greatest, Enlightenment mathematician, was born and educated. AFTER 1569 Flemish mapmaker Gerardus Mercator's map projection allows navigators to plot rhumb-line courses as straight lines on the map. New hope The introduction of supercomputers, computers capable of handling huge amounts of data, in the 1970s revived interest in solving the four-color theorem. The publication of this work marked a turning point for trigonometry. 570 BCE-495 BCE) FIELD Applied geometry BEFORE c. In the Langlands program, numbers are usually manipulated by modular arithmetic. 569 Quotient The result that is obtained when one number is divided by another. He also devised a formula for finding the area of a triangle from the lengths of its sides. Borel concluded that events with a sufficiently small probability will never occur. The work in which al-Karaji developed the binomial theorem is now lost, but later commentators preserved his ideas. [If] a spider climbs so many feet up a wall each day and slips back a fixed number each night, how many days does it take him to climb the wall? Around 200 BCE, Pingala was aware of this pattern in Sanskrit poetry, and the Roman poet Virgil used it in the 1st century BCE. 1815 Augustin-Louis Cauchy, a French mathematician, develops his theory of permutation groups. Its influence, however, never spread further. He had been amazed that he, just a boy, could make sense of it, and yet the best mathematical minute detail this new world of geometric construction, studying and defining the properties of conic sections. A vibrant exploration of the world's most famous and key mathematical ideas, movements, and peopleWhat is an imaginary number? Using only a pair of compasses and a straight edge, the Greeks would superimpose a square on a circle and then use their knowledge of the area of a square to approximate to the area of a circle. In other words, their ratio is a rational number, so it can be expressed as the ratio of integers. Some of those patterns involve the practical triangles required to build pyramids and divide land; other patterns attempt to classify all of the 26 sporadic groups of abstract algebra. Boole proposed that logical propositions could have only two values—true or false—and could not be anything in between. Real number Any number that is either a rational number or an irrational number. The constant c, meanwhile, shows where the line meets the y axis when x is equal to zero. 1749 Leonhard Euler proves that polynomial equations of degree n have exactly n complex roots (where n = 2, 3, 4, 5, or 6). Goldbach also proposed that every odd integer above 5 is the sum of three primes, and concluded that every integer from 2 upward can be created by adding together primes; these additional proposals are dubbed "weak" versions of 306 the original "strong" conjecture, as they would follow naturally if the strong conjecture were true. The first symbolic logic with quantification was produced in 1879 by German logician Gottlob Frege, who objected to Boole's attempts to turn logic into algebra. The blue line shows the rate of change at a given point. The three disks must be moved one disk at a time so as to recreate the starting arrangement on a different pole using the smallest possible number of moves, with the restriction that players can only place a disk on top of a larger disk or on to an empty pole. 140 The early algebraists Scholars at the House of Wisdom produced their own research, and in 830, Muhammad Ibn Musa al-Khwarizmi presented his work to the library—The Compendious Book on Calculation by Completion and Balancing. Groups can also be represented as matrices, enabling difficult problems in group theory to be expressed in terms of matrix algebra, which is more easily solved. As attitudes toward women in academia became less tolerant, mathematics and astronomy would be almost exclusively male preserves until the Enlightenment opened up new opportunities for women in the 1700s. In his sequence listed on the Liber Quadratorum (1225), he tackled several topics right. At the end of the 1500s, Italian physicist and astronomer Galilei used trigonometry to model the trajectories of projectiles on which gravity was acting. Alamy Stock Photo: Science History Images (tr). For probability to be useful, mathematicians needed to determine how an event's outcome could be used to draw conclusions about the probabilities that led to it. Calculus says I can. Just as he was about to give up, he read the paper "Unsolvable Diophantine problems" (1969) by American mathematician Julia Robinson, and a solution fell into place. In mathematics, a group is not simply a collection of how the group members can be used to generate more members, for example, by multiplication, or addition. 484 These physical laws and analytical tools can demonstrate that the Universe is deterministic— if the exact location and condition of an object and all the forces acting upon it are known, it is possible to determine its future location and condition with perfect accuracy. Using a 12,288-sided polygon, he calculated that n is between 3.1415926 and 3.1415927, and suggested two fractions to express the ratio: the Yuelü, or approximate ratio, of 22/7, which had been in use for some time; and his own calculation, the Milü, or close ratio, of 355/113. Written by German mathematician Johannes Müller von Königsberg, known as Regiomontanus, it was a compendium of all known theorems for finding sides and angles of both planar (2-D) and spherical triangles (those formed on the surface of a 3-D sphere). After two decades' work, the government canceled the project in 1842. By slicing that cone in different ways, a series of curves, known as conic sections, can be produced. Al-Khwarizmi and Khayyam had made significant progress in understanding them—work further developed by Sharaf al-Din al-Tusi, a 12thcentury scholar, probably born in Iran, whose mathematics appears to have been inspired by the work of earlier Greek scholars, especially Archimedes. If x is -13, x2 = 169 and 10x = -130. Following their discussions, the two agreed that the logarithm of 1 should be redefined as 0 and the logarithm of 10 as 1. Cylinder A 3-D shape, such as a tin can, with two identical circular ends joined by one curved surface. 556 See also: The mechanical computer • The Turing machine MARYAM MIRZAKANI 1977-2017 At the age of 17, Mirzakani became the first Iranian woman to win a gold medal in the International Mathematical Olympiad. The outside edge is entirely made up of the number 1, and the next set of numbers, in the first diagonal, is a simple number line of 1, 2, 3, 4, 5, and so on. Within 20 years of Lagrange's work, Italian mathematician Paolo Ruffini began to prove that there was no general formula for the quintic equation. Closely related structures include rings and fields, which are also defined in terms of a set with operations and axioms. Lagrange combined the technique of using a related, lower-degree polynomial equation whose coefficients were related to the coefficients of the original equation with a striking innovation—he considered the possible permutations (reorderings) of the roots. See also: Positional numbers • Euler's identity 297 IN a striking innovation—he considered the possible permutations (reorderings) of the roots. CONTEXT KEY FIGURES Abraham de Moivre (1667-1754), Carl Friedrich Gauss (1777-1855) FIELDS Statistics, probability BEFORE 1710 British physician John Arbuthnot publishes a statistical proof of divine providence in relation to the number of men and women in a population. As can be shown on a modern graph that plots a cubic equation as a curve snaking above and below the x axis, a cubic equation has up to three roots. Pythagorean triples, although their existence was known long before Pythagorean triples, although their existence was known long before Pythagorean triples. mathematician Preda Mihăilescu solved the outstanding issues and turned conjecture into theorem. From about 4000 BCE, the Babylonians used elementary geometry and algebra to solve practical problems—such as building, engineering, and calculating land divisions—alongside the arithmetical skills they used to conduct commerce and levy taxes And because anyone can do math, there is no telling who will discover this new math, and where or when. Born in Germany, he moved with his family to Italy when young and studied in Switzerland. Published widely, she recently co-edited Enriching Mathematics in the Primary Curriculum. Arab scholars, who created a vital link between the learning of ancient civilizations and the European Renaissance, introduced magic squares to Europe in the 14th century. It was classified as a type of "logarithmic curve. The Bletchley Park operation was hugely successful, cutting short the length of the war and saving countless lives. The bottom horizontal edge of the square is the x axis and the left vertical edge is the y axis, with a value of 0 in the corners at each end of the curve. All the straight lines described above have the same general equation: y = mx + c, where the coefficient m is the slope of the line, indicating how much bigger (or smaller) y is compared to x. Hardy British mathematician Modern numeration The Hindu-Arabic decimal system used throughout the world today has its origins in India. Meanwhile Swiss mathematician Leonhard Euler, a close friend of Johann Bernoulli's sons, was influenced by their ideas on the subject. Alamy Stock Photo: The History Collection (tr). Other mathematicians took a more theoretical path, inspired by the ideas in the newly available texts. Bombelli also imaginary number equals a negative number: gave practical examples of how to apply his rules for complex number. Application of fractals 506 Fractal geometry has allowed mathematicians to describe the irregularity of the real world. Although German mathematician Heinrich Heesch suggested a method for doing this, he did not have sufficient access to a supercomputer to test it. is the distance from the central point—called the pole, not the origin. The magic total, therefore, is 1/3 of 45, or 15. Groups can be finite or infinite, and their study is called group theory. AFTER 1881 John Venn introduces Venn diagrams to explain Boolean logic. However, this was later found to be impossible, as is demonstrated by multiplying any number by zero (division being defined as finding the missing number in a multiplication). Central to the analysis of waves, such as electromagnetic waves moving along wires, they are also used to describe the diffraction of light, the flow of electricity or heat in a solid cylinder, and the motions of fluids. 136 Steven Wright American comedian Multiplying and dividing Brahmagupta went on to examine zero in relation to multiplication and described how the product of multiplying any number with zero. AFTER 2004 Elliptic curves are first used in cryptography; they use smaller keys but offer the same security as the RSA algorithm. had to be invented—leading to the concept of the imaginary unit (Negative numbers were still mistrusted in the 1500s; imaginary and complex numbers were not widely accepted for many decades. With the equation  $x^2 = 2$ , the solution is not just  $x = but x = \pm .1617$  A spiral rhumb line is named a "loxodrome" by Dutch mathematician Willebrord Snell. From around 1500, as ships began to cross the world's oceans, navigators met a problem—plotting a course across the world that took account of the Earth's curved surface. In 1669, he invented a set of scales known as the Roberval balance. The result could be zero, if zero divided by any number is thought to be zero. This drawing of ancient Greek musicians illustrates two members of the lyre family— the trigonon (left) and the cithara. Aristotle, however, did not find this model entirely satisfactory, so he set about determining a systematic structure for logical argument. In 1966 she was the first woman to chair the District of Columbia State Board of Education. To see a 3-sphere requires 4-D space. Daina Taimina: From Daina Taimina: From Daina Taimina's book Crocheting Adventures with Hyperbolic Planes (cb); Tom Wynne (tr). astronomy. They consist of intersecting circles, each one representing a distinct set. Plato Phi and Fibonacci 182 The golden ratio is also closely related to another well-known mathematical phenomenon— the set of numbers known as the Fibonacci 182 The golden ratio is also closely related to another well-known mathematical phenomenon. elements: the cube (also known as a regular hexahedron with air; and the tetrahedron with air; and tet negative numbers (black rods) was also in tune with the Chinese concept that opposing but complementary forces—yin and yang—governed the Universe. He became one of the most important scholars of the astronomy school founded by Aryabhata, and wrote a commentary, Aryabhatiyabhasya, on Aryabhata's earlier Aryabhatiya treatise. For example, 1, 2, 3, 4, 6, and 12 are all factors of 12. His published mathematical works included a formula for solving cubic equations, an encyclopedic math treatment—Treatise on Numbers and Kanada Kanad NOETHER Born in 1882, Emmy Noether struggled to find education, recognition, and even basic employment in early 20th century academia as a Jewish woman in Germany. Probability is easily measured in the cases shown here. A pleasing rhythm in poetry, for example, is produced when successive lines have 1, 1, 2, 3, 5, and 8 syllables, and there is a long tradition of 6-line, 20-syllable poetry structured in this way. Al-Karaji was instrumental in this development. Order four produces 880 magic squares—with a magic total of 34. Each month, some rabbits mature and others breed. Getty Images: David Williams / Photographer's Choice RF (tr). New frontiers As the century drew to a close, Europe was reeling from political revolutions that had toppled the monarchy in France and given birth to the United States of America. The horizontal line, called the x-axis, and the vertical y-axis are always perpendicular to each other; the origin is the only place they will ever meet. Alamy Stock Photo: James King-Holmes (tr). Brouwer criticized the logical foundations of mathematics as espoused by David Hilbert and Bertrand Russell and helped to found mathematical intuitionism, based on the view of math governed by self-evident laws. He read Cardano's Ars Magna with great admiration. The term "Bacon number" was coined to indicate an actor's degree of separation from Bacon. AFTER 1944 British codebreaker Max Newman builds Colossus, the first digital electronic programmable computer. The product of a fortune and a debt. Jividing zero itself by zero has proved even trickier. 600-C. He wrote: "Multiply 5 + , making 25 -(-15), which is + 15. ] Invented by Leibniz for integration. In the 8th century, Persian mathematician al-Khwarizmi employed a geometric solution for quadratic equations known as completing the square. The Jordan-Hölder theorem, proven around 1889, asserts that, just as all positive integers can be constructed from prime numbers, so all finite groups can be constructed from prime numbers, so all finite groups can be constructed from prime numbers, so all finite groups can be constructed from prime numbers, so all finite groups can be constructed from prime numbers, so all finite groups can be constructed from prime numbers, so all finite groups can be constructed from prime numbers, so all finite groups can be constructed from prime numbers, so all finite groups can be constructed from prime sequence, which can be calculated by adding the numbers on the shallow diagonals (indicated here by the color shading) of Pascal's triangle. Infinitesimals bridge the gap between continuous things and things broken into discrete units. Differential equation An equation that represents a function including the derivative(s) of a given variable. Recurring Any number that is repeated without limit. Ordinal numbers Numbers that denote a position, such as 1st, 2nd, or 3rd. The contestants watch the numbers appear on a big screen, flashing by faster with each round. Nowadays there are many fractalgenerating computer programs, making it possible for the general public to create fractals The elder brother, Jacob, is recognized for his work on number theory, but he also helped develop probability theory, introducing the law of large numbers. Al-Khwarizmi began by studying linear equations, so-called because they create a straight line when plotted on a graph. 1753 Swiss mathematician Daniel Bernoulli shows that a vibrating string consists of an infinite number of harmonic oscillations. Algorithms are widely used in mathematics and computer science for calculation, organizing data, and a multitude of other tasks. 245 The probability of an event is defined as the proportion of times it will happen. Key work 1963 Deterministic Nonperiodic Flow 487 See also: The problem of maxima • Probability • Calculus • Newton's laws of motion • Laplace's demon • Topology • Fractals 488 IN CONTEXT KEY FIGURE Lotfi Zadeh (1921-2017) FIELD Logic BEFORE 350 BCE Aristotle develops a system of logic that dominates Western scientific reasoning until the 1800s. The edition printed a century from now will carry on past page a system of logic that dominates Western scientific reasoning until the 1800s. 325, covering patterns totally alien to us. Emmy Noether, one of the first female mathematicians of the modern era to gain recognition, came to the field of theoretical physics from a perspective of abstract algebra. These are -2 and -1. Constructed in 1941 by German engineer Konrad Zuse, the Z3 used electromechanical relays, or switches, to represent 1s and 0s of binary data. Benoit Mandelbrot After Euclid, scholars and mathematicians modeled the world in terms of simple geometry: curves and straight lines; the circle, ellipse, and polygons; and the five Platonic solids—the cube, the tetrahedron, the dodecahedron, and 501 the icosahedron. Although they were conceived before negative numbers were fully accepted, coordinates now often include both negative values above and to the right of the origin; positive values above and to the right of the origin; positive values above and to the left of the origin; positive values above and to the right of the origin; positive values above and to the left of the origin; positive values above and to the left of the origin; positive values above and to the right of the origin; positive values above and to the left of the origin; positive values above abov wedge marks. A number was set by turning the cogwheels in the column to show the correct digit on each. As yet, only the Poincaré conjecture has been solved; Grigori Perelman's proof was confirmed in 2006. The Egyptians were also able to use their mathematical skills in conjunction with observations of the heavens to calculate and predict astronomical and seasonal cycles and construct calendars for the religious and agricultural year. It has two roots because of the x2 term - 2 is the equation's highest power. In the 5th century, Zu Chongzhi and his son used a 24,576-sided polygon to calculate π as 355/113 = 3.14159292, a level of accuracy (to seven decimal places) not achieved in Europe until the 1500s. Two years later, he published another paper, this time about integration, again using different notation from that of Newton. She needs to decide how many potatoes to precook each day. Such numbers had no place among the wellordered integers and fractions by which Pythagoras claimed the Universe was governed. A rhumb line is all or part of this spiral. In 1905, he was awarded his doctorate by the University of Zurich and published groundbreaking papers on Brownian motion, the photoelectric effect, special and general relativity, and the equivalence of matter and energy. 1000 CE In the Islamic world, mathematicians are using all the various ratios between the sides and angles of triangles. Alamy Stock Photo: Acorn 1 (bc). Later, Christianity, too, embraced the idea of a division between body and soul; and Pythagoras's ideas would become a core part of Western thought. Theirs was a base-20 (vigesimal) number system, which probably evolved from a simple counting method using fingers and toes. A polynomial expression usually includes different powers of a variable, together with constants, for example,  $x^3 + 2x + 4$ . A straight line that runs diagonally from the origin, equidistant from both axes, can be described using algebra as y = x, and has coordinates (0,0); (1,1); (2,2), and so on. This enabled him to extend his algorithm to 9689 sided polygons. Chance and statistics Although mathematics helped to create such a crushing vision of reality, it also helped to dismiss it. 7th century CE Chinese scholar Wang Xiaotong solves a range of cubic equations numerically. In 1722, Abraham de Moivre, a French mathematician, went a step further than Vieté and showed how trigonometric functions could be used in the analysis of complex numbers. 2003 In the US, Jerrold Griggs, Charles Killian, and Carla Savage show that symmetrical Venn diagrams exist for all primes. AFTER 1815 Joseph Fourier's proof that e is irrational is published. The first list of these primes was made by Pascal's contemporary, Marin Mersenne. The exponential function is given in the form f(x) = bx, where b is greater than 0, but does not equal 1, and x can be any real number. For the binomial to the first row of the triangle (the zeroth row is not counted as a row). 820 On the Calculation with Hindu Numerals c. Named after Scottish antiquarian Alexander Henry Rhind, who purchased the papyrus in Egypt in 1858, it was copied from earlier documents by a scribe, Ahmose, more than 3,500 years ago. Any number that can be expressed as a ratio of two integers—a fraction, a decimal that either ends or repeats in a recurring pattern, or a percentage—is said to be a rational number. Irrational numbers can also be found between any two rational numbers. 1700s Swiss mathematician Leonhard Euler calculates that 635,318,657 is the smallest number that can be expressed as the sum of two fourth powers (numbers to the power of 4) in two ways. In The Whetstone of Witte, he used a pair of identical (=) to represent "equals," claiming that "no two things can be more equal" than these. Alamy Stock Photo: History and Art Collection (tr). In everyday life we are used to the base-10 counting system with its familiar ten digits, 0 to 9. He later moved to the US, where he was awarded a doctorate from Yale University in 1960. 300 BCE Euclid's Elements lays the foundations of geometry. The use of computers has made it possible to find more Mersenne primes. In 1816, Möbius was appointed professor of astronomy at Leipzig and stayed there for the rest of his life, writing treatises on Halley's Comet and other aspects of astronomy. Magic squares are discussed in Indian texts dating from 100 CE, and Brihat-Samhita (c. At a young age, he was adopted by the philosopher Parmenides, and was said to have been "beloved" by him. Science Photo Library: Emilio Segre Visual Archives / American Institute Of Physics (tr). Al-Karaji was also an engineer, and his book Extraction of Hidden Waters is the first known manual on hydrology. The more I study [the Analytical Engine], the more insatiable I feel my genius for it to be. Squaring the circle The ancient Greek mathematician Archimedes had used an ingenious method of exhaustion to determine the area between a parabola and a straight line. 31 Ancient roots The history of quadratic equations extends across the world. In the case of handwriting recognition, for example, a binary system would not be sufficiently subtle. Among his mathematical achievements, he provided rigorous proof that pi is an irrational number, and introduced hyperbolic functions, definitions, and the other four postulates; if it could, the fifth was unnecessary. Key works 1636 Perspective 1639 Rough Draft of Attaining the Outcome of Intersecting a Cone with a Plane See also: Pythagoras • Euclid's Elements • Conic sections • The area under a cycloid • Pascal's triangle • Non-Euclidean geometries 242 IN CONTEXT KEY FIGURE Blaise Pascal (1623-62) FIELDS Probability, number theory BEFORE 975 Indian mathematician Halayudha gives the first surviving description of numbers in Pascal's triangle. History mentions only a few pioneering female mathematicians in the ancient world, among them Hypatia of Alexandria. Mihailescu's proof of Catalan's conjecture does not involve any such computation. Although Shannon was not the first to send information digitally, he fine-tuned the technique. Nonetheless, having pondered de Méré's problems, Pascal chose to write to Fermat, communicating his thoughts on these and related issues and asking for Fermat's own views. Among them was the Poincaré conjecture, which had challenged mathematicians for nearly a century. This so-called "method of exhaustion" was taken up by Archimedes in around 225 BCE. Brahmagupta was therefore performing arithmetic with positive and negative quantities, while using fortunes and debts as a way to try to understand what negative numbers represented. They established the study of the principles of arithmetic and geometry as early as 2000 BCE. Level of accuracy Since the Ancient Greeks, the area of a circle has been found by multiplying the square of its radius (r2) with the number pi (n), written as nr2. By 150 BCE, the rod system had developed into alternating horizontal and vertical rods in sets of up to five. 1982 Poincaré's conjecture is proved in four dimensions by American mathematician Michael Freedman. In other words, the average result from a large number of trials will be a close match to the expected value as calculated using probability theory—and increasing the number of trials will result in that average becoming an even closer match. A static object produces a vertical worldline, and the worldline of a moving object is at an angle (see below). 1665-66 Isaac Newton calculates I to 15 digits. Subsequent numbers in the Fibonacci sequence are found by adding the previous two together: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89.... Achilles will get closer to the tortoise, but never actually overtake it. In the 12th century CE, Indian mathematician and astronomer Bhaskara II invented the study of spherical trigonometry. The dot was positioned on the baseline to avoid confusion with the dot notation sometimes 203 used for multiplication. Key work 2nd century BCE Sizes and Distances See also: The Rhind papyrus • Pythagoras • Euclid's Elements • Imaginary and complex numbers • Logarithms • Pascal's triangle • Viviani's tria results, Bernoulli developed the first proof of this relationship by considering a game with two possible outcomes—a win or a loss. Hopper retired from the Navy Reserve in 1966, but was called back on active duty the following year, not retiring until 1986, by which time she held the rank of rear admiral. Now called Noether's theorem, it was praised by Einstein for the way it addressed his theory of general relativity. Zero is the most magical number we know. Math remains crucial in the 21st century as space exploration, medical innovations, artificial intelligence, and the digital revolution press ahead, and more secrets about the Universe are revealed. However, he kept many of his ideas to himself; a great number were discovered in his unpublished papers after his death in 1855. 1874 Russian mathematican Sofia Kovalevskaya is the first woman to be awarded a doctorate in mathematics. But despite the seeming simplicity of the problem, no one was successful, although a century later Leonhard Euler did prove the theory where n = 3. 1884 The Foundations of Arithmetic by German mathematician Gottlob Frege examines the logical principles underpinning mathematics. Milgram studied how many people each of the letters went through to reach their targets. 213 Napier's book describing logarithms was published in 1614, as its title page shows. One of them was the Riemann hypothesis, which is still agreed to be one of the most important unsolved problems in mathematics. Lorenz began investigating climate modeling in the pioneering work of German mathematician and astronomer Johannes Kepler in the early 1600s on the motions of the planets. He continued his academic 73 work in Assos (now in Turkey). This area can also be found with a trigonometric method if the quadrilateral is split into two triangles. 2011 American programmer Jesse Anderson's million virtual monkey software generates the complete works of Shakespeare. He obtained his doctorate in Paris in 1892 and the same year won the Grand Prix des Sciences Mathématiques for his work on primes. Fresh ideas Mathematicians in medieval China and India made further advances in dealing with infinite sums. Getty Images: Smith Collection / Gado / Archive Photos (bc). Even in the 1500s, Italian polymath Gerolamo Cardano solved quadratic and cubic equations without zero. Béla Bollobás Hungarian-British mathematician See also: Diophantine equations • Euler's number • Six degrees of separation • Proving Fermat's last theorem 498 IN CONTEXT KEY FIGURE Roger Penrose (1931-) FIELD Applied geometry BEFORE 4000 BCE Sumerian buildings incorporate tessellations into wall decorations. The story has two alternative endings: in one, the king made Sissa his chief adviser; in the other, Sissa was executed for making the king look foolish. Wolfgang Haken, a former student of Heesch's, became interested in the problem, and began to make progress after meeting computer programmer Kenneth Appel at the University of Illinois. 85 A female personification of Arithmetic judges a contest between the Roman mathematician Boëthius, who uses numbers, and the Greek Pythagoras, who uses a counting board. Bourbaki at the very root of mathematics and lay at the heart of the group's work. 450 The existence of Ta(n) was proved theoretically in 1938 for all values of n. but the search is still on for larger taxicab numbers. The set of integers, and zero, was given the cardinality of 80, the smallest transfinite cardinal, as these are theoretically countable numbers but are actually impossible to count completely. Multiplying complex numbers is simplified when they are treated as polar coordinates, a process that involves multiplying the radial coordinates and adding the bridges arcs (curves or edges) that joined the various points. Peano's first textbook, on calculus, was published in 1884, and in 1891 he began work on the five-volume Formulation of Mathematics), which contained the fundamental theorems of math in a symbolic language largely developed by him. See also: Quadratic equations • Calculating pi • Trigonometry • Algebra 538 BHASKARA I C. The ancient Greeks puzzled over problems relating to areas and volumes of figures bounded by curves. By the early 1800s, Hungarian János Bolyai and Russian Nicolai Lobachevsky independently proved the validity of a "hyperbolic" non-Euclidean geometry in which the PP did not hold but the other four of Euclid's postulates did. ERATOSTHENES Born around 276 BCE in Cyrene, a Greek city in Libya, Eratosthenes studied in Athens and became a mathematician, astronomer, geographer, music theorist, 98 literary critic, and poet. Geometry is knowledge of the eternally existent. If the radius is 1, r2 = 1, so the area is just n; for a quarter circle, divide n by 4 to get approximately 0.78. He also defined some trigonometric functions, produced complete and accurate sine and cosine tables, and calculated solutions to simultaneous quadratic equations. Georges-Louis Leclerc, Comte de Buffon, for example, applied the principles of calculus to probability, and demonstrated the link between pi and probability, while another Frenchman, Abraham de Moivre described the concept of normal distribution, and Englishman Thomas Bayes proposed a theorem of the past. In a Newtonian universe, atoms (and even light particles) follow the laws of motion, and bounce around in a jumble of trajectories. The numerology of the Divine Comedy by Dante (1265-1331)—pictured here in a fresco from the Duomo in Florence, Italy—reflects the influence of Pythagoras, whom Dante mentions several times in his writings. Siméon Poisson, for example, used his knowledge of pure mathematics to develop ideas such as the Poisson distribution, a key concept in the field of probability theory. With later contributions, their technique was developed into the elliptic functions and became a way of analyzing many kinds of complex curves and oscillating systems beyond the simple ellipse. René Descartes 232 3-D Cartesian coordinates can be used to plot an object that has, for instance, width, depth, and height. The paradoxical conclusion is that half a given time is equivalent to double that time. The Chinese abacus (suanpan). Supposedly, when one of Pythagoreans drowned him in disgust. 1510-58) FIELD Number systems BEFORE 250 CE Greek mathematician Diophantus uses symbols to represent variables (unknown quantities) in Arithmetica. Mathematicians knew about Pascal's triangle long before the 1600s. Modular ("clock") arithmetic involves number systems with finite sets of numbers. A sound wave can be understood in terms of the amplitudes of its 340 constituent sine waves, a set of numbers that is sometimes referred to as the harmonic spectrum. Babbage's idea was to create a calculating machine that worked automatically, eliminating human error. He published his findings in 1687 in the three-volume Philosophiae Naturalis Principles of Natural Philosophy), often called the Principia for short. Ring A mathematical structure that is like a group except that it includes two operations rather than one. A parabola emerges if the cut is parallel to the edge of the cone, and a hyperbola results when the plane is vertical. 6000-5000 BCE Sumerians develop early calculation systems to measure land and to study the night sky. The project received crucial input from Ada Lovelace, a brilliant young mathematician. AFTER 1656 Dutch mathematician Huygens bases his invention of the pendulum clock on the curve of a cycloid. Day after day, Wiles tried different approaches to the problem, which proved futile— as his fellow IAS mathematician Peter Sarnak said, "It was like pinning down a carpet in one corner of a room, only for the carpet to pop up in another." Eventually, Wiles turned to a friend, British algebra specialist Richard Taylor, and they worked together on the proof for the next nine months. (A circle is a special ellipse with just one central focus, not two.) In 1609, German astronomer and mathematician Johannes Kepler demonstrated that the orbits of the planets were elliptical, with the Sun being located at one of the foci. Key works 1748 Introductio in analysin infinitorium (Introduction to Analysis of the Infinite) 1862 Meditation upon experiments made recently on the firing of Cannon) Compound interest One of the earliest appearances of e was in calculating compound interest—where the amount saved, rather than being paid out to the investor. It was not until 1920, when Ukrainian mathematician Alexander Ostrowski published his proof, that Gauss's assumptions could all be justified. Srinivasa Ramanujan displayed an extraordinary aptitude for mathematics at an early age. Logarithmic scales The pH logarithmic scale measures alkalinity and acidity. Each plaintext number is raised to the power of a, and then divided by n. AFTER 1936 Alan Turing studies the computability of mathematical functions, with a view to analyzing which problems in mathematics can be decided and which cannot. Their solutions, with a view to analyzing which problems in mathematical functions, with a view to analyzing which problems in mathematical functions, with a view to analyzing which problems in mathematical functions, with a view to analyzing which problems in mathematical functions, with a view to analyzing which problems in mathematical functions, with a view to analyzing which problems in mathematical functions, with a view to analyzing which problems in mathematical functions, with a view to analyzing which problems in mathematical functions, with a view to analyzing which problems in mathematical functions, with a view to analyzing which problems in mathematical functions, with a view to analyzing which problems in mathematical functions, were algebraic but differed from those of today, partly because zero and negative numbers were little used at the time. In 1849, as a result of his correspondence with Augustus De Morgan, Boole was appointed professor of mathematics at the new Queen's College in Cork, Ireland, where he remained until his premature death at the age of 49. As a 316 young mathematics at the new Queen's College in Cork, Ireland, where he remained until his premature death at the age of 49. As a 316 young mathematics at the new Queen's College in Cork, Ireland, where he remained until his premature death at the age of 49. As a 316 young mathematics at the new Queen's College in Cork, Ireland, where he remained until his premature death at the age of 49. As a 316 young mathematics at the new Queen's College in Cork, Ireland, where he remained until his premature death at the age of 49. As a 316 young mathematics at the new Queen's College in Cork, Ireland, where he remained until his premature death at the age of 49. As a 316 young mathematics at the new Queen's College in Cork, Ireland, where he remained until his premature death at the age of 49. As a 316 young mathematics at the new Queen's College in Cork, Ireland, where he remained until his premature death at the age of 49. As a 316 young mathematics at the new Queen's College in Cork, Ireland, where he remained until his premature death at the age of 49. As a 316 young mathematics at the new Queen's College in Cork, Ireland, where he remained until his premature death at the age of 49. As a 316 young mathematics at the new Queen's College in Cork, Ireland, where he remained until his premature death at the age of 49. As a 316 young mathematics at the new Queen's College in Cork, Ireland, Ween's College in Cork, Ireland, Irela formal method to find the function that solved such problems. 2600 BCE. Yet it was this branch of mathematics that would enable Wiles to prove Fermat's last theorem. Turing reached this conclusion using his hypothetical machine, which came in two parts. Laplace's demon was a side note to a career that also encompassed physics and astronomy, where Laplace was the first to postulate the concept of a black hole. Modern algebra is an abstract study of groups, rings, fields, and other algebraic structures. The players contributed equal stakes and agreed, in advance, that the first one to win a certain number of rounds would take the whole stake. Euler and other mathematicians, notably Carl Gauss in Germany, would seek to address this question, eventually concluding that the roots of any polynomial are either real or complex numbers. In 1994, Andrew Wiles used one of Langlands' conjectures to help solve Fermat's last theorem. Alamy Stock Photo: 580 George Oze (br). Soon after Plato's death, antiMacedonian prejudice forced him to leave Athens. Infinite Indefinitely large and without limit. Determining the area of a circle using a polygon with many sides had been proposed at least 200 years before Archimedes, but he was the first person to consider polygons that were both inscribed and circumscribed. Fluctuating fortunes 114 Over a period of several centuries, starting around 200 BCE, the ancient Chinese produced a book of collected scholarship called The Nine Chapters on the Mathematical Art. Alamy Stock Photo: Chronicle (crb). This can be pictured by drawing a tangent (a straight line that touches a curve at only one point). He died around 1030 CE. She developed the Coxcomb chart, a variation on the pie chart, using circle segments of different sizes to display variations in data, such as the causes of mortality among soldiers. In fact, any number to the power of 0 equals 1 for this reason. The new shape was formed from 12 regular pentagons, and known as the dodecahedron—one of the line is set as the zero point, and all other points on the line is set as the zero point, and all other points on the line is set as the zero point.

been a source of speculation. • The Poisson distribution 587 189 191 195 201 209 217 222 235 239 243 251 258 260 262 274 278 282 286 290 298 302 306 308 310 314 318 323 331 335 339 343 347 An indispensable tool in applied mathematics • Bessel functions It will guide the future course of science • The mechanical computer A new kind of function • Elliptic functions I have created another world out of nothing • Non-Euclidean geometries • Group theory Just like a pocket map • Quaternions Powers of natural numbers are almost never consecutive • Catalan's conjecture The matrix is everywhere • Matrices An investigation into the laws of thought • Boolean algebra A shape with just one side • The Möbius strip The music of the primes • The Riemann hypothesis Some infinities are bigger than others • Transfinite numbers A diagrammatic representation of reasonings • Venn diagramm • Topology Lost in that silent, measured space • The prime number theorem MODERN MATHEMATICS 1900-PRESENT The veil behind which the future lies hidden • 23 problems for the 20th century Statistics is the grammar of science • The birth of modern statistics A freer logic emancipates us • The logic of mathematics The Universe is fourdimensional • Minkowski space Rather a dull number • Taxicab numbers A million typewriters • The infinite monkey theorem She changed the face of algebra • Emmy Noether and abstract algebra • sequence • The Turing machine Small things are more numerous than large things • Benford's law A blueprint for the digital age • Information can change the entire cosmos • The butterfly effect Logically things can only partly be true • Fuzzy logic A grand unifying theory of mathematics • The Langlands Program 588 349 351 358 363 368 375 379 383 390 398 402 406 410 412 414 421 425 429 433 441 445 449 453 457 461 465 473 475 477 481 489 493 Another roof, another proof • Social mathematics Pentagons are just nice to look at • The Penrose tile Endless variety and unlimited complication • Fractals Four colors but no more • The four-color theorem Securing data with a one-way calculation • Cryptography Jewels strung on an as-yet invisible thread • Finite simple groups A truly marvelous proof • Proving Fermat's last theorem No other recognition is needed • Proving the Poincaré conjecture DIRECTORY GLOSSARY CONTRIBUTORS QUOTATIONS ACKNOWLEDGMENTS COPYRIGHT 497 499 501 508 512 520 524 531 536 558 574 576 580 583 589 He did this by dropping a needle of length 1 many times onto a series of parallel lines distance d apart, where d is greater than the needle's length (d > 1). The ancient Greeks used a ruler and compasses to attempt constructing a solution to this cubic equation but they never succeeded. Her main contributions were to universal algebraic theories and their models) and algebraic theories and their models) and algebraic theories and their models. For instance, if you draw a line around the center of the strip and then cut along it, the shape does not divide in half. The base10 logarithms calculated by Briggs are known as log10 or common logarithms. Aged only 20, he suggested a notation for the irrational number e, for which Jacob Bernoulli had calculated an approximate value. Ian Stewart British mathematician One plus one is one 392 Despite the resemblance, Boole's true and false binary of 1 and 0 is not the same as binary numbers. See also: Euclid's Elements • Conic sections • Diophantine equations • Emmy Noether and abstract algebra 125 IN CONTEXT KEY FIGURE Zu Chongzhi (429-501 CE) FIELD Geometry BEFORE c. Minkowski turned Einstein's theory into mathematics. PLATO Born around 428 BCE to wealthy Athenian parents, Plato was a student of Socrates, who was also a family friend. The triumph of Cartesian ideas in mathematics... is in no small degree due to the Leiden professor Frans van Schooten. Key work 1982 The Fractal Geometry of Nature Dynamic self-similarities 504 French mathematician Henri Poincaré found that dynamical systems (systems that change over time) also had fractal properties of self-similarity. It is only in the past 100 years that mathematicians have fully explored what are now known as Diophantine equations. This 2 denoted the power of 10 of the original denominator, as 100 is 102. c.1630 Galileo Gali force. The sum of the numbers 1 to 9 is 45, as is the same length; informally, a diamond shape. Henry(bc). For example, 1/3 expressed in decimals is 0.333333..., which can also be described in words as "zero point three recurring." Rhombus A quadrilateral with all four sides the same length; informally, a diamond shape. Henry(bc). Briggs, professor of mathematics at the University of Oxford, recognized the significance of Napier's tables but thought they were unwieldy. De Moivre concluded that it came down to finding the mean deviation (the average difference between the overall mean and each value in a set of figures) of binomial distribution. In 1985, German mathematician Gerhard Frey made a link between the conjecture and Fermat's last theorem. The rules laid down in the Rhind papyrus led ultimately to what would become known more than 3,000 years later as integral calculus. With five items, the total number of permutations is 120, and with six it is 720, making the task of finding all derangements a substantial one. PICTURE CREDITS Disclaimer: Page numbers and positions correspond to the print book. Boolean numbers are entirely different from the mathematics of real numbers. Progress was hindered by the continued use of the abacus for calculation. Political upheaval in Croton led to a revolt against the Pythagoreans. It is difficult to imagine a world without this all-pervasive branch of mathematics, as it would most certainly be one without computers. 2008 Microsoft conducts the first experimental study into the effects of social media on connectedness. 1545 Gerolamo Cardano's Ars Magna, a book of algebra, includes the first published calculation involving complex numbers. 1890 British mathematician Percy Heawood proves that five colors are sufficient to color any map. Another characteristic of 19th-century mathematics was its increasing tendency toward the theoretical. Problem 24, for example, asks what quantity, if added to its seventh part, becomes 19. Euler's number is also significant in a type of permutation called a derangement. Integration The process of performing a calculation in integral calculus. Plotted on a graph, e shows other special properties. 436 Galton built an "anthropometric laboratory" to collect information on human characteristics, including head size and quality of performing a calculation in integral calculus. vision. For example, division is the inverse of multiplication. Acute angle An angle that is less than 90 degrees. A complex number is the sum of any real number (on the number line) and an imaginary number. Deciphering processes Given enough paper and time, it is relatively easy to figure out a Caesar cipher by trying out every possible of any real number (on the number line) and an imaginary number. substitution. Shown here with the example of 32.567 (or 32 + 5/10 + 6/100 + 7/1,000), numbers shift one column to the left or right, crossing over the decimal separator. Mechanical analysis Laplace was inspired by classical mechanics—a field of mathematics describing the behavior of moving bodies, based on Isaac Newton's laws of motion. Although this convention was eventually replaced by René Descartes—in which letters at the beginning of the alphabet represent known numbers and letters at the end represent known numbers at the beginning of the alphabet represent known numbers and letters at the end represent known numbers at the beginning of the alphabet represent known numbers at the end represent known 2, only natural numbers (positive integers) are needed. He later worked as a draftsman at the École Royale, Mézières, and in 543 1780 became a member of the Academy of Sciences. Other social networks In the 1980s, friends of Hungarian mathematician Paul Erdős, who was well known for working collaboratively, coined the term "Erdős number' to indicate his degree of separation from other published mathematicians. Joan Clarke British cryptanalyst Cipher and key The term "cryptography" comes from the Greek for "hidden writing study." For much of history it was used to secure written messages. Factorial The product of any positive integer and all the positive integers that are smaller than it. Al-Karaji explored the multiplication of algebraic terms. Several different systems of coordinates are in use, but the dominant one is the Cartesian system, named after Renatus Cartesius, the Latinized name of French philosopher René Descartes. A ring can be broken into parts called "ideals"—a subset of elements. Largely forgotten until they were republished in 1953, Lovelace's notes confirm that she and Babbage foresaw many of the features of the computer now found in every home and office. A human must turn information into intelligence or knowledge. Escher, while some mathematicians, notably Daina Taimina, have used art and craft techniques to make these "new worlds" intuitively graspable. The Greeks used a numerical system of base-10 (with ten symbols) derived from the Egyptians. Many important sequences have been found in the triangle, and mathematicians believe that it may reflect some truths about relationships that we have yet to understand between numbers. The first particle's progress can be viewed as an arithmetic progression, while that of the second particle is geometric. Scottish mathematician John Napier's discovery of logarithms in 1614 enabled the compilation. In 1494, Italian mathematician Luca Pacioli had 252 suggested that the stakes should be divided in proportion to the number of rounds already won by each player. Key works 1713 Ars Conjectandi (The Art of Conjecturing) 1744 Opera (Collected Works) See also: Probability • Normal distribution • The Poisson distribut 83) FIELD Number theory BEFORE 1618 Logarithms by John Napier. Formula A mathematical rule that describes a relationship between quantities. My methods are really methods of working and thinking; this is why they have crept in everywhere anonymously. 132 The development of zero Any system for recording numbers eventually reaches a point at which it becomes positional; that is to say, digits are ordered according to their value to cope with increasingly large numbers. Not all matrices can be multiplied together; in matrix multiplication, AB can only be calculated if the row count of Barana eventually reaches a point at which it becomes positional; that is to say, digits are ordered according to their value to cope with increasingly large numbers. is the same as the column count of A. GILLES PERSONNE DE ROBERVAL Born in 1602, in a field near Roberval in northern France, where his mother was bringing in the harvest, Gilles Personne de Roberval was tutored in classics and mathematics by the local priest. main job as imperial mathematician was to make astronomical tables. At the same time, the ancient civilizations of India and China independently developed their own numerical systems. In 1939, American mathematician Nathan Jacobson established the bold capital C to signify the set of complex numbers, {a + bi}, where a and b are real and i = While working on the electrical circuitry for one of the world's first computers, Shannon realized that Boole's two-value binary system could be the basis of logic gates (physical devices that an 2.82 oz egg should be boiled for both four and six minutes (with degrees of almost 0 and 490 almost 1 respectively). AFTER 1843 Irish mathematician William Hamilton extends the complex plane by adding two more imaginary units to create quaternions—expressions that are plotted in a 4-D space. Alamy Stock Photo: Peter Horree (clb). Many countries use a comma as the decimal separator instead of a point. For the different kinds of averages, see mean, median, and mode. Table of contents : HOW TO USE THIS EBOOKINTRODUCTIONANCIENT AND CLASSICAL PERIODS 6000 BCE-500 CE Numerals take their places • Positional numbers The square as the highest power • Quadratic equations The accurate reckoning for inquiring into all things • The Rhind papyrus The sum is the same in every direction • Magic squares Number is the cause of gods and daemons • Pythagoras A real numbers The quickest runner can never overtake the slowest • Zeno's paradoxes of motion Their combinations give rise to endless complexities • The Platonic solids Demonstrative knowledge must rest on necessary basic truths • Syllogistic logic The whole is greater than the part • Euclid's Elements Counting pi is like exploring pi is like explored. geometrical tour de force • Conic sections The art of measuring triangles • Trigonometry Numbers can be less than nothing • Negative numbers The very flower of arithmetic • Diophantine equations An incomparable star in the firmament of wisdom • Hypatia The closest approximation of pi for a millennium • Zu ChongzhiTHE MIDDLE AGES 500 1500 A fortune subtracted from zero is a debt • Zero Algebra is a scientific art • Algebra freeing algebra from the constraints of geometry • The binomial theorem Fourteen forms with all their branches and cases • Cubic equations The ubiquitous music of the spheres • The Fibonacci sequence The power of doubling • Wheat on a chessboardTHE RENAISSANCE 1500-1680 The geometry of art and life • The golden ratio Like a large diamond • Mersenne primes Sailing on a rhumb • Rhumb lines • The equals sign and other symbology Plus of minus times plus o multiplication into addition • Logarithms Nature uses as little as possible of anything • The problem of maxima The fly on the ceiling • Coordinates A device of marvelous invention • The area under a cycloid Three dimensions made by two • Projective geometry Symmetry is what we see at a glance • Pascal's triangle Chance is bridled and governed by law • Probability The sum of the distance equals the altitude • Viviani's triangle theorem The swing of a pendulum • Huygens's tautochrone curve With calculus I can predict the future • Calculus The perfection of the science of numbers • Binary numbers • Bin Newton's laws of motion Empirical and expected results are the same • The law of large numbers that are creatures of their own • Euler's number Random variation makes a pattern • Normal distribution The seven bridges of Königsberg • Graph theory Every even integer is the sum of two primes • The Goldbach conjecture The most beautiful equation • Euler's identity No theory is perfect • Bayes' theorem Simply a question of algebra • The algebra • The fundamental theorem of algebra • The fundamental theorem of algebra • The algebra • The algebra • The fundamental theorem of algebra • The fundamental theorem of algebra • The algebra • The algebra • The fundamental theorem of algebra are coordinates on a plane • The complex plane Nature is the most fertile source of mathematical discoveries • Fourier analysis The imp that knows the positions of every particle in the Universe • Laplace's demon What are the chances? 123RF.com: Antonio Abrignani (tr). Book VI contains more advanced plane geometry. 6th century BCE In ancient Greece, Pythagoras discovers his theorem relating to the geometry of triangles. This is known as a "periodic tessellations, in which the pattern does not repeat, are harder to find, although some regular shapes can be combined to create nonperiodic tessellations." Nonperiodic tessellations of the event is Flash Anzan<sup>™</sup>, a feat of mental arithmetic in which the players imagine operating an abacus to add 15 three-digit numbers—no physical abacus is allowed. 1684 Gottfried Leibniz publishes New Method for Maximums and Minimums, his first work on calculus. At the time, mathematicians did not have constructed the circles and Khayyam would have constructed the circles and the ci and parabola geometrically. 2009 An anonymous computer scientist mines the first Bitcoin, a cryptocurrency without a central bank. Awarded his first degree in mathematics at Merton College, Oxford, and his doctorate at Clare College, Cambridge, he took up a post at the Institute for Advanced Study in Princeton in 1981, and was appointed professor there the following year. 267 Fluxion model In 1665-66, English mathematician Isaac Newton developed his "method of fluxions," a method for calculus. She has been a member of the British Society for the History of Mathematics for more than 30 years The main number system we use today is the base-10 or decimal system, where the number of parents of a male and a female from the same generation of bees is three. Boole's legacy It was not until some 70 years after Boole's death that the potential of his ideas was fully grasped. Alamy Stock Photo: Chronicle (tr); Peter Horree (cla). His mathematical achievements so far had been impressive, but not direction, in contrast to a vector. Eratosthenes also recognized that Earth is a sphere and calculated its circumference by comparing the angles of elevation of the Sun at noon at Aswan in southern Egypt and at Alexandria in the north of the country. While useful to later mathematicians, these were also unsatisfactory. In 1658, English architect Christopher Wren calculated the length of an arc of a cycloid as four times the diameter of the generating circle. In 1947, Cartwright became the first female mathematician to be elected a Fellow of the Royal Society in London. He noted that the new way of thinking of algebra as a kind of arithmetic at be elected a Fellow of the Royal Society in London. He noted that the new way of thinking of algebra as a kind of arithmetic at be elected a Fellow of the Royal Society in London. He noted that the new way of thinking of algebra as a kind of arithmetic at be elected a Fellow of the Royal Society in London. the known." Al-Samaw'al continued al-Karaji's work on polynomials, but also developed the laws of indices, which led to much later work on logarithms and exponentials, and was a significant step forward in mathematics. 416 Euler's formula, V + F - E = 2, works for most polyhedra, including a cube. Blaise Pascal 244 Pascal's triangle is created by adding together two adjacent numbers (as shown by the arrows) to give the sum in the next row. Aged just 13, he entered St. Andrews University and became passionately interested in theology. Revisiting logic The late 19th and early 20th centuries saw the emergence of another field of mathematics—the foundations of mathematics. This led, in 1980, to the creation of the Mandelbrot set. 1895 Henri Poincaré lays the foundations of algebraic topology in Analysis Situs (Analysis of Position). Key work 1693 Traité des Indivisibles) See also: Euclid's Elements • Calculating pi • Mersenne primes • The problem of maxima • Pascal's triangle • Huygens's tautochrone curve Calculus 238 IN CONTEXT KEY FIGURE Girard Desargues (1591-1661) FIELD Applied geometry BEFORE c. The Bat Country, a jungle gym project by American artist Gwen Fisher, is a Sierpinski tetrahedron featuring softball bats and balls. Abstract algebra The branch of algebra, developed mainly in the 1900s, that investigates abstract mathematical structures such as groups and rings. All whole numbers are rational as they can be shown as fractions divided by 1. Other achievements include his text on Hindu numerals, which, in its Latin translation, introduced Europe to Hindu-Arabic numerals. A dodecahedron (12 faces and 20 vertices), and an icosahedron (20 faces and 12 vertices) form another dual pair. Goldbach believed he had observed something remarkable—that every even integer can be split into two prime numbers, such as 6 (3 + 3) or 8 (3 + 5). A regular icosahedron is one of the five Platonic solids. Tetrahedron A 3-D polyhedron that is made up of four triangular faces. Turing then took this thought experiment further by imagining that you could use the machine M\* to test whether its own algorithm, M\*, would halt. What marks algebra from arithmetic is the use of symbols, such as letters, to represent variables (unknown numbers). He showed how space and time hashes (and time hashes). He showed how space and time hashes (unknown numbers) are parts of a four-dimensional spacetime, where each point in space and time hashes (unknown numbers). a position. As long as the determinant is not zero, the system will have a unique solution. Matrix (plural matrices) A square or rectangular array of numbers or other mathematical quantities that can be treated as a single object in calculations. The sequence has also been used in music. Lovelace also foresaw how such machines would be limited by the quality of the input. After graduating from Smith College, Massachusetts, with a math degree in 1914, she then embarked on a teaching career, and in 1930 established the math department at Miner Teachers College, which later merged with the University of the District of Columbia. Hilbert's first problem is substantially resolved, although set theory (the study of the properties of sets) is a complex subject, and much more work on it remains to be done. some of the values for p and q being used in every good mathematician is at least half a mathematician. The modern quadratic equation has four terms: ax2, bx, c, and 0. Yet studies continue, as the fascination of these equations is to ensure communications is to ensure communications is to ensure communications is to ensure communications and 0. Yet studies continue, as the fascination of these equations is largely theoretical. The Treviso Arithmetic multiplies the number 56,289 by 1,234. 512 One main reason people have needed to encrypt communications is to ensure communications is to ensure communications is to ensure communications is to ensure communications and 0. Yet studies continue, as the fascination of these equations is a state of the equation of the equati financial transactions can happen without banking information falling into the wrong hands. The first mechanical aids to calculating device, which was a first step toward true computing devices. The Chinese had introduced various step toward true computing devices are computed without banking information falling into the wrong hands. forms of decimal time over some 3,000 years, but finally abandoned it in 1645 CE. This opened up a completely new approach to geometry, paving the way for the nascent field of topology (the study of space and surfaces) which was also influenced by the possibility of more than three dimensions offered by William Hamilton's discovery of quaternions. For the record, my own definition of math as "the sort of things that mathematicians enjoy doing," while delightfully circular, is largely unhelpful. ROBERT LANGLANDS Born near Vancouver, Canada, in 1936, Robert Langlands did not plan to go to study at a university until a teacher "took up an hour of class time" to publicly implore him to make use of his talents. For example, in the 1600s, Galileo formulated laws about the way pendulums swing and how objects fall; Johannes Kepler showed how planets sweep through space as they orbit the Sun; and Isaac Newton combined this knowledge with physical laws covering gravity and motion. 300 BCE) FIELD Geometry BEFORE c. The lengths therefore must be doubled from the false positions of 4 and 3 to reach the solutions: 8 and 6. Dorling Kindersley: The Science Museum (tl). Argand's discovery As Argand began to plot complex numbers, he discovered that the imaginary number i does not get bigger if raised to higher powers. He imagined joining two Möbius strips together to create a shape that has only one surface, is nonorientable (has no "left" or "right"), and, unlike a Möbius strip, has no edge or boundary curve. He masterfully overcame the limitations that Hamilton encountered by using a technique called surgery, in effect cutting out the singularities, and was able to prove the conjecture. Periodic function A function whose value repeats periodically, as seen, for example, in the graph of a sine function, which is in the form of a repeating series of waves. This logarithmic scale fits well with the way we hear things, as a sound must become 10 times more intense to sound twice as loud to the human ear. In 1967, Stanley Milgram studied how many intermediate acquaintance links were needed to connect strangers in the US. If the answer was Yes, the algorithm M\* will halt, then the machine M\* would not halt. New dimensions Van Schooten and Fermat had both suggested extending Cartesian coordinates into the third dimension. He found that a 96-sided polygon allowed a calculation of n as 3.14, but by repeatedly doubling the number of sides up to 3,072, he reached a value of 3.1416. 1970 Mathematicians in Russia show that there is no algorithm that can solve all Diophantine equations. Trigonometry, a term based on the Greek words for "triangle" and "measure," is of immense importance in both the historical development of mathematics and in the modern world. By noticing that harmonious musical notes were connected to numerical ratios, Pythagoras was the first to uncover the relationship between mathematics and music. Key works 1892 Determination of the Number of Primes Less than a Given Number 1910 Lesson on the Calculus of Variations See also: Euclid's Elements • Mersenne primes • Imaginary and complex numbers • The Riemann hypothesis 424 425 INTRODUCTION In 1900, as the arms race that led to World War I intensified, German mathematician David Hilbert attempted to anticipate the directions that mathematics would take in the 20th century. He returned to India in 1919 and died a year later—probably as a result of amoebic dysentery contracted years earlier. Those weighing 6 and 12 units sounded the same notes at different pitches; in today's music terminology they would be said to be an octave apart. No device has survived, but scholars think that the ancient Olmec people invented it 3,000 years ago. It was introduced by Leonardo of Pisa, or Fibonacci, in his 1202 book Liber Abaci (The Book of Calculation). This is also the area of the large triangle, which is 1/2 ha, and so h = p + q + r. John Napier The impact of logarithms had an immediate impact on science, and on astronomy in particular. In 1738, Leonhard Euler similarly proved a case in which the only powers allowed were squares and cubes. 368 Galois' approach to this problem was to relate it to a question in another area of mathematics. Sissa's concept is an example of what is known as a geometric series, in which every successive term is the previous one multiplied by two: 1 + 2 + 4 + 8 + 16, and so on. 1976 Croatian mathematician anko introduces the sporadic simple group Janko Group 4, the last finite simple group to be discovered. Science Photo Library: Royal Astronomical Society (br). The orrery, a "clockwork universe" showing the movement of the celestial bodies in the Solar System, became a popular device after the publication gravity. There are different kinds, including proof by induction and existence proofs. This was the first of the letters between Pascal and Fermat in which the mathematical theory of probability was developed. At that time, meteorologists used linear statistical modeling to forecast weather, and they often failed. This apparent contradiction is at the heart of chaos theory. Key work 1868 Über die Hypothesen, welche der Geometrie zu Grunde liegen (On the Hypotheses Which Lie at the Foundation of Geometry) See also: • Mersenne primes • Imaginary and complex numbers • The complex plane • The prime number theorem 405 IN CONTEXT KEY FIGURE Georg Cantor (1845-1918) FIELD Number theory BEFORE 450 BCE Zeno of Elea uses a series of paradoxes to explore the nature of infinity. AFTER 1996 The Bacon number • The Complex plane • The Möbius strip • Topology • The butterfly effect • The four-color theorem 305 IN CONTEXT KEY FIGURE Christian Goldbach (1690-1764) FIELD Number theory BEFORE c. They were distrustful of the creative guesswork, as they saw it, of older mathematics for physics. This has enabled investigators to use the law to detect financial fraud. Mathematics had suffered a long period of stagnation in Europe but, as the Renaissance took hold in the 1300s, renewed interest in the subject led to fresh ideas about motion and the laws governing distance and speed. 300 BCE Euclid introduces the concept of a power to describe squares c. Alamy Stock Photo: Peter Horree (br). It was written in eight volumes, of which seven survive: books 1-4 in Greek, and books 5-7 in Arabic. He began studying mathematics after Euclid's great work Elements had emerged and he employed the Euclidian method of taking "axioms"—statements taken to be true—as starting points for further reasoning and proofs. Any combination of shapes in a plane, however complex the pattern, can be colored in using just four colors so that no two adjacent shapes have the same color. He then spent his whole working life teaching mathematical logic, but just as importantly paved the way for the language of computers almost a century later. In an attempt to solve this problem, the 22nd General Conference on weights and measures—a meeting of delegates from 60 nations of the International Bureau of Weights and Measures—decided in 2003 that, although either a point or comma on the line could be used as the decimal separator, the delimiter was to be a space rather than either of the previous symbols. The formula dramatically sped up the process of finding new Pythagorean triples. Getty Images: Werner Forman / University, in New Jersey, managed to prove Frey's conjectured link. This number of permutations divided by e, rounded to the nearest whole number. 300 BCE Euclid defines a triangle in his book Elements and proves many theorems concerning triangles. Postulate 5 talks about a point of intersection, but that is between two lines, and not two circles. In other practical areas, physicists use complex numbers in the study of electromagnetism, fluid dynamics, and quantum mechanics, while engineers need them for designing electronic circuits, and for studying audio signals. A supporter of the Parliamentarian cause, Wallis 541 deciphered Royalist dispatches during the English Civil War. The rows and columns in a Latin square contain figures or symbols that appear only once in each row and column. The musical "rave" scene in the US and UK in the late 1980s and early '90s was linked to a surge of interest in fractal art. French composer Claude Debussy (1862-1918) employed Fibonacci numbers in several compositions. Turning around by 2 × pi (2π) radians is the same as turning 360 degrees (that is, in a complete circle). Laplace never used the word "demon" himself; it was introduced in later retellings, evoking a supernatural being made godlike by mathematics. 1800 BCE The Moscow papyrus provides solutions to 25 mathematical problems, including the calculation of the surface area of a hemisphere and the volume of a pyramid. 200 BCE In Conics, Apollonius describes the properties of conic sections. 526 Wiles's investigation of Fermat's last theorem began with his study of elliptic curves, which are described by the equation  $y_2 = x_3 + Ax + B$ , where A and B are constants (fixed). Mathematics is a more powerful instrument of knowledge than any other that has been bequeated to us by human agency. For example, the real number 1 is the complex number 1 + 0i, while the number i is 0 + i. The fuzziness of AI helps to give the illusion of a self-directing intelligence, but in reality fuzzy logic processes data to smooth out uncertainty. These are very different problems in terms of both usefulness and complexity, but both types of pattern have become the obsession of mathematicians throughout the ages. He was appointed Minister of the Marine in 1792, and also worked to reform France's education system, helping to found the École Polytechnique in Paris in 1794 and contributing to the founding of the metric system of measurement in 1795. Arriving in a cab with the number 1,729, Hardy remarked, "Rather dull number, don't you think?" Ramanujan disagreed, then explained that 1,729 is the smallest number that is the sum of two positive cubes in two different ways. Until 1536, mathematicians believed that all prime numbers for n, when employed in the equation 2n - 1, would lead to another prime as the solution. Albert Einstein 35 Structure of equations Modern quadratic equations usually look like ax2 + bx + c = 0. An outstanding scholar (and violinist), he studied in Berlin and Göttingen. Euler was a contemporary and friend of Daniel and Nicholas Bernoulli, Johann's sons, and at an early age proved himself a worthy successor to Jacob and Johann. Boole had studied the algebraic properties of numbers and realized that the set {0, 1}, together with operations such as addition and multiplication, could be used to form a consistent algebraic language. Although Newton devised his theory of fluxions in 1665-66, he did not publish it until 1704, when it was added as an appendix to his work Opticks. But the hypothesis does pin down the blend of predictability and randomness the primes obey. Instead, he would have drawn the shapes as accurately as possible and carefully measured to be justified by reference to the definitions, the postulates, and the axioms. To do this, it needed to describe the motion of atoms and molecules inside matter. Hence this product is 40." This is the 197 first recorded calculation involving complex numbers, but the significance of this breakthrough escaped Cardano; he branded his work strengthened the connection between algebraic equations and graphs—between mathematical symbols and visual representations. Mathematicians gradually developed rules to handle a wider range of general problems. Later attempts to prove the FTA included those of Leonhard Euler and JosephLouis Lagrange. This system worked well for recording amounts but not for doing calculations. Each problem that I solved became a rule which served afterwards to solve other problems. Compounding interest yields a bigger total sum. Instruction books The Butterfly effect 550 JOHN VON NEUMANN 1903-57 The son of affluent Jewish parents in Budapest, Hungary, von Neumann was a child prodigy, able to divide eight-digit numbers in his head at the age of six. There, it met some initial opposition from Christian religious leaders, who found the concept of zero satanic because they associated nothingness with the devil. Large-scale weather systems, such as major cyclonic flows, for instance, repeat themselves on much smaller scales, right down to gusts of wind. The structure and evolution of the Universe can also be modeled on fractals, as can the fluctuations of economic markets. The Königsberg?" Euler's answer is that such a graph must have at most two nodes of odd degree, but the Königsberg graph has four odd degree nodes. In statistics, the Poisson distribution is used to model the number of times a randomly occurring event happens in a given interval of time or space. If a key falls into the wrong hands, the encryption fails. 320 Buffon's needle experiment demonstrated how probability can be connected to pi. Henderson himself had learned the technique from pioneering American topologist William Thurston. Both sets run from the outside to the center: one clockwise, and the other counterclockwise. Langlands taught at Princeton, Berkeley, and Yale before moving to the Institute for Advanced Study (IAS), where he still occupies Einstein's old office. As well as his theorem, Pythagoras and his close-knit community made numerous other advances in mathematics, but carefully guarded that knowledge. For division, one logarithm is subtracted from another and the result is converted back. Double base Around the 2nd century CE, abaci had become a common tool in China. Samuel Morse's dot-dash telegraph code, which revolutionized communication in the 1800s, and the on/off encoding in a modern computer both have parallels with Bacon's cipher. The Romans regarded mathematics as a practical tool rather than worthy of study. Two matrices of the same size (with the same number of elements in their respective rows and columns) are added by simply adding corresponding elements. Profit is based on total revenue minus production cost; companies create a quadratic equation known as the profit function with these variables to work out the optimal sale prices to maximize profits. Octahedra and icosahedra are also formed with equilateral triangles, while cubes are created from squares, and dodecahedra are constructed with regular pentagons. In 1876, Sylvester returned to the US as a math professor at Johns Hopkins University, Maryland, where he founded the American Journal of Mathematics. But computers also provided a powerful tool for mathematicians. The differential equations devised by Poincaré to prove his theory implied the existence of dynamical states that possess self-similarity much like fractal structures. Binomial distribution (used to describe outcomes based on one of two possibilities) was first shown by Bernoulli in Ars Conjectandi (The Art of Conjecturing), published in 1743. She graduated from the University of Oxford in 1923 with a degree in mathematics. If you are trying to find the fifth Fibonacci number (f5), for example, you must add together f4 and f3. 296 American architect Eero Saarinen in 1947. Each Fibonacci number (step 2), for example, you must add together f4 and f3. 296 American architect Eero Saarinen in 1947. Each Fibonacci number is the sum of the previous two, so the first two have to be stated before the third can be calculated. For example, a three-by-three magic square is said to have an order of three. We use our ten fingers to count. In 1836, Babbage realized he too could use punched cards—to control his own machine but also to record results and calculation sequences. This theorem states that all polynomial equations of degree n have n roots. This in turn opened up new ways of looking at how to prove many other mathematical conjectures. The Merton scholars were devoted to solving physical and philosophical problems using calculations and logic, and were interested in the quantitive analysis of phenomena such as heat, color, light, and velocity. In these six types, the letters a, b, and c all represent known numbers, and x represents the unknown quantity. Complex number A number that is a combination of a real number and an imaginary number. Representing larger numbers could require 50 or more hieroglyphs. At the time, Wiles was employed at the Institute for Advanced Study (IAS) in Princeton, home to some of the world's finest mathematicians and an imaginary number. One of the main rudiments of mathematics is the ability to use equations to vork out solutions to real-world problems. Cubit A measure of length used in the ancient world, based on the length of the human forearm. Her mathematical provess had been recognized, however, and when the Bletchley Park project was established to decipher the German Enigma Code, Clarke was recruited. Early applications The process of mathematical discovery began in prehistoric times, with the development of ways of counting things people needed to quantify. This volume of the book was published in Latin in 1621. AFTER 1653 In Traité du triangle arithmétique (Treatise on the Arithmetical Triangle), Blaise Pascal reveals the triangular pattern of coefficients in the bionomial theorem in what is later called Pascal's triangle. His work was rediscovered and republished in 1864. In 2001, computer scientist Ray Kurzweil wrote an influential essay describing the exponential growth in technology over previous years. Key works 1827 The Calculus of Centers of Gravity 1837 Textbook of Statics 1843 The Elements of Celestial Mechanics See also: Graph theory • Topology • Minkowski space • Fractals 401 IN CONTEXT KEY FIGURE Bernhard Riemann (1826-66) FIELD Number theory BEFORE 1748 Leonhard Euler defines the Euler product, linking a version of what will become the zeta function to the sequence of prime numbers. When two numbers between 0 and 1 are chosen at random as the x and y coordinates, whether the point will lie inside the quarter circle (success) or outside it (failure) can be , where a is the x coordinate and b is the y deduced by examining coordinate. SuperStock: Album / Oronoz (tr). An arctangent (arctan) series is a way of determining the angles in a triangle from knowledge of the length of its sides, and involves radian measure, where a full turn is 2n radians (equivalent to 360°). With similar rigor, Diophantus pioneered the use of symbols to represent unknown numbers in his equations; this was the first step toward the symbolic notation of algebra. 457 Noether was not the first to focus on abstract algebra. For example, on a log10 scale, every unit up the scale represents a 10-fold increase in whatever is being measured. This area of topology." The field of topology." The field of topology is now vast, embracing abstract algebraic structures far removed from a simple notion of "shape." It has wide-ranging applications in areas such as genetics and molecular biology, such as helping to unravel the "knots" created around DNA by certain enzymes. Joseph Fourier was attempting to solve the problem of how heat diffused through a solid object. There are two ways of combatting frequency analysis. Viviani's research included work on the speed of sound, which he measured to within 82 ft (25 m) per second of its true value. This rule simplified the process of finding the determinant of a very large matrix by breaking it down into the determinant of employed as a senior mathematician at a computer company, she developed the Common BusinessOriented Language (COBOL), which became the most widely used programming language. Instead, they were instruction manuals for use in trade, accounting, construction, and other activities that involved measurement and calculation. The straight edge-and-compass methods of his day, however, had their limitations. See also: Diophantine equations • 23 problems for the 20th century MARY JACKSON 1921-2005 An aerospace engineer, Mary Jackson (born Winston) worked on the US space program and campaigned for better opportunities in engineering for women and people of color. Key works c. More developments followed in the 1600s with the work of English theologian and mathematician Isaac Barrow and Italian physicist Evangelista Torricelli, followed by that of Pierre de Fermat and René Descartes, whose analysis of curves advanced the new area of graphical algebra. Alamy Stock Photo: Chronicle (tl). What the theorem says The prime number theorem is designed to calculate how many primes there are less than or equal to a real number x. There's no reason why these problems shouldn't be easy, and yet they turn out to be extremely intricate. He was familiar with Pythagoras's theorem, which states that the square of the hypotenuse in a 59 right-angled triangle is equal to the sum of the squares of the other two sides. 6th century BCE The Greek scientist Thales travels to Egypt and studies its mathematical theories. After moving to Rome in his youth, he received training from the artist-mathematical theories. artistic perspective, and architecture. He left a tantalizing note in the margin: "I have discovered a truly marvelous proof, which this margin is too small to contain here." 524 Fermat's note related to Diophantus's discussion of Pythagoras's theorem—that in a right-angled triangle the square of the hypotenuse (the side opposite the right angle) is equal to the sum of the squares on the other two sides, or x2 + y2 = z2. If squared and cubed numbers are lined up in order of their values, the difference between each value becomes clear. 360 Elliptic functions are used to define the trajectories of spacecraft such as the Dawn probe, which explored the dwarf planet Ceres and the asteroid Vesta in the asteroid belt. He was a student of polymath Qutb al-Din al-Shirazi, himself a pupil of Nasir al-Din al-Tusi (see above), and, like them, was a member of the Maragheh school of mathematician-astronomers. While there, he met Richard Hamilton, whose work influenced his proof of the Poincaré conjecture. It also revolutionized algebra by turning equations into lines (and lines into equations). Mathematics of relativity In 1904, Dutch mathematical function. It is said that, while walking past a 55 blacksmith's forge, he noticed that different notes were produced when hammers of unequal lengths of metal. NASA: Images courtesy of NASA / JPL-Caltech / Space Science Institute (crb). Impressive Möbius strip buildings are being constructed to minimize the impact of the sun's rays. See also: Calculus • Newton's laws of motion • Non-Euclidean geometries KAREN KESKULLA UHLENBECK 1942- In 2019, Uhlenbeck became the first woman to be awarded the Abel Prize for Mathematics. Projectile flight, for example, can be modeled with quadratic equations. 116 Accepting the negative Throughout the Middle Ages, European mathematicians remained unsure of negative quantities as numbers. One type of network is a "six degrees of separation" social network diagram, which measures how connected people are to each other. In fact, base-20 number systems were used across the world, in Europe, Africa, and Asia. He also took his vows as a Franciscan friar, combining monastic pursuits with teaching. Lagrange applied his method, which Euler named the calculus of variations, to study a wide range of physical phenomena, including the vibration of strings. Euler's number makes it possible to calculate the number of derangements in any set. Adding 1 to both sides of the equation produces Euler's identity, ein + 1 = 0. In about 550 BCE, Pythagoras discovered that if you take two taut strings of the same material and the resulting notes will be an octave apart. Vera Rubin American astronomer Chi-squared test, now one of the cornerstones of statistics. 86 See also: Positional numbers • Pythagoras • Zero • Decimals • Calculus 87 IN CONTEXT KEY FIGURE Archimedes (c. Similarly, adding zero to a positive number. He also began his long solo attempt to prove Fermat's last theorem. Getty Images: DEA / M. Until Einstein's theory of relativity, the whole of mechanical physics was based on Newton's three laws of motion. In 1891, for example, German Karl Weierstrass created a method— 329 now known as the Durand-Kerner method, due to its rediscovery by mathematicians in the 1960s—for simultaneously finding all of the roots of a polynomial. There, she campaigned tirelessly for better hygiene, earning the nickname "The lady with the lamp." Back in Britain, Nightingale became an innovator in the use of graphs to display statistical data. Space, industry, and art The Möbius strip shape sometimes occurs naturally, such as in the movement of magnetically charged particles within the Van Allen radiation belts that surround Earth and in the molecular structure of some proteins. Mathematical renaissance City-states in Italy were quick to trade with the Islamic Empire, and it was an Italian, Leonardo of Pisa, nicknamed Fibonacci, who spearheaded the revival of mathematics in the West. When he published this finding in 1620 in his book Ephemerides novae motuum coelestium, Kepler dedicated it to Napier. Dreamstime.com: Antonio De Azevedo Negrão (cr). Key works 1847 Mathematical Analysis of Logic 1854 An investigation into the laws of thought 1859 Treatise on differential equations • Venn diagrams • The Turing machine • Information theory • Fuzzy logic 397 IN CONTEXT KEY FIGURE August Möbius (1790-1868) FIELD Applied geometry BEFORE 3rd century CE A Roman mosaic of Aion, Greek god of eternal time, features a zodiac shaped like a Möbius strip. For a point on a plane (a flat two-dimensional surface), two number lines are needed. She had a long association with Girton College, Cambridge, from 1930 to 1968, during which time she taught, researched, and served as Mistress of the college. While an algorithm could use traditional logic to divide a set of eggs into precise weight ranges and assign exact cooking times, fuzzy logic achieves this result. 1806 Jean-Robert Argand creates a geometrical interpretation of complex numbers by plotting them as coordinates to create the complex plane. During the French Revolution, he lost his private fortune, but in 1794 he published Eléments of Geometry), which remained a key geometry textbook for the next century, and he was then appointed a math examiner at the École Polytechnique. Differentiation can be used to find the rate of change at a given point in time. The second is 28 (1 + 2 + 4 + 7 + 14 = 28), the third 496, and the fourth 8,128. Numerator The upper number in a fraction, such as the 3 in 3/4. At the age of 60, Pythagoras is said to have married a young member of the community, Theano, and perhaps had two or three children. A prominent pacifist and social critic, in 1918 he was 443 jailed for six months, during which he wrote his Introduction to Mathematical Philosophy. For 10 numbers, Euler found that the probability of getting a derangement is 1/e to an accuracy of six decimal places. Hilbert wanted to know if there was a way to predetermine whether an algorithm—a method for solving a specific mathematical problem using a given set of instructions in a given order—would arrive at a solution to the problem. French mathematician Pierre-Simon Laplace used normal distributions in a given set of instructions in a given set of instructions in a given order—would arrive at a solution to the problem. of a normal curve. In contrast, the tall barrels from Kepler's homeland on the Rhine River held much less wine. Two millennia after Pythagoras, in 1638, Pierre de Fermat enlarged on this idea when he asserted that any number si the sum of up to 3 triangularity of a normal curve. numbers, up to 4 square numbers, or up to 5 pentagonal numbers, and so on. The rows and columns can be extended indefinitely, which enables matrices to store vast amounts of data in an elegant and compact manner. Gwen Fisher: Bat Country was selected as a 2013 Burning Man Honorarium Art Project. A fuzzy rule is then applied, with large eggs boiled for six minutes and small eggs for four. A year after his death in 1630, the transit of Mercury was observed as he had predicted. AFTER c. Following the war, Turing worked at Manchester University, where he designed the Automatic Computing Engine (ACE) and developed further digital devices. Which is why so many mathematicians of Mercury was observed as he had predicted. benefit from his presence. For example, "HELLO" might become "IFMMP." Going from plaintext to this ciphertext requires a cipher and a key. 1596 German astronomer Johannes Kepler proposes a model of the Solar System, explaining it geometrically in terms of Platonic solids. Coordinates can also be used to examine the relationship between two quantities. There is no definitive way to organize all of mathematics, but looking at it chronologically is not a bad way to go. However, in 1809, German mathematician Carl Friedrich Gauss found an equation to create a "best fit" line, which would show the relationship between the variables. Where the coefficients (a, b, and c, which multiply the variable x) are real numbers, rather than complex numbers, the equation will have at least one real root and up to three roots in total. The first Fibonacci number is written as f1, the second as f2, and so on. Bhaskara II's work, along with the ideas in Ptolemy's Almagest, were valued by the Islamic scholars of the medieval world, who had begun studying trigonometry well before Bhaskara II. However, it was Hungarian mathematician John von Neumann who contrived a real-life version of Turing's hypothetical device using a central processing unit (CPU) that converted an input to an output by calling up information to be saved If the power (n in the equation) is greater than 2, the equation has no integer solutions for x, y, and z—as Fermat asserted in a marginal note in 1657 and British mathematician Andrew Wiles finally proved in 1994. He went on to search for a general solution to cubic equations—in which the highest power of x is x3—but was unable to find one. 92 The advent of calculators and electronic computers in the 1900s made finding the digits of n much easier. 360 BCE Philebus c. See also applied mathematics. Parts of it were also used to denote fractions whose denominators were powers of 2. Hypotenuse The longest side of a right-angled triangle, located on the opposite side from the right angle. In a derangement, none of the items can remain in their original position. Hardy, then a professor at Trinity College, Cambridge. Getty Images: DE AGOSTINI PICTURE LIBRARY (bl). It was not until after Simon Stevin's De Thiende (The Art of Tenths) was published in 1585 that a decimal system became commonplace. At the heart of many mathematical provide answers to almost all problems. 10th century CE Egyptian scholar Abu Kamil Shuja ibn Aslam uses negative and irrational numbers to solve guadratic nial theorem. Gradient The slope of a line. New logic, new millennium For a while, it seemed electronic computing could potentially equations. 240 In the 1630s, Desargues began investigating which properties were unchanged (invariant) when an image is projected onto a surface (perspective mapping). Key works 1585 De Thiende (The Art of Weighing) to the Art of Weighing). Key works 1585 De Thiende (The Art of Tenths) 1585 De Thiende (The Art of Weighing). Introducing decimals Finding that conventional fractions were both time-consuming and prone to errors, Stevin began using a decimal system. Returning to Germany, in 1862 he started work at the Technical High School in Braunschweig, where he remained for the rest of his working life. The New York Public Library: (tr). Pascal's step-by-step methods and Fermat's considered replies provide some of the earliest examples of using expectations when reasoning about probability. In 1894, Alfred Dreyfus, a Jewish relative of Hadamard's wife, was falsely accused of selling state secrets and was sentenced to life in prison. Each starting point evolves into a looping line that swings from one quadrant of the space to another—indicating, for example, a change from wet and windy weather to hot, dry conditions, and all states in between. 1799 Carl Gauss determines the key characteristics of elliptic functions, but his work is not published until 1841. The result was a series of breakthroughs that remain influential today. Parabolic objects have special reflective properties. These were outlined in his major work on calculation by "completion and balancing"—methods he devised that are still used today. Mathematicians searched for a proof for this deceptively simple theorem for more than 120 years, making it one of the most enduring unsolved theorems in mathematics. Beyond the arts, the golden ratio has also appeared in modern geometry, particularly in the work of British mathematician Roger Penrose, whose Fibonacci tiles incorporate the golden ratio in their structure. Monte Carlo methods went on to be useful in modern applications, especially once computers made it far less time-consuming to repeat a probability experiment over and over again. Joyce Carol Oates American novelist AUGUST MÖBIUS Born near Naumberg in Saxony, Germany, in 1790, August Ferdinand Möbius was the son of a dance teacher. AFTER 1806 Robert Argand publishes the first rigorous proof of the FTA that allows polynomials with complex coefficients. These numbers all form part of the Fibonacci sequence. The introduction of decimal measures for length and weight (using meters and kilograms) arrived in Europe some 200 years later, during the French Revolution. The continuum hypothesis also stated that there was no infinity lying between these two infinities. Fibonacci died c. This was very different from Euclid's "synthetic geometry," in which shapes are defined by the way they are constructed using a 227 ruler and pair of compasses. 1915 Einstein describes gravity as curvature in spacetime in his general theory of relativity. It was important for them to be able to calculate the dimensions of fields for growing and irrigating crops and to work out the volume of buildings to store grain. SuperStock: Melvyn Longhurst (b). Sofya Kovalevskaya Russian mathematician Arithmetic and algebra So began the history of mathematicians that form the bulk of this book. Quaternions were the brainchild of William Rowan Hamilton, an Irish mathematician who was interested in how to model movement mathematically in three-dimensional space. Getty Images: Steve Jennings / Stringer / Getty Images: Steve Jennings / Steve Jennings / Steve Jennings Statistical Bureau of the Seine, where he pursued his studies in mathematical physics, including work on the Fourier series (a series of sine waves that characterize sounds). Napier died at Merchiston Castle in 1617. The final chapter concerns applications of the "Gouqu" (equivalent to Pythagoras's theorem), and the solving of guadratic equations. 211 JOHN NAPIER Born into a wealthy family in 1550 at Merchiston Castle, near Edinburgh, John Napier would later become 8th Laird of Merchiston. Certain numbers were endowed with characteristics and spiritual significance in what amounted to a kind of number worship, and Pythagoras and his followers sought mathematical patterns in everything around them. In this method, the mathematician selects a convenient number that is usually easy to calculate, then works out what the solution to the equation would be using that 32 number. Keith Devlin British mathematician selects a convenient number that is usually easy to calculate, then works out what the solution to the equation would be using that 32 number. Cardano and Niccolò Fontana Tartaglia found that solving cubic equations required a square root of a negative number. In the 5th century BCE, Hippasus's attempts to resolve x2 = 2 and his realization was irrational (neither a whole number nor a fraction) are said to have that led to his death for betraying Pythagorean beliefs. Alamy Stock Photo: Science Photo Library (br). The method used to find the area of a circle was to find the area of a square with sides that are 8/9 of the circle's diameter. He was awarded the Abel Prize in 2018 for his "visionary" Program. However, others believe they were merely replicating patterns in nature, such as on a turtle's shell. 1876 French mathematician Édouard Lucas verifies that 2127 - 1 is a Mersenne prime. This system is both a place value system and a base-10 or decimal system. Some 800 years later, Diophantus had no knowledge of negative numbers, so could not accept an equation where x is negative, such as 4 = 4x + 20, where x is -4. The collaboration was hugely productive: in addition to the taxicab numbers, Ramanujan also developed a formula for obtaining the value of pi to a high level of accuracy. This is because the derivative (rate of change) of ex is, in fact, ex, and the derivative is used to find the tangent. The product of two debts is a fortune. Pythagoras Ancient Greek mathematician Geometry and calculus A third major field of mathematics, geometry, is concerned with the concept of space, and the relationships of objects in space: the study of the shape, size, and position of figures. Sissa asked for some grains and explained the quantity he desired using the squares on the 8 × 8 chessboard. Fractals and the arts Under the Wave off Kanagawa by Japanese artist Katsushika Hokusai (1760-1849) employs the concept of self-similarity to dramatic effect. In contrast, the hammers of weights 9 and 8 were dissonant, as 9:8 is not a simple mathematical ratio. The year-computed to a highly accurate 365.24 days-began at the vernal equinox in March, when the center of the visible Sun is directly above the equator Indian mathematicians made significant advances in the fields of geometry, which had practical applications in astronomy, navigation, and engineering, but the most far-reaching innovation was the development of a character to represent the number zero. However, in 1975, Polish-born mathematician Benoit Mandelbrot drew attention to fractals — nonuniform shapes that echo larger and smaller shapes in a structure such as a jagged mountaintop. Wiles was close to having to admit that he had claimed a proof prematurely. They owe their name to an anecdote from 1919, when British mathematician G. Sofya ended her career as a professor of math at the University of Stockholm, where she died of influenza aged just 41. In Iran, it is known as Khayyam's triangle after Omar Khayyam, but he was just one of many Islamic mathematicians to have studied it between the 7th and 13th century—a golden age for learning. Key works 2002 "The entropy formula for the Ricci flow and its geometric applications" 2003 "Finite extinction time for the solutions to the Ricci flow on certain 3manifolds" See also: The Platonic solids • Graph theory • Topology • Minkowski space • Fractals 535 536 DIRECTORY In addition to the mathematicians covered in the preceding chapters of this book, many other men and women have made an impact on the development of mathematics. Then, in 1984, while working for the Digital Equipment Corporation (DEC), she invented the Spanning Tree Protocol (STP), which ensures there is only one active path between two network devices; this would later prove crucial for the development of the internet. This can be visualized on the complex plane. Proposition 6 If in a triangle two angles are equal to one another, the sides separated from the third side by these angles will also be equal. With an understanding of these ideas, and an insight into the people and societies in which they were discovered, we can gain an appreciation of not only the ubiquity and usefulness of mathematics, but also the elegance and beauty that mathematicians find in the subject. SuperStock: Stocktrek Images (crb). In the 1600s, new translations of Elements reached Europe, and Giovanni Saccheri showed that if the PP was untrue, then the sum of angles in a triangle was always either less than or greater than 180°. They are rotation (ρ) through 120°, 240°, and 360° and reflection (σ) through a vertical line through A, B, or C. 481 The idea that a butterfly flapping its wings in one part of the world could alter atmospheric conditions and eventually produce a tornado elsewhere has captured the popular imagination. Later attempts to analyze the movement of the Moon to improve navigation were plaqued by inaccuracies. A doughnut and an orange, however, are not homeomorphic because of the hole in the doughnut. However, in the graph representing Königsberg, A is the endpoint of three. In 1972, Edward Lorenz, an American meteorologist and mathematician, delivered a talk titled "Does the flap of a butterfly's wings in Brazil set off a tornado in Texas?" This was the origin of the term "butterfly effect," which refers to the idea that a tiny change in atmospheric conditions (which could be caused by anything, not just a butterfly) is enough to alter weather patterns somewhere else in the future. LUCA PACIOLI Luca Pacioli was born in 1445 in Tuscany. Light provided the mathematical prompt. Groups can also have a further attribute known as commutativity. In 1569, Mercator maps—on which lines of longitude are drawn parallel, so that all rhumb lines are straight—were introduced. Getty Images: Science & Society Picture Library (bc). In the ancient world, scholars considered problems in a geometric way. Gottlob Freqe Visualizing results One way of visualizing Boole's algebra is in the form of diagrams invented by British logician John Venn. His major contribution to axes, establishing the new field of analytic geometry, in which lines and shapes are described in terms of algebraic equations. EVARISTE GALOIS Born in 1811, Evariste Galois lived a brief but fiery and brilliant life. In 1679, he described a calculating machine that worked on binary principles, with open or closed gates to let marbles fall 275 through. ARISTOTLE The son of a physician at the Macedonian court, Aristotle was born in 384 BCE, in Stagira, Chalkidiki. The function produces the curve y = ex, which cuts the y axis at (0,1), and gets exponentially steeper. (clb). Others reconstructed his results and explained them in the Asian Journal of Mathematics. The result is often called exponential growth. 395 Logic gates, which are physical electronic devices implementing Boolean functions, form an important part of computer circuitry. The main drawback to this was that waves become weaker the further they 475 travel, and increasing background interference creeps in. Using calculus, the relationships between any complex variables can — in theory—be predicted by solving a particular differential equation. In the 1600s, a general theory of polynomial equations, now called the fundamental theorem of algebra. Exponent The superscript number to which a number or quantity has been raised, such as the 2 in x2 (x × x). Our destinies are infinite, and infinitely recurring. 69 Like Plato, later scholars sought Platonic solids in nature and the Universe. Tones have frequency, which determines pitch, and amplitude, which determines pitch, and amplitude, which determines pitch and amplitude of the transformation matrix contains information about the area or volume of the transformation about the area or volume. even further. Their work was likely to be controversial, so they adopted the pseudonym Nicolas Bourbaki. 1967 American social psychologist Stanley Milgram conducts experiments on the interconnectedness of social networks. The result of the calculation is another function called the differential or derivative of the first function. A new dawn in the East Greek dominance was eventually eclipsed by the rise of the Roman Empire. Fertilized eggs develop into female bees, which can either be queens or workers. AFTER 1799 The metric system is introduced for French title of The Art of Tenths. In the early 1800s, German mathematician and astronomer Friedrich Wilhelm Bessel gave solutions to a particular differential equation, the so-called Bessel equation. From each of these possibilities, the vertex B has two available destinations. A tesseract has four cubes, six squares, and four edges meeting at every corner. Hadamard, who was also Jewish, worked tirelessly on behalf of Dreyfus and he was eventually freed. Pearson realized that by uniting the mean and the standard deviation, he could calculate Galton's regression precisely. The slide rule, used here in 1941 by a member of the Women's Auxiliary Air Force, is marked with logarithmic scales that facilitate multiplication, division, and other functions. In the book, he explored a paradox, known as Russell-Zermelo paradox, after German mathematician Alexander Gelfond shows that en is transcendental, that is, irrational and still irrational when raised to any power. If the interest is calculated on a yearly basis, an investment of \$100 at an interest rate of 3% per year would produce \$100 × 1.03 = \$103 after one year. Among the unsolved for the foreseeable future. Power series A mathematical series where each term has a greater power than the previous one, such as x + x2 + x3 + x4 + ... Although these early mathematicians were not so concerned with the practical applications of their discoveries, they did not restrict their studies to mathematics alone. Born in Cleveland, Ohio, in 1942, she gained a PhD in mathematics from Brandeis University, Waltham, Massachusetts in 1968, and went on to achieve notable breakthroughs in mathematical physics, geometrical analysis, and topology. In the examples above, the original shape is the green quadrilateral. 598-668 CE) derived square root approximations of π using regular polygons with 12, 24, 48, and 96 sides: , , , and respectively. He realized he could plot its position, using numbers to describe where it was in relation to the two adjacent walls. Greek scholars quickly assimilated the mathematical ideas of the Babylonians and Egyptians. The binomial  $(x + y)^2 = 1x^2 + 2xy + 1y^2$  has 246 the coefficients 1, 2, and 1, as on the second row of Pascal's triangle. The Renaissance thirst for discovery also accelerated a "Scientific Revolution" - classic texts of mathematics, philosophy, and science had become widely available, and inspired a new generation of thinkers. Before the 1500s, predicting the outcome of a future event with any degree of accuracy was thought to be impossible. Out of this grew the mathematical field of topology, which by the early 1900s had moved far from the notion of "shape" to embrace abstract algebraic structures. In 1743, he stated that Newton's third law of motion is as true for freely moving bodies as it is for fixed bodies (d'Alembert's principle). 1979 The first Sudoku-style puzzle is published by Dell Magazines in New York. 3000 BCE South American Indians record numbers • Algebra • Decimals • Logarithms • Calculus 194 IN CONTEXT KEY FIGURE Rafael Bombelli (1526-72) FIELD Algebra BEFORE 1500s In Italy, Scipione del Ferro, Tartaglia, Antonio Fior, and Ludovico Ferrari compete publicly to solve cubic equations. Positive and negative numbers were represented by red and black rods respectively or different cross sections (triangular). Naturally occurring data that extends over several orders of magnitude, from hundreds to millions, for example, fulfils the law better than data that is more closely grouped. 1478 The Treviso Arithmetic explains in simple language how to perform addition, subtraction, multiplication, and division calculations. Experimenting with the Möbius strip produces other

unexpected results. For example, in the UK, the commas in the number 2,500,000 are delimiters and are used to make it easier both to read the number and to recognize its size. The resulting series of musical pitches forms the basis of the musical scale. A body moving a certain distance, it says, would have to reach the halfway point before it arrived at the end, and in order to reach that halfway mark, it would first have to reach the quarter-way mark, and so on ad infinitum. However, friction introduced a larger error than the one Huygens was trying to resolve. When del Ferro died in 1526, Fior decided the time had come for him to unleash del Ferro's formula upon the world. The answer to Ma modn is the ciphertext (C), and in this example it is 2. Matrices have special rules for addition and multiplication. Cube A 3-D geometrical figure whose faces are six identical squares. 1824 In Norway, Niels Henrik Abel completes Paolo Ruffini's 1799 proof that there is no general formula for the quintic equation. Robert Recorde tested the equals sign (=) in his own calculations, as seen here in one of his exercise books. A"taxicab" number, Ta(n), is the smallest number that can be expressed as the sum of two positive cubed integers (whole numbers) in n (number of) different ways. Admitting that he could not prove the theorem, De Morgan shared it with Irish mathematician William Hamilton. It does this by asking: does A halt (have a solution)? At the age of 18, he entered the University of Leipzig to study mathematics, physics, and astronomy, and later studied in Göttingen under the great German mathematics and astronomy. algebra-became increasingly important. Byron left England a few months after her birth, and Lovelace never saw her father again. Science Photo Library: Emilio Segre Visual Archives / American Institute of Physics (bl). In these triangles, the square of the lengths of the two shorter sides. Early computers used multiple bits as "addresses" for sections of memory— showing where the processor should look for data. To qualify for an Erdős number, a person has to have written a mathematical paper—someone who coauthored a paper with Erdős number, a person has to have written a mathematical paper. Chevalley, Jean Delsarte, Jean Dieudonné, and René de Possel—agreed to create a new body of work that covered all fields of mathematics. Invented in 1622, it was a vital mathematical tool before the advent of pocket calculators. 1602 Galileo discovers that the time taken for a pendulum to complete a swing does not depend on the swing's width. Creating a musical scale 56 Although scholars have questioned the story of the forge, Pythagoras is also widely credited with another musical discovery. He came from a family of wealthy lawyers who owned several properties, including a manor and a small chateau with fine vineyards. used by search engine algorithms for ranking web pages. For example, a radioactive isotope decays into another atomic form at an exponential 173 rate, and that results in a half-life, where half the material takes the same amount of time to decay, irrespective of the starting quantity. The quest led to many mathematical innovations, including new types of numbers—such as 314 negative and complex numbers—as well as modern algebraic notation and group theory. Around the age of 21, Pascal developed a mechanical adding and subtraction machine, the first ever marketed. Initially fascinated by theology, he was persuaded to change his degree to mathematics by Carl Gauss, under whom he then studied at the University of Göttingen. The Babylonian base-60 numbers 1 to 9, and the 10 symbol, repeated for 20, 30, 40, and 50. The Indian mathematician Aryabhata (474-550 CE) pursued the study of chords to produce the first table of what is now known as the sine function (all the possible values of sine/cosine ratios for determining the unknown length of the side of a triangle are known). Widely known as Pythagoras's theorem, it states that a 2 + b 2 = c 2, where c is the longest side of the 49 triangle (the hypotenuse), and a and b represent the other two, shorter sides that are adjacent to the right angle. Hippasus may have encountered irrational numbers while exploring the relationship between the length of the side of a pentagram formed inside it. Her doctorate was awarded for three papers, the most significant being on partial differential equations. Mathematical innovation A major advance in the business of calculation came with the adoption of the Hindu-Arabic number system and an increase in the use of symbols to represent functions such as equals, multiplication, and division. Others, notably Henri Poincaré, explored the newly established field of topology, the offshoot of geometry dealing with surfaces and space. The 19th century was a creative period for mathematics, and the exciting new field of topology spawned many new geometrical shapes. In order to achieve this, he developed a type of linguistic algebra, in which the operations of ordinary algebra, such as addition and multiplication, were replaced by the connectors that were used in logic. 1878 Georg Cantor advances the continuum hypothesis, later Number 1 on Hilbert's list. Along with their mathematically gifted children, the Bernoullis were the leading mathematicians of the early 1700s, making their home town of Basel in Switzerland a center of mathematical study. In fact, the origins of the triangle date back to ancient India in 450 BCE (see The ancient triangle). SuperStock: Fine Art Images / A. Once the key was uncovered, the two individual wheels could be transmitted digitally. His work was then rediscovered in Europe during the Renaissance, leading mathematicians to develop the analytic geometry that helped to fuel the scientific revolution. However, no one had any idea how to prove it. Different-sized squares The number of rows and columns in a magic square is called its order. A new world record was set when computer scientist Emma Haruka Iwao calculated π to more than 31 trillion decimal places in March 2019. Fuzzy logic is now a ubiquitous part of computer-controlled systems. For a start, it provides a quick way of multiplying out binomial expressions to high powers, which would otherwise take a very long time. See also: Positional numbers • The Rhind papyrus • Decimals • Logarithms • The mechanical computer • Boolean algebra • The Turing machine • Cryptography 277 278 INTRODUCTION By the late 1600s, Europe had become established as the cultural and scientific center of the world. David Hilbert Challenges for the future Hilbert's remarkable achievement was to accurately predict what would concern mathematicians in the 1900s and beyond. Perpendicular At right angles to something else. In his view, if a machine appeared to be thinking for itself, then it was. Key works 1614 Mirifici Logarithmorum Canonis Description of the Marvellous Rule of Logarithms) 1617 Rabdologiae Improving the method It took Napier 20 years to complete his calculations and to publish his first logarithm tables as Mirifici Logarithmorum Canonis Descriptio (A Description of the Marvellous Rule of Logarithms). One particularly appealing property of Pascal's triangle is the "hockey stick" pattern, which can be used for addition. In 1887, Poincaré won a prize from King Oscar II of Sweden for his partial solution of the many variables involved in determining the stable orbit of three planets around one another. In his Book of Rare Things in the Art of Calculation, Abu Kamil attempted to solve indeterminate equations (those with more than one solution). The radius of the two circles is the same, so AC = AB and BC = AB; this means that AC = BC, which is Euclid's first axiom (things that are equal to one another, and, if the equal straight lines are equal to one another, and, if the equal straight lines are extended below the base, the angles under the base will also be equal to one another. 428-348 BCE FIELD Geometry BEFORE 6th century BCE Pythagoras identifies the tetrahedron, cube, and dodecahedron. Impressed by these numerals' simplicity compared with the lengthy Roman numerals used in Europe, he discussed them in Liber Abaci (The Book of Calculation), which he wrote in 1202. 164 Leonardo traveled widely with his diplomat father and studied at a school of accounting in Bugia, North Africa. A few decades later, Eratosthenes and Euclid work perfectly and can be proven always to do so, but they did not conform to a formal definition. If hackers ever do figure out a simple way of determining the prime factorization of very large numbers, a new system will need to be devised. Read Euler, read and pence, which stemmed from the Latin system. Over the centuries, many mathematicians have refuted the paradoxes. This means that e is irrational. He began to publish major mathematical papers in his late teens and started teaching math at the University of Berlin aged 24. Relying completely on computing power—the first proof in the history of mathematics to do so-they examined around 2,000 cases, involving billions of calculations and using 1,200 hours of computer time. The gradient or steepness of this line will be the rate of change of the curve at that point. Her conjecture concerned images of distant astronomical light sources. Using ancient Greek methods, he worked out the area by dividing the ellipse into strips of infinitesimal width. See also: Calculus • The fundamental theorem of algebra • Non-Euclidean geometries KARL WEIERSTRASS 1815–97 Born in Westphalia, Germany, Weierstrass developed an interest in mathematics at an early age. Getty Images: DEA PICTURE LIBRARY / De Agostini (t). Public key encryption scrambles data with an encryption key available to anyone. Algebraic topology allows us to read qualitative forms and their transformations. Christian scholars now gained access to Greek and Indian philosophical and mathematical texts, and to the work of the Islamic scholars. In the 1900s, British mathematician John Fauvel maintained that the meaning of the Greek word for "element," stoicheia, changed over time, from "a constituent of a line," such as an olive tree in a line of trees, to "a proposition used to prove another," and eventually evolved to mean "a starting point for many other theorems." This is the sense in which Euclid used it. Searching for solutions The Babylonians and ancient Greeks used geometrical methods to solve problems that are now usually expressed by quadratic equations. He once commented that "Every good mathematician is at least half a philosopher, and every good philosopher at least half a mathematician." Frege mixed little with students or colleagues and remained largely unrecognized in his lifetime, although he was a major influence on the work of Bertrand Russell, Ludwig Wittgenstein, and other mathematical logicians. 50 CE Heron of Alexandria defines a formula for finding the area of a triangle from its side lengths. Continuing to multiply by i results in more 90° rotations, which is why every four multiplications arrive back at the start point. Funnily, of the 20 data sets that Benford collected, six of the sample sizes have leading digit 1. The ancient Egyptians had symbols for 2/3 and 3/4, but other fractions were expressed as the sum of unit fractions, for example as 1/3 + 1/13 + 1/17. His breakthrough, which he called the "golden theorem," was published by his nephew in 1713 in the book Ars Conjecturing). The stadium paradox concerns three columns of people, each containing an equal number of people; one group is at rest, while the other two run past each other at the same speed in opposite directions. In other words, squares, cubes, and higher powers of natural numbers are almost never consecutive. AFTER 1665 In England, Isaac Newton develops infinitesimal calculus, which introduces ideas such as limits, functions, and derivatives. One of the earliest examples is the method used by the Greek geometer Euclid to calculate the greatest common divisor of two numbers—the largest number that divides both of them without leaving a 466 remainder. Infinite because numbers can be added infinitely. Although these were plausible, they were not rigorously proved in Gauss's work. Each row of Pascal's triangle gives the binomial coefficients for a particular power. Number 3, for instance, asks if one of any two polyhedra of the same volume can always be cut into finitely many bits that can be reassembled to create the other polyhedron. Late 1900s Quantum Monte Carlo methods are used to explore particle interactions in microscopic systems. J. 1522 Portuguese navigator Ferdinand Magellan's ship completes the first voyage around the world. This proved that there is only a finite number of powers that can be consecutive. On the other hand, if the Taniyama-Shimura conjecture were true, Fermat's last theorem would follow. See also: The mechanical computer • The Turing machine MARJORIE LEE BROWNE 1914-79 551 Only the third African-American woman to earn a PhD in math, Browne was born in Tennessee at a time when it was hard for women of color to pursue an academic career. AFTER 1202 Leonardo of Pisa's Liber Abaci uses the Hindu-Arabic number system. Wellcome Collection (tr). Writer and amateur mathematician Antoine Gombaud, who styled himself Chevalier de Méré, understood the 1/6 odds of an ace with one throw of a die, and sought to calculate the odds of throwing a double ace using a pair of dice. The illustration above shows how the formula uses a, b, and c to find the value of x. 240 BCE Archimedes investigates the volume and surface area of spheres in his Method Concerning Mechanical Theorems. Khayyam suspected two, but would not have considered negative values. in the sequence produces the following: 1/1 = 1, 2/1 = 2, 3/2 = 1.5, 5/3 = 1.666..., 8/5 = 1.6, 13/8 = 1.625, 21/13 = 1.61538..., 34/21 =Engineering at the University in 1600. The resulting distribution is written as b(n, p). The development of calculus allowed mathematicians to deal with infinitesimal quantities without resulting in contradiction. They discuss a game between a player attempting to throw at least one ace in eight throws and a "banker" who takes the pot if the player is unsuccessful. He is known as the father of geographical language used today. Wiles had first read about Fermat's challenge when he was ten. AFTER 1748 Italian mathematician Maria Agnesi writes the first textbook to explain differential and integral calculus. This was known as the "problem of points," and it had a long history. In the early civilizations, these new discoveries in mathematics, and specifically the measurement of objects in space, became the foundation of the field of geometry, knowledge that could be used in building and toolmaking. The ancient triangle Myanmar's Hsinbyume pagoda represents the mythical Mount Meru, whose staircase inspired another name for Pascal's triangle. Fermat could not prove this conjecture; it was only in 1813 that French mathematician Augustin-Louis Cauchy completed the proof. This measure was arrived at by dividing c by b, or 144/36 in the example below. Gauss's idea of the "intrinsic curvature" of a surface or space was an important tool in establishing this new world, but he left little evidence of having developed non-Euclidean geometry himself. Even with the benefits of computer calculations, mathematicians have not yet moved beyond Uwe Hollerbach's discovery of Ta(6). In this work, Maclaurin applies calculus to the study of infinite series of algebraic terms. Satellite dishes are parabolic for this reason. In handwritten Spanish, an upper comma (similar to an apostrophe) is also common. One example involves a quarter circle, inscribed in a square, which curves from the top left corner of the square to the bottom right. He is said to have considered it a holy science, rather than something simply to be used for entertainment. He is committed to inclusion in education, and the idea that people of all ages learn in different ways. From 2 onward, these numbers are all powers of 2: 1 + 2 + 22 + 23 + 24, and so on. A system controlled by fuzzy logic, however, allows for degrees of truth that can better analyze complex phenomena, including human actions and thought processes. Its exact significance is debated, but the information appears to include Pythagorean triples (three positive numbers representing the lengths of sides of a right-angled triangle), alongside another set of numbers that resemble the ratios of the squares of sides. This was slowly refined and by the 1900s, the soroban was a 1:4 abacus (with 1 upper bead on each rod, and 4 lower beads). Khayyam's contribution 159 While Archimedes, working in the 3rd century BCE, may well have examined the intersection of conic sections in a bid to solve cubic equations, what marks Khayyam out is his systematic approach. Base (1) In a number system, the base is the number of items in a set with a cardinality of \$0 is ω. Buffon then counted the number of times the needle crossed the line as a proportion of total attempts (p) and came up with the formula that π is approximately equal to twice the length of the needle l, divided by the distance (d) multiplied by the proportion of needles crossing the line:  $\pi \approx 2l \div dp$ . Aristotle notes in Posterior Analytics that, even in a valid syllogistic argument, a conclusion cannot be true unless it is based on premises accepted as true, such as self-evident truths or axioms. after the other and usually following a set pattern. In particular, he used the triangle to help lay the foundations of probability theory in his correspondence with fellow French mathematician Pierre de Fermat. This gave him a "graph" that represented the relationships between the land and the bridges. This was a problem faced by Italian mathematician Gerolamo Cardano and his peers in the 1500s; while working on cubic equations, they found that some of their solutions involved square roots of negative numbers. 1930 Polish logicians Jan Łukasiewiecz and Alfred Tarski define a logic with infinitely many truth values. including light waves. They can also be used to forecast profits and loss in the world of business. Argand's additions Gauss published an improved proof of the FTA in 1816 and a further refinement at his golden jubilee lecture (celebrating 50 years since his doctorate) in 1849. The ancient civilizations each established the foundations of mathematics through this interdependent process of application and discovery, but also developed a fascination with mathematics for its own sake, so-called pure mathematics, is full of symmetries, and group theory is proving a powerful tool for both understanding and prediction. The pattern is clear: the number of members in each generation of ancestors forms the Fibonacci sequence. 486 The geodesic path of a planet orbiting a star in a predictable way is shown in the left-hand image. Polynomials and roots In the 1700s, one of the most studied areas of mathematics involved polynomials equations. Because of his Jewish background, he did not return to Germany after Hitler came to power in 1933, but settled in the United States, becoming a citizen there in 1940. This process is called binomial coefficients. The Egyptian concept of a fraction was closest to a unit fraction—that is, 1/, where n is a whole number. See also: Irrational numbers • The Platonic solids • Syllogistic logic • Calculating pi • Trigonometry • The golden ratio • Projective geometry 58 IN CONTEXT KEY FIGURE Hippasus (5th century BCE) FIELD Number systems BEFORE 19th century BCE) FIELD Number systems BEFORE 19th century BCE (19th century BCE) FIELD Number systems BEFORE 19th century BCE (19th century BCE) FIELD Number systems BEFORE 19th century BCE) FIELD Number systems BEFORE 19th century BCE) FIELD Number systems BEFORE 19th century BCE (19th century BCE) FIELD Number systems BEFORE 19th century BCE) FIELD Number systems BEFORE 19th century BCE (19th century BCE) FIELD Number systems BEFORE 19th century BCE) FIELD Number systems BEFORE 19th century BCE (19th century BCE) FIELD Number systems BEFORE 19th century BCE) FIELD Number systems BEFORE 19th century BCE (19th century BCE) FIELD Number systems BEFORE 19th century BCE) FIELD Number systems BEFORE 19th century BCE (19th century BCE) FIELD Number systems BEFORE 19th century BCE) FIELD Number systems BEFORE 19th century BCE (19th century BCE) FIELD Number systems BEFORE 19th century BCE (19th century BCE) FIELD Number systems BEFORE 19th century BCE) FIELD Number systems BEFORE 19th century BCE (19th century BCE) FIELD Number systems BEFORE 19th century BCE) FIELD Number systems BEFORE 19th century BCE (19th century BCE) FIELD Number systems BEFORE 19th century BCE) FIELD Number systems BEFORE 19th century BCE (19th century BCE) FIELD Number systems BEFORE 19th century BCE) FIELD Number systems BEFORE 19th century BCE (19th century BCE) FIELD Number systems BEFORE 19th century BCE) FIELD Number systems BEFORE 19th century BCE (19th century BCE) FIELD NUMBER 19th century BCE) FIELD NUMBER 19th century BCE (19th century BCE) FIELD NUMBER 19th century BCE) FIELD NUMBER 19th century BCE (19th century BCE) FIELD NUMBER 19th century BCE) FIELD NUMBER 19th c understood their properties. The Boolean commands have simply become silent: a search for "George Boole," for example, has an implied AND between the two words, so that only web pages containing both names will appear in the results. Each player has staked 32 pistoles, so the stake is 64 pistoles. Time is a relative, not an absolute, quantity running at different speeds in different places and woven together with space. It could also be 1, as any number divided by itself is 1. The work outlines the basic rules for adding, subtracting, and multiplying imaginary numbers in terminology that differs from that used by mathematicians today. By about 1000 CE, the Aztec people knew it as the nepohualtzintzin—the "personal accounts counter"—and wore it on the wrist as a bracelet. Pythagoras's followers were also aware of some of the regular polyhedra; these are the perfectly symmetrical three-dimensional shapes (such as the cube) that were later known as the Platonic solids. It is a Latin word derived from the ancient Greek, abax, which means "slab" or "board"— a surface that would have been covered in sand and used as a drawing board. His Treatise on Demonstration of Problems of Algebra (1070) notably includes a fresh perspective on the difficulties within Euclid's postulates, a set of geometric rules that are assumed to be true without requiring a proof. In 1852, he asked his brother Frederick, who was studying under mathematician Augustus De Morgan in London, if his theory could be proved. In addition, he produced the first world map that featured meridian lines, the Equator, and even polar zones. Yet Jefferson's view did not hold sway for measurement, and inches, feet, and yards are still used today. Two centuries later, Swiss mathematician Leonhard Euler introduced the symbol i to denote the imaginary unit. In 1858, the two men independently investigated the twisted strip, which Listing is said to have discovered first. 300 BCE) FIELD Number systems BEFORE c. 1202 CE Fibonacci introduces his famous sequence. ancient world. The abscissa (value of y) to create the tuple (x,y). Babbage built several small working models with just several small work it a "transcendental" number—a real number that cannot be computed by solving an equation. While there, he also gave mathematics tuition, one of his students being Leonardo da Vinci, who illustrated Pacioli's Divina Proportione. Diophantine equations of the kind xn + yn = zn may look simple to calculate, but only those with squares are solvable. After the fall of the Roman Empire, mathematics in the Mediterranean area declined, but the spread of Islam from the 7th century had a revolutionary impact on algebra. The first volumes produced little that was not known to Euclid, but the later works were a significant advance in geometry. However, the attractor shows that as a whole, the system is stable. Named after 19th-century German mathematician August Möbius, a Möbius strip can be created in seconds by twisting a strip of paper through 180°, then joining its two ends together. His visual images of attractors and oscillators, along with Benoit Mandelbrot's fractals, became icons of these new fields of study. There was no need for zero, or indeed negative numbers (numbers less than 0), as the Greeks did not have a positional number system (lengths cannot be nonexistent or negative). Carl Benjamin Boyer American historian of mathematics Islamic trigonometry Brahmagupta had already created a table of sine values, but in the 9th century CE, Persian astronomer and mathematician Habash al-Hasib ("Habash the Calculator") produced some of the first sine, cosine, and tangent tables to calculate the angles. Every year, computers using AI try to win the prize. The 8-bit byte was convenient because 8 bits have 28 permutations (256), and can encode numbers from 0 to 255. Instead, it follows a fourstep pattern that repeats infinitely: i1 = i; i2 = -1; i3 = -i, i4 = 1; i5 = i, and so on. 1120 Omar Khayyam creates an early version of Pascal's triangle. He was killed by a Roman soldier during the Siege of Syracuse in 212 BCE, despite orders that his life be spared. The area of a circle could be determined, for example, by using slender pie-slice triangles. Induction A way of obtaining a general conclusion in mathematics by establishing that if a statement is true for one step in a process, it is also true for the next step in a process, it is also true for the next step in a process, it is also true for the next step in a process and all those that follow. Quadratic equations often equal 0, because this makes them easy to work out on a graph; the x solutions are the points where the curve crosses the x axis. For example, the number of primes up to 100 gives π(100) = 25. More recent developments in algebra. The curved path of any pendulum would have to be adjusted so it traveled in a cycloid. The paradoxes are known from Aristotle's treatise Physics, which lists nine of them. In the 1760s, French mathematician Jean d'Alembert questioned whether it was possible to have a very long string of occurrences in a sequence in which occurrence and non-occurrence are equally likely—for example, whether a person flipping a coin might get "heads" two million times in a row Ancient Egyptians used vertical lines to denote the numbers 1 to 9. The Elements gave rise to what is called Euclidean geometry; spherical geometry; spherical geometry; spherical geometry is the first example of a non-Euclidean geometry; spherical geometry is the first example of a non-Euclidean geometry; spherical geometry is the first example of a non-Euclidean geometry. Bernhard Riemann, devised new geometrical coordinate systems that extended beyond the limits of the 2-D and 3-D systems devised by René Descartes. The point is chosen at random, so could be anywhere in the square. Written as a puzzle, it suggests he married at the age of 35, and five years later had a son, who died at the age of 40 when he was The two additional guaternion units, j and k, share similar properties to i and are imaginary. Alamy Stock Photo: Massimo Dallaglio (tl). Weldon, followed by the world's first university department of statistics at University College, London, in 1911. The problem with using the golden ratio to define human beaut looking hard enough for a pattern, you'll almost certainly find one. A three-rotor Enigma had over 158,962,555,217 billion possible settings, which were changed daily. Around 1800 BCE, the Babylonians recorded sets of Pythagorean numbers on the Plimpton 322 clay tablet; these show that triples become more spread out as the number line progresses. In the 1860s, Scottish scientist James Clerk Maxwell found that the speed of light is the same whatever the speed of its source. 1890 Henri Poincaré shows there is no general solution to the three-body problem, which predicts the motion of three celestial bodies kept in contact by gravity. Marjorie gained a reputation for being an excellent teacher, and for her research, especially in topology. Wherever groups disclosed themselves, or could be introduced, simplicity crystallized out of comparative chaos. See also: Decimals • Projective geometry • Pascal's triangle ADRIEN-MARIE LEGENDRE 1752–1833 Legendre taught physics and math at the École Militaire in Paris from 1775 to 1780. In acoustics, sound intensity is measured in decibels. For the sake of brevity, we will always represent this number, 2.718281828... by the letter e. On the other hand, for A OR B, there are three possible combinations in which that statement is true, as it will only be false if both A and B are false. One way to see the symmetries of the triangle is to consider all of the possible permutations of the vertices. He was among the first to state that a cubic root could be a complex number, something not possible with "real" numbers. An inscription from 36 BCE shows that they used a shell-shaped symbol to denote zero, which was widely used by the 4th century. In using these measurements for practical purposes, people found that certain patterns were emerging, which could in turn prove useful. Until the 10th century, geometric methods were were often used, as quadratic equations were used to solve real-world problems involving land rather than abstract algebraic challenges. Starting in the 9th century BCE in China, the story then bounces around via Indian texts from 100 CE, Arab scholars in the Middle Ages, Europe during the Renaissance, and finally modern Sudoku-style puzzles. 1637 French philosopher and mathematician René Descartes develops a way to plot algebraic expressions as coordinates on a grid. The most common letter in the English language is "e," so a cryptanalyst would find the most common letter in the ciphertext and designate that as e. A sound 1,000 times louder is assigned a decibel value of 10; a sound 10 times louder is assigned a decibel value of 10; a sound 1,000 times louder is assigned a decibel value of 10; a sound 100 times louder is assigned a decibel value of 10; a sound 100 times louder is assigned a decibel value of 10; a sound 1,000 times louder is assigned a decibel value of 10; a sound 100 times louder is assigned a decibel value of 10; a sound 1,000 times louder is assigned a decibel value of 10; a sound 100 times louder is assigned a Much of this impetus came from German mathematicians, including Möbius and Johann Listing. An equation means nothing to me unless it expresses a thought of God. See also: Positional numbers • Pythagoras • Trigonometry • Algebra • The golden ratio • Pascal's triangle • Benford's law 170 IN CONTEXT KEY FIGURE Sissa ben Dahir (3rd or 4th century CE) FIELD Number theory BEFORE c. The word "algebra" comes from al-jabr. In their exploration of the properties of numbers, shapes, and processes, they discovered universal rules and patterns that raised metaphysical questions about the nature of the cosmos, and even suggested that these patterns had mystical properties. This involved the correct three rotors being inserted and set to the right starting positions, and the 10 plugs being connected correctly on the board. 1762 Leonhard Euler describes the use of logic circles, now known as "Euler circles." AFTER 1963 American mathematician David W. 394 This logic module is used for teaching how logic gates function in electronic circuits. A cube number is one that is obtainable by multiplying a smaller number together twice — for example 8, which is 2 × 2 × 2 (23). Dividing stakes Pascal was asked to look into probability in 1652 by a notorious French gambler. In Latin translations of Elements, deductions end with the letters QEF (quod erat faciendum, meaning "which was to be [and has been] done." Logical proofs end with QED (quod erat demonstrandum, meaning "which was to be [and has been] demonstrated"). Newton went on to develop his theory further by considering the converse problem—if the rate at which a variable itself? In the 4th century BCE, the ancient Greek philosopher Aristotle suggested in Metaphysics that Earth was created by atoms coming together entirely by chance. The idea of perspective had been addressed two centuries earlier by Renaissance artists and architects. This number gets closer to 1 the greater the number of spins. For example, a doughnut and a teacup are topologically similar because they are both shapes that have one hole going through the mathematicians gather in the library of a mosque in an illustration from a manuscript by the 12th-century poet and scholar Al-Hariri of Basra. 178 IN CONTEXT KEY FIGURE Luca Pacioli (1445-1517) FIELD Applied geometry BEFORE 447-432 BCE Designed by the Greek sculptor Phidias, the Parthenon is later said to approximate the golden ratio. A 3-manifold, such as the 3-sphere, is a purely theoretical concept: it has a 3-D surface and exists in a 4-D space. See also polynomial. BROUWER 1881-1966 Luitzen Egbertus Jan Brouwer (known to his friends as "Bertus") was born in Overschie, Netherlands. His rules for calculating with positive and negative numbers, which he called "fortunes" and it has been applied extensively in guantum mechanics (dealing with the motion of subatomic particles). Unlike his first geometric approach, his second and third proofs were more algebraic and technical in nature. Alamy Stock Photo: Interfoto (bl). Breaking the brotherhood's rule of secrecy by revealing their knowledge of the dodecahedron would therefore have been an especially heinous crime, punishable by death. 2015 Möbius strips are used in laser beam research, with potential application in nanotechnology. He analyzed the motion of bodies under the action of forces and posited gravitational attraction to explain the movement of the tides, projectiles, and pendulums, and the orbits of planets and comets. Kepler's solution was to accept the notion of "infinitesimals"—the thinnest slices that can exist without vanishing. If the parabola does not touch or cross the x axis, there are no real roots. This expresses the quadratic equation x2 + 3x + 2 = 0. In 1848, he ran unsuccessfully for parliament as a liberal candidate and the offended king temporarily withdrew his support. This can be a powerful strategy when the other area is well understood. The symmetry groups of polygons can be thought of as permutations of a set of elements. In the Treviso description, zero is just a placeholder number, which itself was still a new notion. Briefly imprisoned for his religious beliefs, de Moivre emigrated to England upon his 300 release. In 1854, after the outbreak of the Crimean War, Nightingale went to work among wounded soldiers at The Barrack Hospital in Scutari, Turkey. 335 After centuries of suspicion, mathematicians finally embraced the concept of negative numbers in the 1700s. After Galileo's death in 1642, Viviani gathered together his master's work, editing the first collected edition in 1655-56. The same is then done for multiples of 3, 5, and 7. In Boolean terms, such statements can be given as, for example: (A OR B) = (B OR A); NOT (NOT A) = A; or even NOT (A OR B) = (B OR A); NOT (A O Chapters on the Mathematical Art, reveals the most important mathematical discovery in the last 400 years. For the side of the smallest square, 3 is used because this length is 3/4 of the side of the other small square. 1914 British philosopher Bertrand Russell, who described Zeno's paradoxes as immeasurably subtle, states that motion is a function of position with respect to time. Polyhedron Any 3-D shape whose faces are polygons. Under Pythagoras's leadership, the community gained considerable political influence. He was also barred from working on codebreaking for the government. This "small world theory" predated Milgram. Alamy Stock Photo: Idealink Photography (cra). De Triangulis Omnimodis (On Triangles of all Kinds) was published in 1533. 550 CE), a book of divination, includes the first recorded magic square in India, used to measure out quantities of perfume. The other two appendices were on light and the weather. In other words, it is a question of noting how many ways the event can occur, and dividing this by the total number of possibilities. A golden spiral can be inscribed within a golden rectangle. Ten years later, Mirzakani was both the first woman and the first woman and the first woman and the first woman and the first woman surfaces. For example, if two quantities are in inverse proportion, the larger one of them becomes, the smaller the other one will become; for example, if one quantity is multiplied by 3. This cursive style was more compact and practical than drawing complex hieroglyphs. The results of a statement in Boolean algebra can also be assessed using a truth table, in which all possible input combinations are tried and written out. Operation Any standard mathematicians René Descartes and Pierre de Fermat, but he did get close to coordinate representations of his conic curves. 1800 BCE The Babylonian Plimpton 322 tablet contains a list of Pythagorean triples, long before Pythagorean triples, long before Pythagorean triples are still used occasionally in China, just as Roman numerals are sometimes used in Western society. The symbols used for such operations are called operators. Its foundations were laid in the late 1800s, principally by British polymaths Francis Galton and Karl Pearson. This is referred to as the golden mean. The process of reduction (simplifying the symbols used for such operations) are called operators. Its foundations were laid in the late 1800s, principally by British polymaths Francis Galton and Karl Pearson. an equation) could be done by rejoining (al-jabr)—moving subtracted terms to the other side of an equation—and then balancing the two sides of the equation. Variable A mathematical quantity that can take on different values, often symbolized by a letter such as x or y. In the 1800s, however, alternatives to classical Euclidean geometry were proposed, opening up new areas of study, including topology, which examines the nature and properties not only of objects in space, but of space itself. 581 Getty Images: Eduardo Munoz Alvarez / Stringer / Getty Images: Eduardo Munoz Alvarez / Stringer / Getty Images News (cra); ullstein bild Dtl. AFTER 1742 Leonhard Euler refines the Goldbach conjecture. They invited six fellow former students from the Ecole Normale Supérieur to lunch in Paris, hoping to persuade them to take part in an ambitious project to write a new treatise that would revolutionize mathematics. In his Elements, Euclid stated many properties of both composite numbers (integers above one that can be made by multiplying other integers) and primes. In each column there are two beads above the bar worth five each and five beads below the bar worth one each. While he was studying set theory, Cantor aimed to create definitions for every number or quantity by which another number or quantity is being divided. JOSEPH FOURIER Jean-Baptiste Joseph Fourier was born in Auxerre, France, in 1768. 497 The Erdős in their published work. He returned to Norway in 1828, but died from tuberculosis the following year at the age of 26, days before a letter arrived offering him a prestigious math professorship at the University of Berlin. They noted, for instance, that Euclid did not justify or explain the existence of C, the point of intersection of the two circles. Stephen Strogatz American mathematicians alike, the threebody problem is largely theoretical. All physical reality is held within it, as nothing can travel faster than light. His first proof was based on assumptions about algebraic curves. Although little is known of Zeno's life, the ancient Greek biographer Diogenes claimed he was beaten to death for trying to overthrow the tyrant Nearchus. great contribution to mathematics, Hilbert's death in 1943, during World War II, went largely unnoticed. He worked with more than 500 others in the global mathematics, including number of permutations that are possible in a collection of objects. The conservation of electric charge, for example, is related to rotational symmetry. The continuum hypothesis, the first problem on Hilbert's list, pointed out that the set of natural numbers (the positive integers) was infinite, but so was the set of real numbers between 0 and 1. In any set of scientific data, most errors are likely to be very small, and only a few are likely to be very large. Between any two rational numbers, another rational numbers, another rational number for example, the number 752 would use a vertical 7, then a horizontal 5, followed by a vertical 2. It is a concept within topology. It requires only 10 symbols—nine digits (1, 2, 3, 4, 5, 6, 7, 8, 9) and a zero as a placeholder. It is an easy code to break, but Bacon realized that a and b do not have to be letters—they can be any two different objects— "... as by bells, by trumpets, by lights and torches... and any instruments of like nature." It was an ingeniously adaptable cipher, which Bacon could use to "make anything signify anything." A secret message could be hidden in a group of objects or numbers, or even musical notations for the field of probability. Each set is given its own circles are overlapped. His parents wanted their son to have a career in administration, so 545 he was sent to study law and economics at his university, but left without gaining a degree. With the invention of the movable-type printing press in the 1400s, books of all sorts, including the Treviso Arithmetic, became widely 130 available, spreading the newfound knowledge across Europe. Although he entered university to study for the ministry, he switched to mathematics which now forms the underlying basis of many other branches of mathematics. Expression Any meaningful combination of mathematical symbols, such as 2x + 5. Conic sections are figures formed from the intersection of a plane and a cone and they may be circular, elliptical, or parabolic in shape. A negative makes a positive. Quadratic equation An equation containing at least one variable multiplied by itself once (for example y × y, also written y2), but containing no variables raised to higher powers. In this sequence, every number is the sum of the previous two (0, 1, 1, 2, 3, 5, 8, 13, 21, 34, and so on). The attempt was so unpopular that it was dropped after just one year. Getty Images: DEA PICTURE LIBRARY / De Agostini (crb). Each particle left the same starting position at the same time and at the same velocity. Books VII to IX are devoted to numbers. In 1631, Fermat moved to Toulouse and worked as a lawyer. Key works 1629 De tangentibus linearum curvarum (Tangents of Curves) 1637 Methodus ad disquirendam maximam et minimam (Methods of Investigating Maxima and Minima) The Pascal-Fermat letters During the 1600s, it was common for mathematicians to meet at academies— scientific societies. Hailing from Samos, which was not far from Miletus in western Anatolia (present-day Turkey), he may have studied at the school of Thales of Miletus under the philosopher Anaximander. Stevin's circled numbers for tenth powers were also done away with, meaning that 32 + 5/10 + 6/100 + 7/1,000 could now be written as 32.567. He later opposed the Vietnam War. For a whole summer Katz went through the proof line by line, querying and 528 questioning until the meaning was clear. His interests were wideranging, and his output was immense—on topics ranging from ship-building to natural history and astronomy. Her mother, Lady Byron, was mathematically gifted—Byron called her his "Princess of 356 Parallelograms"— and insisted Lovelace study mathematics, too All these shapes possess self-similarity, which is a key property of fractal geometry. Newton said that the rate of change of momentum (mass - velocity) of a body is equal to the force acting on it. The "Fourier trigonometry series" has been used widely in scientific fields such as optics, electromagnetism, and, more recently, quantum mechanics. rarity of an Erdős number higher than 10 indicates the degree of collaboration within the mathematical community. And no doubt that will be true again 100 years from now. It involved constructing a square with the same area as a given circle. Schrödinger's wave equation, a differential equation published in 1925 by Austrian physicist Erwin Schrödinger, models a particle as a wave whose state can only be determined by using probability. While there have been many different types of musical scales in use by different cultures, the long tradition of Western music dates back to the Pythagoreans and their quest to understand the relationship between music and mathematical proportions. The ancient method was limiting; Descartes' new method opened up all sorts of new possibilities. If any quadratic equation is drawn on a graph, these roots can be easily found: they are where the line touches the x axis. He then went to Cambridge and came second in the university examinations, but was not allowed to graduate because, as a Jew, he would not swear allegiance to the Church of England. The principles behind his logarithm tables were published in 1619, two years after his death. If the denominator is only divisible by 2 or 5 and no other prime numbers-as is the case for 10-then the decimal will terminate. Mathematicians hoping to solve this "three-body problem" have abstracted it to consider imaginary bodies moving around surfaces and spaces with specific curvature. It was only in the 1750s that the Italian Joseph-Louis Lagrange arrived at a solution, where the height of the curve needs to be in proportion to the square of the length of the curve needs to be in proportion. she moved to the US, working at Bryn Mawr College and at the Institute for Advanced Study until her death in 1935. Some historians believe the Maya 383 people manipulated numbers in rows and columns to solve equations, and cite gridlike decorations on their monuments and priestly robes as evidence. Perspective makes the parallel lines on sides of this flat-roofed building appear as though they will eventually meet. 376 Since quaternions can model and control the motion of objects in three dimensions, they are particularly useful in virtual reality games. I had this rare privilege of being able to pursue in my adult life what had been my childhood dream. This means that if x2 is the highest power in an algebraic expression made up of a single variable (such as x) and real coefficients (numbers multiplying the variable), the expression has a degree of two, and also two roots; roots are values of x that make the polynomial equal to zero. 1891 Russian crystallographer Evgraf Fyodorov proves there are only 17 possible groups that form periodic tilings of the plane. This graph plots the points (in black) of the quadratic function where a = 1, b = 3, 36 and c = 2. In India, mathematician who has become almost a household name is Pierre de Fermat, whose claim to fame rests largely on his enigmatic last theorem, which remained unsolved until 1994. In the mid-1500s, Niccolò Tartaglia, another prominent mathematician, had noted that such a division would be unfair if the game was interrupted, say, after only one round. In 1843, she arranged the publication of her translation of a pamphlet about the Analytical Engine written by Italian engineer Luigi Menabrea, to which she added extensive explanatory notes. Pierre-Simon Laplace A basis for Euler's work The push for accurate log tables spurred mathematician, and astronomer Thales of Miletus deduces that the angle inscribed inside a semicircle is a right angle. The fact that 2 + 2 = 4 is true, independent of human existence; the rules of mathematics, like the laws of physics, are universal, eternal, and unchanging. He died there in 322 BCE. Or, for any number between 1 and x, the probability of it being a prime is approximately 1 ÷ ln(x). See also irrational number. It can ascertain where changing phenomena reach a maximum or minimum value, and at what rate they travel between the two. 20 Geometry in particular chimed with Greek culture, which idolized beauty of form and symmetry. For centuries, algebra had just been a tool to solve geometric problems, but now became a discipline in its own right, where calculating increasingly difficult equations was the end goal. A5 is also a simple group. I have often admired the mystical expressions or quantities are equal to each other. There are only 25 prime numbers up to 100-starting with 2, 3, 5, 7, and 11, and ending with 97—all identified by simply removing every multiple of 2, 3, 5, and 7. With a parabolic mirror, any ray of light parallel to its line of symmetry will reflect off the surface to the same fixed point (A). 529 In many ways, it is not the proving of Fermat's last theorem that has significance, but rather the proofs used by Wiles. (bl). The group aimed to strip mathematics back to basics and provide a foundation from which it could go forward. The division can continue indefinitely. By adding these points at infinity. He did, however, consider that the Universe might be non-Euclidean. The boundaries between these regions were self-replicating, or fractal. A fervent Protestant, he also wrote a prominent book attacking Catholicism. However, binary was the language of the first Turing-complete machine ever built, the Z3. The Monte Carlo method This problem is an example of a wider class of experiments that employ a statistical approach called the Monte Carlo method, a code name coined by Polish-American scientist Stanislaw Ulam and his colleagues for the random sampling used during secret work on nuclear weapons in World War II. 1050, Jia Xian created a similar triangle to show coefficients. The Chinese version, which is still used today, has a central bar and a varying number of vertical wires to separate ones from tens, hundreds, or more. New developments The incremental steps toward the end of the 16th century. But computing science was based on a binary system of logic first proposed by George Boole in the 1800s, and the polar opposites of onoff, true-false, 0-1, and so on could not describe how things are in the real world. The algebra developed by al-Khwarizmi 150 years or so previously had used a system of symbols to work out unknown quantities and was limited in scope. 300 BCE that is etched with horizontal lines. Forecasting the outcomes of games of chance proved a useful way of approaching probability, which, by definition, measures the likelihood of 251 something occurring. Egyptian engineers, for example, used mathematics in the building of pyramids. Positional number An individual numeral whose value depends on its position within a larger number. which swapped 10 pairs of letters, thus scrambling the message further. Imaginary numbers are a fine and wonderful refuge of the divine spirit. This is why all positive numbers have two square roots (a positive, and a negative numbers have numbers have two square roots). squared is also positive. Other notable discoveries in number theory at this time include the creation of set theory and the description of an "infinities" of Georg Cantor, Eugène Catalan's conjecture about the powers of natural numbers, and the application of elliptic functions to number theory at this time include the creation of set theory at this time include the creation of set theory and the description of an "infinities" of Georg Cantor, Eugène Catalan's conjecture about the powers of natural numbers, and the application of elliptic functions to number theory at this time include the creation of set theory at this time include the creation of elliptic functions to number theory at this time include the creation of an "infinites" of Georg Cantor, Eugène Catalan's conjecture about the powers of natural numbers, and the application of elliptic functions to number theory at this time include the creation of elliptic functions to number theory at the powers of natural numbers, and the application of elliptic functions to number theory at the creation of elliptic functions to number theory at the powers of natural numbers, and the application of elliptic functions to number theory at the powers of natural numbers, and the application of elliptic functions to number theory at the powers of natural numbers, and the application of elliptic functions to number theory at the powers of natural numbers, and the application of elliptic functions to number theory at the powers of natural numbers, and the application of elliptic functions to number theory at the powers of natural numbers, and the application of elliptic functions to number theory at the powers of natural numbers, and the application of elliptic functions to number theory at the powers of natural numbers, and the application of elliptic functions to number theory at the powers of natural numbers, and the applications to number theory at the powers of natural numbers, and the applications to number theory at the powers of natural numbers, and the applicat doctorate was awarded by The Catholic University of America in 1943 for a dissertation on set theory. For example, these functions appear in the 1995 proof of Fermat's last theorem, and the latest public-key cryptography systems. In 1918, German mathematician Felix Hausdorff proposed the existence of fractional dimensions. This is the essence of chaos theory. 110 In the 1700s, Joseph Fourier applied trigonometry to his research into different forms of waves and vibrations. However, his view of a conic curve as a set of ordered parallel lines emanating from an axis looked toward the later creation of 101 coordinate system geometry. Only the start and end points can have an odd number. 243 While it might seem that such a simple rule for arranging numbers could only lead to simple patterns, Pascal's triangle is fertile ground for several branches of higher mathematics, including algebra, number theory, probability, and combinatorics (the mathematics of counting and arranging). One story suggests that his fear of irrational numbers drove his followers to drown a fellow Pythagorean—Hippasus— for revealing their existence when attempting to find . Today, ratios derived from the sequence are used as an analytical tool to forecast the point at which stock market prices will stop rising or falling. Initially referred to as "discrete variables," in 1948 the 1s and 0s in computer code were renamed "bits," short for binary digits. Described as the "father of engineering drawing," Monge invented descriptive geometry, the mathematical basis of technical drawing, and orthographic projection. Less well known are his contributions to the development of calculus, number theory, and analytic geometry. 491 See also: Syllogistic logic • Binary numbers • Boolean algebra • Venn diagrams • The logic of mathematics • The Turing machine 492 IN CONTEXT KEY FIGURE Robert Langlands (1936-) FIELD Number theory BEFORE 1796 Carl Gauss proves the quadratic reciprocity theorem, relating the solvability of quadratic reciprocity theorem. Ladislaus Bortkiewicz uses the Poisson distribution to study the number of Prussian soldiers killed by horse kicks. From passively observing and recording the patterns in the movement of celestial bodies, scholars began to model that movement mathematically so that they could predict future astronomical events with ever greater accuracy. It necessarily requires many more symbols than a base-10 system. Arguably, Gauss's first, geometric proof suffered for being premature—in 1799, 328 the concepts of continuity and of the complex plane, which would have helped him explain his ideas, had not yet been developed. Galton and Pearson's ideas are tainted by the doctrine of eugenics and racial improvement, but the techniques that they developed have found applications elsewhere. The unknown author makes 0 the 10th numbers to increase their value. They had also started using numbers and symbols, rather than writing equations in words. If you take a diagonal down from any of the outer 1s, then stop anywhere, you can then find the total sum of all of the numbers in the diagonal by taking one step further in the opposite direction. It was a breakthrough in the modeling of probability and is still used today in areas as diverse as locating lost aircraft and testing for disease. In so doing, he not only gave a definitive answer to a historical quest that had begun in ancient Babylon, but also laid the foundations of abstract algebra. It becomes known as the modularity theorem. Shapes in the Universe? Quintic Referring to equations or expressions of the fifth degree, where the highest power contained in them is 5, for example, x5. 300 BCE Euclid's Elements sets down ideas that will later constitute Euclidean geometry. The settings became the encryption key. Mathematicians now describe division by zero as "undefined." Some have suggested that the required answer to n/0 is "infinity," but infinity is not a number and cannot be used in calculations. Several thousand, mostly dating from between 1800 and 1600 BCE, are housed in museums around the world. Robert Kanigel American science writer Ancient texts Determining pi's exact value is not straightforward and the quest continues to find pi's decimal representation to as many places as possible. Each number would have its equivalent "artificial number" as he initially termed it. A man provided with paper, pencil, and rubber, and subject to strict discipline, is in effect a universal machine. NASA: NASA CONTEXT KEY FIGURE Pedro Nunes (1502-78) FIELD Graph theory BEFORE 150 CE The Greco-Roman mathematician Ptolemy establishes the concepts of latitude and longitude. Regression, on the other hand, looks for the best equation for the graph line for two variables, so that changes in one variable can be predicted from changes to the other. Zero is perhaps the most important number of all. The Alexandrian scholar Hypatia, depicted here in an 1889 painting by Julius Kronberg, was revered as a heroic martyr after her murder. Geometry is knowledge of what always exists. Applied mathematics The use of mathematics to solve problems in science and technology. His Liber Abaci (1202), which brought many useful Arabic ideas to the west, also introduced a new notation for fractions to Europe that would form the basis of the notation used today. I have not thought it necessary to undertake the labor of a formal proof of the theorem in the general case of a matrix of any degree. Such spirals were called loxodromes by Dutch mathematician Willebrord Snell in 1617; they became a key concept in the geometry of space. In 1569, Flemish cartographer Gerardus Mercator devised as the surface of the globe transferred onto a surrounding cylinder. Fourier's studies of heat distribution showed that no matter how complex a waveform, it could be broken down into its constituent sine waves, a process that is now called Fourier analysis. 225 BCE On the Sphere and the Cylinder c. Without the list of code words, deciphering the message's full meaning is impossible. Today's mathematicians face further challenges, but aspects of Hilbert's problems especially those that are still unsolved - remain relevant. Johnson then worked for the National Aeronautics and Space Administration (NASA) from 1958 as part of its Space Task Group. Like the Babylonians, they used a calendar of 360 days plus festivals, to make 365.24 days based on the solar year; their calendars helped them work out the growing cycles of crops. Dreamstime.com: Gavin Haskell (tr). He wanted to use logic to answer questions about God's existence and believed that the binary system captured his view of the Universe's creation, with 0 representing God. Harmonic series 1 + 1/2 + 1/3 + 1/4 + 1/5 + ... An important advance in the arithmetical use of negative numbers came around 400 years later from India, in the work of the mathematician Brahmagupta (c. Galton noticed that very short parents, while very short parents, while very short parents, while very short parents tend to have children who are shorter than their parents. If the real numbers are drawn along the x axis and the imaginary numbers are drawn along the y axis, then the whole plane between them becomes the realm of the complex numbers would have areas of 16 and 9 respectively, which when added together give a total area of 25. It would take a supercomputer thousands of years to work out p and q by trial and error, making RSA and similar protocols practically unbreakable. Although her mathematics— from 1908 to 1923 she received no pay. Stevin placed circles after each number; these were shorthand for the denominator of the original decimal fraction. A new cultural movement, known as the Renaissance ("rebirth"), was inspired by renewed interest in the arts and philosophy of the Graeco-Roman Classical period. 1820s French mathematician Augustin Cauchy clarifies the rules for calculus, inaugurating a new rigor in mathematics. All imaginary roots of polynomials, and called numbers are multiples of i. Although l'Hôpital was an accomplished mathematician, many of his ideas were not original. A network of triangulation stations such as this stone "trig point" in Wales was launched by the Ordnance Survey in 1936 to accurately map the island of Great Britain. Often mathematics was therefore seen as a complementary discipline to philosophy-many of the greatest mathematicians through the ages 13 have also been philosophers, and vice versa-and the links between the two subjects have persisted to the present day. From about the 5th century CE, India began a "Golden Age" of mathematics, building on its own long tradition of scholarship, but also on ideas brought in by the Greeks. Despite this, many people have subsequently attempted to relate the golden ratio to the notion of attractiveness in people (see box). Once they had been made public, Langlands' conjectures became influential across mathematics, and continue to shape research 50 years later. For subtraction, the units of the first number are entered, then bead values are adjusted downward in each column as further subtracted numbers are entered. It was thought that their combined area would eventually converge toward the true area of the shape. As new applications for these mathematical discoveries were found—in astronomy, navigation, engineering, bookkeeping, taxation, and so on-further patterns and ideas emerged. It described the atomic world in terms of random chance-something at odds with the mechanical determinism of Laplace's demon. In 1614, he published the first description and table of logarithms; a logarithm of a given number is the exponent or power to which another fixed number (the base) is raised to produce that given number. The method was developed into a full geometry by Poncelet in 1822. Cubic equation An equation containing at least one variable multiplied by itself twice (for example, y × y × y, also written as y3), but no variable multiplied more times than this. He died in Athens around 348 BCE. There he met Prince Maurice, the heir of William of Orange, and they became friends. The side of one of the smaller squares is equal to one half plus a quarter of the side of one of the smaller squares is equal to one half plus a quarter of the smaller squares is equarter of the smaller squares is equares is equares i The parallel postulate is not true for spherical geometry, where all pairs of lines have points in common, nor for hyperbolic geometry, where a formula for solving quadratic equations that could be applied to equations in the form ax2 + bx = c. The Monster There are precise descriptions of 18 families of finite simple groups, with each family related to symmetries of certain types of geometrical structure. They developed early notions of area and volume, although these tended to be in the form of very specific examples, such as in the Rhind papyrus, where one problem involves the area of a round field with a diameter of 9 khet (a khet being an ancient Egyptian unit of length). 429 It requires a special technical brilliance and self-confidence to predict relevant problems for the next hundred years, but this is what German mathematician David Hilbert did in 1900. 1763 Thomas Bayes develops a way of predicting the chance of an outcome by taking into account the starting conditions related to that outcome. 533 A 3-sphere is the 3-D equivalent of a spherical surface, or 2-sphere, such as the ball shown here. At each increase of knowledge, as well as on the contrivance of every new tool, human labor becomes abridged. With the ABC triangle, the identity is the rotation of 0°. They are arranged so that a tilted square is formed in the middle, by the hypotenuses (c sides) of the four triangles. JOHANNES KEPLER Born near Stuttgart, Germany, in 1571, Johannes Kepler witnessed the "Great Comet" of 1577 and a lunar eclipse, and remained interested in astronomy throughout his life. Then, in September 1994, he had a revelation. In 1872, German mathematical concept of "continuous function," meaning that changes in the output. Unlike traditional Euclidean geometry, where all 2-D figures and objects belong in the same plane, projective geometry is concerned with how the apparent shape of an object is altered by the perspective from which that object is viewed. Their many uses include solving several equations simultaneously, describing vectors, calculating transformations in the shape and position of geometrical figures. that is formed on a flat 2-D surface by repeated copies of one or more regular geometrical shapes that cover the surface without any gaps in between them cancel out, each pushing on the other with an equal force, but in opposing directions. The link he made between the two areas is now called Galois theory. It was the need for this that led Turing to create his "virtual machine." In 1937, Turing published his first paper as a fellow of King's College, Cambridge, "On Computable Numbers, with an Application to the Entscheidungsproblem." It showed that there is no solution to Hilbert's decision problem. some algorithms are not computable, but there is no universal mechanism for identifying them before trying them. 4th century BCE Euclid sets out the foundations of geometry in Elements. Of course, numbers are still very useful tools to understand even the most esoteric areas of mathematics, but the point is that they are not the most interesting aspect of it. It is impossible to be a mathematician without being a poet of the soul. In the early 1800s, French physicist and mathematician Pierre-Simon Laplace developed electromagnetic theory with the help of calculus. After graduating with a degree in mathematician without being a poet of the soul. In the early 1800s, French physicist and mathematician Pierre-Simon Laplace developed electromagnetic theory with the help of calculus. statistics. Graph (1) A chart on which data is plotted using, for instance, lines, points, curves, or bars. The Babylonians worked with a more complex sexagesimal (base-60) number system that was probably inherited from the earlier Sumerians and is still used across the world today for measuring time, degrees in a circle (360° = 6 × 60), and geographic coordinates. Being able to model these diverse phenomena using fractal geometry enables us to better understand their behavior and evolution, even if that behavior is not entirely deterministic. Our lives are Möbius strips, misery and wonder simultaneously. Kurzweil argued that this rate of growth in technology would eventually lead to the singularity, which is defined in physics as a point at which a function takes 174 an infinite value. See also: Calculus • The logic of mathematical equivalent of a Nobel Prize) in 1966 for solving the first of David Hilbert's list of 23 unresolved mathematical problems—that there is no set whose number of elements is between that of the integers and that of the real numbers. Velocity is a vector as it has exactly those values (a quantity of speed and a direction of motion), while other vectors, such as Oresme's heat and expansion, are visualized in this way to make it easier to add and subtract different sets of values or to manipulate them in another way. Similarly, in Renaissance times, when insurance was calculated for ships, premiums were based on an intuitive estimate of risk. It seemed a small insight, but it made all the difference. Cartesian coordinates remain an ubiquitous tool, however, able to plot anything from simple survey data to the movements of atoms. 1820 Mathematicians such as Carl Friedrich Gauss, János Bolyai, and Nicolai Ivanovich Lobachevsky begin to move toward hyperbolic non-Euclidean geometry. Watts and mathematician Steven Strogatz random graph model to measure connectedness. The graph shows the path of the gum as the wheel rotates, creating a cycloid shape, which, as de Roberval discovered, has an area three times that of the wheel. 598-668). In Practica Geometriae (1220), he solved problems associated with surveying, such as finding the height A page from the original of a tall object using similar triangles (triangles that manuscript of Liber Abaci shows the Fibonacci have identical angles, but different sizes). 262 The development of calculus, the branch of mathematics that deals with how things change, was one of the most significant advances in the history of mathematics. Alamy Stock Photo: Science History Images (bl, cra). Kepler later acknowledged he was wrong, but his calculations led him to discover that planets have elliptical orbits. See also: Pythagoras • Syllogistic logic • Calculus • Transfinite numbers • The infinite monkey theorem 66 IN CONTEXT KEY FIGURE Plato (c. There can be very little... science and technology that is not dependent on complex numbers. His work was also criticized by the clergy, but Cantor, who was deeply religious, saw his research as a glorification of God. In 1897 he taught the young Albert Einstein in Zurich. 1859 Bernhard Riemann states a hypothesis about the distribution of prime numbers. We've tended to forget that no computer will ever ask a new question. He also developed an interest in mathematics, studying the work of ancient and contemporary mathematicians. Hipparchus compiled what was effectively the first true table of trigonometric values. Despite being an academic and a foreigner, Lagrange survived the French Revolution and Reign of Terror, and died in Paris in 1813. For example, a right-angled triangle with two shorter sides of lengths 3in and also wrote books that made math more accessible to young students, including Philosophy and Fun of Algebra (1909). Getty Images: AFP Contributor (br). 545 BCE Thales lived in Miletus, an ancient Greek city in what is now Turkey. See also: Topology EVELYN NELSON 1943-87 The Krieger-Nelson Prize, awarded by the Canadian Mathematical Society for outstanding research by a female mathematician, is named in honor of Evelyn 555 Nelson and fellow Canadian Cecilia Krieger. Henderson 366 See also: Euclid's Elements • Projective geometry • Topology • 23 problems for the 20th century • Minkowski space 367 IN CONTEXT KEY FIGURE Évariste Galois (1811-32) FIELDS Algebra, number theory BEFORE 1799 Italian mathematician Paolo Ruffini considers the sets of permutations of roots as an abstract structure. 434 Francis Galton invented the guincunx (sometimes called the Galton board) to model the bell curve.

primes (Mn). Observing this method firsthand may have encouraged Pythagoras to study and prove the underlying mathematical theorem. Fermat's work, for the mathematics of probability, but its relevance does not stop there. More emails followed, before the truth emerged—Katz had found a flaw at the heart of time. He is thought to have visited Phoenicia Persia, Babylon, and Egypt, and may also have reached India. Wiles recruited the help of these colleagues for the final step in compiling his proof. There are thousands of ways in which to arrange the numbers 1 to 9 in a three-bythree grid. This notion would be formalized in the late 1600s by Isaac Newton and Isaac Barrow in England, Gottfried Leibniz in Germany, and Scottish mathematician James Gregory. In geometry, it is considered a classic example of a "nonorientable" surface. Undeterred, Achilles runs another meter; however, in the same time, the tortoise runs one-hundredth of a meter, so it is still in the lead. Geographic coordinates plot points on the globe as angles from preset great circles—the Equator and the Greenwich Meridian. Armed with a binary code arranged in sets of eight digits—and later even longer strings—software could be produced for any conceivable application. Alfred North Whitehead British mathematician Paving the way for others In his description of these four geometric objects, Apollonius used no algebraic formulae and no numbers. De Moivre had realized that binomial outcomes cluster around their mean—on a graph, they plot an uneven curve that gets closer to the shape of a bell (normal distribution) the more data is collected. However, the full machine was hugely expensive to develop, and tested the technological capability of the day to its limits. Assuming that the chance of a heads or tails result is equal, the law dictates that after many tosses, half (or very near it) will have landed on heads, and half on tails. This phenomenon is known as normal distribution. The three rotor wheels sit behind the lampboard, and the plugboard is at the front. I is a subaltern of A and O is a subaltern of E. Because meridians get closer toward the poles, rhumb lines bend around into a spiral. 300 BCE Euclid's Elements describes geometry in terms of logical deduction from axioms. He further explored this topic in his Book of Birds, in which he posed a miscellany of bird-related algebra problems, including: "How many ways can one buy 100 birds in the market with 100 dirhams?" Algebra is but written geometry and geometry is but figured algebra. An example of a polynomial equation is  $x^2 + x + 41 = 0$ . Binomial distribution b(n, p) can take values from 0 to n, centered on a mean of np. The British encryption device, the Typex, was a modified version of Enigma that could encode a letter as itself. The RSA algorithm is special because it ensures that any third party monitoring the connection will be completely unable to figure out any private details. For example, the mean of the four numbers 1, 4, 6, and 13 is 1 + 4 + 6 + 13 = 24 divided by 4 = 6. 115 Temperature readings on the Celsius scale display negative numbers to show when something such as an ice crystal is colder than 0°C—the point at which water freezes. Pythagoras 53 In The School of Athens, painted by scholars eager to learn from him. Graph theory A branch of mathematics that studies how graphs made up of points and lines are connected. The identity element in ordinary multiplication, for example, is 1, as  $1 \times x = x$ , and in the addition of real numbers, it is 0, as 0 + x = x. By shortening the labors, [Napier] doubled the life of the astronomer. Sumerian clay tablets from as early as the 6th millennium BCE include symbols denoting different quantities. The word "graph" is most often used to describe a Cartesian system of coordinates with points plotted using x and y axes. The models were a success, breaking the stereotype of mathematics that this seemingly unremarkable number should produce such aesthetically pleasing proportions in art, architecture, and nature. The results were published in 1624 with the logarithms calculated to 14 decimal places. The quadratic equation  $x^2 + 4x - 12 = 0$ , for example, has two roots, x = 2 and x = -6, both of which produce the answer 0. Tetrahedra, which have four faces and four vertices, are said to be self-dual. 850–930 CE). whose Book of Algebra was designed to be an academic treatise for other mathematicians, rather than for educated people who had a more amateur interest. Only in the 1600s did European mathematicians begin to fully accept negative numbers. These chunks of bits became known as "bytes," spelled this way to avoid confusion with "bits." In the early decades of computing, bytes generally contained 4 or 6 bits, but the 1970s saw the 471 rise of Intel's 8-bit microprocessors, and byte became the unit for 8 bits. The problem was solved in 1572 when another Italian, Rafael Bombelli, set out the rules for an extended number system that included numbers. The first states that integration and differentiation are opposites—for any continuous function (one that can be defined for all values), there exists an "anti-derivative" (or "integral"), whose derivative (a measure of the rate of change) is the function itself. AFTER 2010 When Perelman rejects the Clay Millennium Prize, the £1 million award is used to set up the Poincaré Chair for gifted young mathematicians. He had hoped for a university teaching position, but he still faced some discrimination as a Frenchman in England. The physically impossible event is therefore the one that has infinitely small probability, and only this remark gives substance... to the theory of mathematical probability. Leonhard Euler first used i to denote the imaginary unit () in his attempts to prove the fundamental theorem of algebra (FTA). A similar system, using celestial coordinates, describes the location of stars in an imaginary sphere centered on Earth and missiles into the atmosphere today. 7th century In India, Brahmagupta establishes zero as a number in its own right and not just as a placeholder. Pascal's triangle built from a very simple arrangement of numbers in everwidening rows. 183 Leonardo da Vinci supposedly used golden rectangles in his composition of The Last Supper (1494-98). In the early days of the internet, the AND, OR, and NOT commands were commonly used to filter results to find the specific thing being search using more natural language. In 1948, Claude Shannon, an American mathematician and electronics engineer, published a paper called A Mathematical Theory of Communication. The most noticeable thing about prime numbers is that the larger they are, the more widely spread out they get. He wanted to resolve the navigational problem of determining a ship's longitude. Two years later, she was appointed chair of the mathematics department at North Carolina Central University. 1674 Robert Hooke writes An attempt to prove the motion of the Earth and hypothesizes what will become Newton's first law. A young German, Carl Friedrich Gauss, published his fundamental 280 theorem of algebra, marking the beginning of a spectacular career and a new period in the history of mathematics. These can be simplified to the quadratic equation (3/4 y)2 + y2 = 100 to find the length of a side on each square. Charles Babbage Delayed legacy Lovelace's plans to develop Babbage himself was tired, ill, and disillusioned by the lack of support for his Difference Engine. See also: Quadratic equations • Diophantine equations • Cubic equations 539 NASIR AL-DIN AL-TUSI 1201-74 Born in Tus, the Persian mathematician Joseph Liouville proves that a number can be transcendental—have an infinite number of digits arranged with no repeating pattern and without an algebraic root. Cultures throughout history had used many different bases for expressing parts of a whole. The golden spiral was first described by French philosopher, mathematician, and polymath René Descartes in 1638 and was studied by Swiss mathematician. increasingly complex and detailed abstract equations, without using geometry. A logarithmic table is a small table by the use of which we can obtain knowledge of all geometrical dimensions and motions in space. In the 2nd century BCE, the astronomer and mathematician Hipparchus, generally regarded as the founder of trigonometry, was particularly interested in triangles inscribed within circles and spheres, and the relationship between angles and lengths of chords (straight lines drawn between two points on a circle—or on any curve). Volume The amount of space inside a 3-D object. 1700 BCE-c. Aristotle approached logic as a branch of philosophy, but in the 1800s, scholars began to study logic as a mathematical discipline. While their work sparked a brief fad in the 1960s, it proved too radical for teachers and pupils alike. An extension of complex numbers, quaternions are used to model, control, and describe motion in three dimensions, which is essential in, for example, creating the graphics of a video game, planning a space probe's trajectory, and calculating the direction in which a smartphone is pointing. Science Photo Library: Oona Stern (clb). It guarantees that, over the long term, the outcomes of future events can be predicted with reasonable accuracy. Recurring pattern Benford's law is evident in many situations, from the lengths of rivers to share prices and mortality rates. The Egyptians used a method called "false position" to determine the solution. In 343 BCE, Philip II recalled him to Macedonia to head the school at the court; one of his students was Philip's son, later known as Alexander the Great. The lower row of this table is a geometric sequence (progressing powers of 2), while the top row is an arithmetic sequence that reveals the exponents (powers) by which 2 is raised to arrive at the numbers in the lower row. Descartes originally published the book in French so it could be read by less well-educated people. A gambler might assume that toss number 11 is likely to be a tail because the number of heads and tails must balance out, but the probability of heads or tails is the same in every toss, and the outcome of one toss occurs independently of any other. Although taken prisoner by invading Mongols in 1255, al-Tusi was appointed a scientific advisor by his captors and later established an astronomical observatory in the Mongol capital Maragheh, now in Iran. In 1687, he became a professor of mathematics at the University of Basel, a position he held for the rest of his life. The Mayans' number system was in use in Central America until the Spanish conquests in the 16th century. Key work 5th century BCE Mystic Discourse See also: Positional numbers • Quadratic equations • Pythagoras • Imaginary and complex numbers • Euler's number 62 IN CONTEXT KEY FIGURE Zeno of Elea (c. de Jong Hanedoes, Amsterdam (bl). Unlike the base-10 system with its 10 symbols, binary has just two: 1 and 0. 2014 Iranian mathematician Maryam Mirzakhani is the first woman to win the Fields Medal. 365 Crochet models of hyperbolic surfaces created by Daina Taimina are more tactile than paper models. The law of large numbers is one of the foundations of probability theory and statistics. He died in 1865, following a severe attack of gout. PierreSimon Laplace, for example, applied the theories of calculus to celestial mechanics. His painstaking work advanced early algebra, while later mathematicians have continued to refine its expression and scope. So, for example, if n were 10 and Ma were 12, that would give the answer 2. Luca Pacioli, an early Renaissance mathematics of the golden ratio that was so important in Classical art, and the innovative use of perspective in painting inspired Girard Desargues to explore the mathematics of the golden ratio that was so important in Classical art, and the innovative use of perspective in painting inspired Girard Desargues to explore the mathematics of the golden ratio that was so important in Classical art, and the innovative use of perspective in painting inspired Girard Desargues to explore the mathematics of the golden ratio that was so important in Classical art, and the innovative use of perspective in painting inspired Girard Desargues to explore the mathematics of the golden ratio that was so important in Classical art, and the innovative use of perspective in painting inspired Girard Desargues to explore the mathematics of the golden ratio that was so important in Classical art, and the innovative use of perspective in painting inspired Girard Desargues to explore the mathematics of the golden ratio that was so important in Classical art, and the innovative use of perspective in painting inspired Girard Desargues to explore the mathematics of the golden ratio that was so important in Classical art, and the innovative use of perspective in painting inspired Girard Desargues to explore the mathematics of the golden ratio that was so important in Classical art, and the innovative use of perspective in painting inspired Girard Desargues to explore the mathematics of the golden ratio that was so important in Classical art, and the innovative use of perspective in painting inspired Girard Desargues to explore the mathematics of the golden ratio that was so important in the innovative use of perspective in painting inspired Girard Desargues to explore the mathematics of the golden ratio the innovative use of the golden ratio the golden ratio the golden ratio the golden ratio the go behind it and develop the field of projective geometry. Finding it hard to get full recognition locally, he took the bold step of sending some of his results to G. Fermat also helped Dutch mathematician Frans van Schooten to understand Descartes' ideas. they were close to its value. 50 The graphic above demonstrates why the Pythagorean equation (a<sup>2</sup> + b<sup>2</sup> = c<sup>2</sup>) works. See also: Positional numbers • The complex plane 138 IN CONTEXT KEY FIGURE Al-Khwarizmi (c. These were later dubbed "natural logarithms" because they can be used to mathematically describe many processes in nature, but with algebraic notation still in its infancy, Napier saw logarithms only as an aid to calculation involving the ratio of distances covered by moving points. Longer messages were vulnerable to another decryption technique called frequency analysis. providing the ability for [people] to communicate and interact with each other in a totally anonymous manner. Mathematicians started to develop non-Euclidean geometric frameworks, while some considered time as a potential dimension. 1990 A 16-bit binary code is used to code pixels on a computer screen, allowing it to display more than 65,000 colors. Getty Images: Xavier Laine / Getty Images Sport (bc). Fuzzy control systems can work effectively with uncertainties in the everyday world, and are therefore used in artificial intelligence (AI) systems. This "antidifferentiation" entailed working out areas under the curve. See also: Non-Euclidean geometries • Topology JOHN NASH 1928-2015 American mathematician John Nash is best known for establishing the mathematical principles of game theory (see John von Neumann). The book divided reality into the two opposing poles of yin and yang—one represented as a broken line. Many notable mathematicians have puzzled over how primes are distributed. The second law explains what is happening when objects are in motion. Every finite simple group either belongs to one of the 18 families or is one of the 18 families or is one of the 26 sporadic groups. 72 Seeking a rigorous proof Implicit in his discussion of valid syllogistic logic is the process of deduction, working from a general rule in the major premise, such as "All men are mortal," and a particular case in the minor premise, such as "Aristotle is a man," to reach a conclusion that necessarily follows—in this case, "Aristotle is mortal." This form of deductive reasoning is the foundation of mathematical proofs. Cardano published a solution to cubic and quartic equations, acknowledged the existence of imaginary numbers (based on the square root of -1), and is alleged to have forecast the exact date of his own death. As it has no intersections, the shape can only truly exist in four-dimensional space. To find the volume of a barrel or any other 3-D shape, Kepler imagined it as a stack of thin layers. Ciphertext without any of the letters in those words was a potential ciphertext of them. In order to describe these differing infinities, he introduced "transfinite" numbers. AL-KHWARIZMI Born in c. It is 517 made up of two numbers, n and a. 7th century CE Brahmagupta solves the quadratic equation. Robert Recorde ROBERT RECORDE Born in Tenby, Wales, around 1510, Recorde grew up to study medicine first at Oxford University, then at Cambridge, where he qualified as a physician in 1545. They are also important in computer science and cryptography. Logarithm of a number is the power to which another number. This must be exchanged by the original number. physical means—written in a military code book or whispered in the ear of a spy at a secluded rendezvous. See also: The fundamental theorem NIELS ABEL 1802-29 Abel was a final blow. In ancient Rome, fractions were based on a system of twelfths, and written out in words: 1/12 was semis, and 1/24 was semiuncia, but this cumbersome system made it difficult for people to do any calculations. Alamy Stock Photo: Classic Image (bl). His list of the 23 unsolved problems he considered crucial was influential in identifying the fields of mathematics that could be fruitfully explored by mathematicians. Langlands' conjectures are significant in that they allowed problems from number theory to be reframed in the language of harmonic analysis. Multiplying a negative imaginary unit, for example, equals a positive imaginary unit by a negative imaginary unit equals a negative integer. These groundbreaking ideas formed the foundations of complex analysis. 521 This Cayley graph shows all 60 elements (different orientations) of the group A5 (the group A5 relate to each other. Bernoulli came close to working this out in his calculations, when he identified e as the limit of (1 + 1/)n as n approaches infinity  $(n \rightarrow \infty)$ . Based on a five-digit vigesimal, or base-20, counting system, it used corn kernels threaded on strings to represent numbers. It indicates the number of degrees of separation an actor has from American actor Kevin Bacon. Nothing else can be taken as obvious, and intuition is regarded as potentially suspect. A code changes an entire word or phrase in the plaintext before it is encrypted. 441 The common perception that mathematics is logical, with fixed rules, evolved over millennia, dating back to ancient Greece with the works of Plato, Aristotle, and Euclid. 564 Infinitesimal calculus Another term for calculus, generally used in the past when calculus was viewed as involving the adding up of infinitesimals (infinitesimals (infinitesimals (infinitesimals (infinitesimals)). 1957 In the US, the B. The equation's simplicity has led mathematicians to describe it as "elegant," a description reserved for proofs that are profound yet also unusually succinct. With no friction or drag in space, the craft keeps going due to Newton's first law, and exits the Solar System in 2012. Getty Images: Bettmann (tl). Starting with a single pair of rabbits, he asked his readers to work out how many pairs there would be in each successive month. Antoine Gombaud, the Chevalier de Méré, wanted to know how to divide stakes fairly if a game of chance was suddenly broken off. 36 BCE A shell-shaped zero is recorded on a Mayan stela (stone slab) in Central America. This was driven not just by their 105 monumental building program, but also by the annual flooding of the Nile River, which required them to mark out the areas of fields each time the floods subsided. Although five colors can seem to be necessary, there is always a way of recoloring the map using only four colors. The next prime, 3, has only two factors, so all the other multiples of 3 cannot be primes. In 1814, Pierre-Simon Laplace, a French mathematician who combined mathematics and science with philosophy and politics, presented a thought experiment now known as Laplace's demon. However, this begs the question: if the barber shaves himself, to which is why it is known in China as Yang Hui's triangle. Galton found that the smallest pea seeds had larger offspring and the largest seeds produced smaller offspring. It failed to offer a means of measuring or calculating continuous change. For by the ultimate velocity is meant that, with which the body is moved, neither before it arrives at its last place, when the motion ceases nor after but at the very instant when it arrives. René Descartes 228 A geometric shape such as the curve of a roller-coaster can be mapped on to a graph and described in relation to the x and y axes. He found that it was impossible to express it as a ratio between two whole numbers. 61 HIPPASUS Details of Hippasus's early life are sketchy, but it is thought that he was born in Metapontum, in Magna Graecia (now southern Italy), around 500 BCE. He produced theorems on conic sections, simplified the calculation of the orbits of comets, and created several new map projections. See also: Calculus • Newton's laws of motion GIUSEPPE PEANO 1858-1932 Brought up on a farm in the northern Italian region of Piedmont, Peano studied at the University of Turin, where he gained his doctorate in math in 1880. And philosophically, we have used mathematics as an exercise in pure thought to explore patterns and logic. 347 To use the Poisson distribution to calculate n, conditions must be met: orders must occur randomly, singly, and uniformly—on average, the same number of potatoes are ordered each day. In the 7th century CE, another great Indian mathematician and astronomer, Brahmagupta, made his own contributions to geometry and trigonometry, including what is now known as Brahmagupta's formula. Mathematician and poet Omar Khayyam, for example, was interested in solving problems using the relatively new discipline of algebra, but employed both geometrical and algebraic methods. This term was coined by Claude Shannon, a leading figure in information theory—the field of mathematician Karl Weierstrass develops a general theory of elliptic functions. showing that they can be applied to problems in both algebra and geometry. A particular branch of geometry, trigonometry (the study of the properties of triangles), proved to be especially useful in these pursuits. AFTER 1903 German mathematician Edmund Landau simplifies Hadamard's proof of the prime number theorem. In 1812, French mathematicians Augustin-Louis Cauchy and Jacques Binet proved that when two square matrices of the same as the product of their individual determinants: detAB = (detA) = (det Greek philosophers and mathematicians. 250 BCE In On Conoids and Spheroids, Archimedes deals with the solids created by the revolution of conic sections about their axes. The Möbius strip is the simplest nonorientable, two-dimensional surface that can be created in three-dimensional space. If you substitute 0 for a and 1 for b, this becomes a binary sequence. For example, the set of odd integers are an ideal in the ring of integers. Even so, the group made important contributions across a wide range of mathematical topics, particularly in set theory and algebraic geometry. However, many sets of numbers—a list of populations for US villages, towns, and cities, for example—show a distinctly different pattern. 1965 American co-founder of Intel Gordon Moore observes how the number of transistors on a microchip doubles roughly every 18 months. Hannah Fry British mathematician Laying down the theory In other letters, Pascal and Fermat discuss other cases of interrupted games, such as when the play alternates between two players until one is successful. When these elements include shapes, groups can be thought of as encoding symmetry. All the members of a group is simple if it cannot be broken down into smaller groups. 102 It is thought that all eight volumes of Conics were compiled while Apollonius was in Egypt. If the weights of the hammers were in exact and particular proportions, their resulting notes were harmonic. Kevin Bacon See also: Logarithms • Graph theory • Topology • The birth of modern statistics • The Turing machine • Social mathematics • Cryptography 480 IN CONTEXT KEY FIGURE Edward Lorenz (1917-2008) FIELD Probability BEFORE 1814 Pierre-Simon Laplace ponders the consequences of a deterministic universe where knowing all present conditions were geometry, and the solutions were geometry. It was tied to the rules of geometry was built in the capital Isfahan, and Khayyam assembled a team of eight astronomers to assist him with the work. Sanders, Robin Thomas, and Paul Seymour provide a simpler proof of the four-color theorem. Fuzzy truth values also require fuzzy logical operator, which outputs the minimum of the two inputs. In the dramatic climax of his Cloches à travers les feuilles (Bells Through the Leaves), the ratio of total bars in the piece to climax bars is approximately 1.618. From the ancient Egyptians, Babylonians, and Greeks to the medieval scholars of Persia, India, and China and the city-state rulers of Renaissance Europe, those looking to build, trade, fight wars, and manipulate money realized that measuring and calculating were crucial. Some of his pamphlets were later expanded into published papers. Turing's thought experiment had, therefore, created a paradox which could be used as a form of mathematical proof. The theorem named after him states that where the longest side of a triangle contained within a circle is the diameter of the circle, that triangle has to be a right-angled triangle. See also deduction. Within a few years, it was solved —by a little-known Russian mathematician, Grigori Perelman. See also deduction within a circle is the diameter of the circle, that triangle has to be a right-angled triangle. numbers • The complex plane 162 IN CONTEXT KEY FIGURE Leonardo of Pisa, also known as Fibonacci (1170-c. The result of the calculation — 7 plus 7/7 (or 1)—is 8, not 19, so a scaling factor is needed. Dreamstime.com: Mohamed Osama (tr). Langlands himself has helped to develop the means for working on the Program, including generalizing functoriality—a way of comparing the structures of different groups. Here was the chance he had been waiting for—if he could prove the seemingly impossible Taniyama-Shimura conjecture, then he would also prove Fermat's last theorem. It was no longer merely a branch of astronomy, but a key component of geometry. In India, mathematicians explored some of them from around 800 BCE, as the ancient Shulba Sutras texts reveal. In June 1993, at a conference at the University of Cambridge, Wiles delivered his results. A seemingly impossible problem about integers had been solved by marrying number theory to algebraic geometry, using new and existing techniques. In a letter to Euler, he proposed the conjecture for which he is best known, that every even integer greater than 2 can be expressed as the sum of two primes, which remains unproven to this day. 284 Newton made significant discoveries in the fields of gravitation, motion, and optics, where he developed a rivalry with eminent English scientist Robert Hooke. The hypothesis = i, which does not appear on the real number x axis at all, but on the imaginary y axis. The first person to formulate the four-color theorem, for instance, states that the intersections of lines connecting pairs of six points on a conic all lie on a straight line. SIMON STEVIN Born in 1548 in Bruges, now in Belgium, Simon Stevin worked as a bookkeeper, cashier, and clerk before entering the University of Leiden in 1583. Complex plane on which complex number as its only factor is 1. Key works 628 Brahmasphutasiddhanta (The Correctly Established Doctrine of Brahma) 665 Khandakhadyaka (Morsel of Food) The Nadi Yali yantra is part of an 18th-century observatory in Ujjain, India. As the corner moves around the circle, the short sides of the triangle vary in length, but the hypoteneuse does not because it is always the radius of the circle. Rachel Fewster Statistical ecologist, New Zealand See also: The Fibonacci sequence • Logarithms • Probability • Normal distribution 474 IN CONTEXT KEY FIGURE Claude Shannon (1916-2001) FIELD Computer science BEFORE 1679 Gottfried Leibniz develops the ancient idea of binary numbering. Pearson then found the variance —the average of the squared differences from the mean. For example, 6,765 ÷ 4,181 = 1.61803. Place value system The standard system for writing numbers, where the value of a digit depends on its place in a larger number. 79 However, it seems that Euclid did not explicitly show that the lines AB, BC, and CA lie in the same plane. However, as the proof reached its final stages, in the seventh year of working on it, Wiles realized he needed help. In Vietnam, for example, a price of two hundred thousand Vietnamese dong is often written as 200.000. By finding new links between mathematical fields that seem profoundly different, the Program has revealed some of the structures at the heart of mathematics. Since Babylonian times, scholars have sought 323 solutions to equations, periodically encountering seemingly insoluble examples. JOSEPH-LOUIS LAGRANGE Born Giuseppe Lodovico Lagrangia in Turin in 1736, Lagrange embraced his family's French heritage and went by the French version of his name. 40 In the workings for problem 24, one-seventh is easiest to find for the number 7, so 7 is used first as a "false" value for the variable. In the case of a damped pendulum (one that is losing energy because of friction), the oscillatory motion will diminish until the imaginary body reaches the fixed point—when it stops moving. However, Harriot's ideas remained unpublished until long after his death in 1621. Wiles was under immense pressure. As well as his mathematical contributions, Pascal played an important role in many scientific developments of the idea of air pressure: the scientific unit of pressure is called the Pascal. 2000 BCE). AFTER 1930 Dutch mathematician Bartel Leendert Van der Waerden writes the first step is to make the data fuzzy—every egg is regarded as both large and small, belonging to both sets to different degrees. The Rhind papyrus, thought to have been intended for trainee scribes, describes how to calculate the volumes of cylinders and pyramids and also the area of a circle. Common groupings of letters, such as "th" and "ion" could also provide a way into revealing the cipher. 3 ÷ 12 = 0.25, so the investment after a year would be \$100 × 1.002512 = \$103.04. Bourbaki helped make the link the other way, turning algebra into geometry to create 463 algebraic geometry, which is perhaps their lasting legacy. Each disk is represented by a binary digit, or bit. However, his pursuit of this goal showed how mathematics had progressed since the time of the ancient Greeks. Khayyam saw that such tools were not enough to solve all cubic equations, and set out his use of conic sections and other methods in his treatise on algebra. Bob uses n and a to encrypt a plaintext message (M), which is a string of numbers (or letters ciphered into numbers). 1621 In England, William Oughtred invents the slide rule, which simplifies the use of logarithms. (2) In a circle, the area between a chord and the outside edge (circumference). In 1694, he had offered the Swiss mathematician Johann Bernoulli 300 livres a year for information on his latest discoveries and an agreement that he would not share them with other mathematicians. MATT PARKER, FOREWORD Originally a math teacher from Australia, Matt Parker is a now a stand-up comedian, mathematics communicator, and a prominent math YouTuber on the Numberphile and Stand-up Maths channels, where his videos have had more than 100 million views. Alamy Stock Photo: Beth Dixson (bl). We define the art of conjecture... as the art of conjecture... as the art of evaluating... the probabilities of things, so that in our judgments and actions we can always base ourselves on what has been found to be the best. Objects in each set are then arranged in the circles so that objects that belong in more than one set are placed where the circles overlap. In his work Symbolic Logic (1881), Venn developed Boole's theories employing what became known as Venn diagrams. Hadamard died a year later. Techniques such as machine learning, in which AIs program themselves by a process of trial and error, and expert systems, in which the AI draws upon a database of knowledge provided by human programmers, have greatly extended the abilities of AI. In the early 1900s, French mathematician Emile Borel explored improbability— when events had very small chance of ever occurring. Among them was Christian Goldbach, a German contemporary of Euler's. Napier returned to Scotland in 1571 and devoted much time to his estates, where he devised new methods of agriculture to improve his land and livestock. If the Universe is "simply connected," then you would be able to gather in the whole string, like a loop following the smooth contours of a sphere; if you had passed through holes or gaps, then the string around its girth, the string would get caught. Mathematicians now utilized the plus or minus symbol, ±, in solving quadratic equations. In the 1st to 4th centuries CE, the use of place value. 300 BCE Euclid's 13-volume Elements sets out the propositions that form the basis of plane geometry. He is best known for his work in astronomy, especially his laws of planetary motion and his astronomical tables. By the 1500s, most mathematicians accepted negative solutions and were comfortable with surds (irrational roots - those that cannot be expressed exactly as a decimal). He was one of the small group of scholars whose meetings led to the establishment of the Royal Society of London in 1662. By showing that information could be expressed as binary digits, he launched the theory of information—with implications stretching into every field of science, and into every field of science, and into every field an aeolipile, a winc wheel that could operate an organ, and a vending machine that dispensed "holy" water. Finally, in the 1800s, the first two months, he said, there would only be the original pair: by the end of three months, there would be a total of two pairs; and at the end of four months, there would be three pairs, as only the original pair was old enough to breed. Identity element In a set of numbers or other terms unchanged after the operation has been carried out. The potential of binary numbers was finally realized by German mathematician and philosopher Gottfried Leibniz. The examples below show how a \$10 principal investment accrues interest if the yearly interest rate is 100 percent, versus compound interest paid at shorter intervals. The term for the x-axis is abscissa, while the y-axis is the ordinate. After lecturing in Lille for four years, he returned to the École, where he dazzled fellow mathematicians with a series of brilliant papers. A solution 404 In 2018, British mathematicians with a series of brilliant papers. 24 is known as "false position." This technique— used well into the Middle Ages—is based on trial and improvement, choosing the simplest, or "false," value for a variable and adjusting the value using a scaling factor (the required quantity divided by the result). In 1991, Japanese mathematician Mitsuhiro Shishikura proved that the boundary of the Mandelbrot set has a Hausdorff dimension of 2. Having learned all that school could teach him by the age of 12, he had to wait until he was 16 to be allowed to attend Berlin University, and spent the intervening years teaching himself mathematics. Katz could find no errors, so Wiles decided to go public. 1982 Israeli engineer Dan Shechtman discovers quasi-crystals whose structure is similar to Penrose tilings. It is this notion of proof and rigor that is the Greeks' greatest contribution to mathematical work of all time. Truth tables show the possible outcomes of various inputs into the gate. Number theory branch of mathematics that studies the properties of numbers), their patterns, and their relationships. Getty Images: Print Collector / Hulton Archive (cla). See also: Calculus • Binary numbers • Boolean algebra 476 IN CONTEXT KEY FIGURE Michael Gurevitch (1930-2008) FIELD Number theory BEFORE 1929 Hungarian writer Frigyes Karinthy coins the phrase "six degrees of separation." AFTER 1967 American sociologist Stanley Milgram designs a "small world experiment" to investigate people's degrees of separation. The FTA stated that anyone faced with solving an equation built from real numbers could be sure that they would find their solution within the realm of complex numbers. After graduating from Hampton University, Darden taught at Virginia State University before moving in 1967 to NASA's Langley Research Center. Limitations One drawback in Boole's system of algebra was that it contained no method of quantification: there was no simple way of expressing a statement such as "for all x," for example. 250 CE Arithmetica See also: The Rhind papyrus • Pythagoras • Hypatia • The equals sign and other symbology • 23 problems for the 20th century • The Turing machine • Proving Fermat's last theorem 123 IN CONTEXT KEY FIGURE Hypatia of Alexandria (c. An itinerant teacher of considerable mathematical ability, but with few financial resources, Tartaglia discovered a general method for solving cubic equations independently of del Ferro. Bhaskara I was the first person to write numbers in the Hindu-Arabic decimal system with a circle for zero. Power The number of times a quantity or number has been multiplied by itself. He is said to have died around 495 BCE. A pretty result which I had not noticed before. Later in life, al-Karaji moved to "mountain countries" (possibly the Elburz mountains near Karaj), where he spent his time working on practical projects for drilling wells and building aqueducts. If Ma were 2, it would also give an answer of 2, because 10 goes into 2 zero times with a remainder of 2. Alamy Stock Photo: Steve Taylor ARPS (bc). Diophantus is then said to have lived a further four years, dying at the age of 84. Some of these—the "trivial zeros"—are easy to find; they are all the negative even integers (-2, -4, -6, and so on). In the 1600s, the pace of mathematical discovery accelerated as never before, and several pioneering modern mathematicians emerged. Terms are the parts separated by operators; they can be a number or variable, or a combination of both. It already has real-world applications such as in programming codes in cell phones and smart cards. Polynomials During the 10th and early 11th centuries, a more abstract theory of algebra was developed, which was not reliant on geometry—an important factor in establishing its academic status. Key works 1853 Lectures on Quaternions 1866 Elements of Quaternions 1866 Elements of Quaternions 1866 Elements of Participation and complex numbers • Coordinates • Newton's laws of motion • The complex plane 378 IN CONTEXT KEY FIGURE Eugène Catalan (1814-94) FIELD Number theory BEFORE c. Studies to date, however, are inconclusive and often contradictory; there is little scientific basis for believing that the golden ratio makes a face more attractive. 513 Cipher wheels, such as this British example from 1802, sped up the decryption of Caesar ciphers. Negative numbers are evidence of inconsistency or absurdity. Then a and z are calculated from p and q using a formula which ensures that the modulo calculations work. Magic squares—arrangements of numbers where the sum in each row, column, and diagonal is always the same—are one of the oldest 9 areas of recreational mathematics 1992 The International Society for Bayesian Analysis (ISBA) is founded to promote the application and development of Bayes' theorem. An inspirational teacher, she was appointed head of the city's Platonist school in 400 CE. AFTER 1901 Bertrand Russell's barber paradox exposes the weakness of set theory's ability to define numbers. The corresponding sides of each triangle can be extended to meet at three points on a line (axis of perspectivity), and a line from each vertex to its corresponding vertex and beyond will meet at a point (the center of perspectivity). As with del Ferro, the condition was that the method should never be published. He defined single terms called monomials"x, x2, x3, and so on—and showed how they can be multiplied or divided. The circle passed through the origin (0,0) and its center was on the x axis at (2,0). In the 5th century CE, the Greek philosopher Proclus wrote in his history of mathematicians that Euclid taught at Alexandria during the reign of Ptolemy I Soter (323-285 BCE). In 1496, Pacioli moved to Milan to work as a payroll clerk. She graduated from Tehran's Sharif University of Technology, before earning a doctorate from Harvard University in 2004 and taking up a professorship at Princeton University of the "simpler" area (the theory of groups) in order to tackle the more difficult problem (solubility of equations). The principle was based on Socrates' dialectal method of questioning assumptions to expose inconsistencies and contradictions. For example, the set 1, 2, 3 can be arranged as 1, 3, 2, or 2, 1, 3, or 2, 1, 1, or 3, 1, 2, or 3, 2, 1. The figure of 8/9 is used as an approximation for the proportion of the area of a square that would be taken up by a circle if it were drawn within the square. Alamy Stock Photo: World History Archive (tc). AFTER 1822 German geometer Karl Wilhelm Feuerbach publishes a proof for the ninepoint circle, which passes through the midpoint of each side of a triangle of a triang Today, Sierpinski triangles are popular for use in knitting patterns and in origami, where a Sierpinski triangle is converted into three dimensions to create a Sierpinski tetrahedron. In 1816, she was the first woman to win a prize from the Academy of Sciences in Paris, for a paper on the elasticity of metal plates. Matrices are used to represent electrical circuits for solving problems about voltage and current. They were studied by the ancient Greeks as one of the conic sections. For example, he wrote out the equation (x/3 + 1)(x/4 + 1) = 20 as: "A quantity: I multiplied a third of it and a dirham being a single coin, used by al-Khwarizmi to signify a single unit. Group theory continues to develop in its own right and has many applications. In 1667, after 272 completing his university studies, Leibniz became an advisor on law and politics to the Elector of Mainz, a role that enabled him to travel and meet other European scholars. The "laws" of Boole's algebra allow statements that would not be permitted by other forms of algebra. There are also 26 individual groups, the largest of which is called the Monster, which has 196,883 dimensions and approximately 8 × 1053 elements. Key works 1903 The Principles of Mathematical Logic as Based on the Theory of Types 1910-13 Principia Mathematica (with Alfred North Whitehead) See also: The Platonic solids • Syllogistic logic • Euclid's Elements • The Goldbach conjecture • The Turing machine 444 IN CONTEXT KEY FIGURE Hermann Minkowski (1864-1909) FIELD Geometry BEFORE c. Such growth can be plotted on a graph and will appear as a curve. Napoleon later made him a baron, and then a count. Parallel rays come from a point at infinity (such as the Sun). Proposition 3 Given two unequal straight 81 line equal to the less. In 2008, Microsoft conducted research to show that everyone on Earth is separated from every other person by only 6.6 people on average. 600 CE) FIELD Number systems BEFORE c. 193 To avoid the tedious repetition of these words, is equal to, I will set, as I do often in work use, a pair of parallels. AFTER 1806 Jean-Robert Argand uses a coordinate plane to represent complex numbers. Slope The angle of a line to the horizontal, or an angle of a tangent to a curve to the horizontal. In the scientific revolution of the 16th and 17th centuries, the lack of a reliable calculating tool hampered progress in areas such as navigation and astronomy, where the potential for error was greater because of the lengthy calculations involved. 315 An algebraic equation is made up of variables and coefficients. Fibonacci explored the sequence in his 1202 book Liber Abaci (The Book of Calculation). Notating algebra 192 While the earliest algebraic techniques date back more than two millennia to the Babylonians, most calculations before the 16th century were recorded in words— sometimes abbreviated, but not in a uniform way. He points out that a player with a 7-5 lead in a game to 10 aces has the same chance of eventually winning as a player with a 17-15 lead in a game to 20. Like Kepler and Galileo, Newton was interested in studying moving bodies with motion on Earth. 1200 The magnetic compass is used by navigators in China, Europe, and the Arab world. His proof is not as formidable as Andrew Wiles's proof of Fermat's last theorem, but it is still highly technical. In its basic form, algebra is the study of the underlying rules of how those symbols are used in mathematics—in equations, for example. The sequence has important forecasting applications in nature, geometry, and business. I found at length some excellent brief rules. AFTER 1858 Arthur Cayley formally defines matrix algebra, and proves results for 2 × 2 and 3 × 3 matrices. After many of Pythagorean triples is a triangle with side lengths destroyed in a 6th-century BCE political purge, 3, 4, and 5. It is created by splitting the rectangle into squares and a smaller golden rectangle, then repeating the process in the smaller rectangle. More complex statements can also be assessed by drawing truth tables. See also induction. The factorial of an integer is the product of the integers below it: 2 (2 × 1), 3 (3 × 2 × 1), 4 (4 × 3 × 2 × 1), 5 (5 × 4 × 3 × 2 × 1) and so on, adding one more term in the product each time. As in earlier periods, the types of problems discussed were largely geometrical. The study of calculus is underpinned by the fundamental theorem of calculus, specifying the relationship between differentiation and integration, both of which rely on the concept of infinitesimals. The legacy of great mathematical philosophers such as Pythagoras, Plato, and Aristotle was consolidated by Euclid, whose principles of mathematics based on a combination for some 2,000 years. Euclid did not use it until Proposition 29, in which he stated that one condition for a straight line crossing two parallel lines was that the interior angles on the same side were equal to two right angles. A student of mathematics and astronomy, he broke with the tradition of using mythology as a way of explaining the world. In 1551, Recorde was given charge of the Dublin mint, which included silver mines in Germany. He moved to Bordeaux to lecture at the university, and there proved the prime number theorem. Catenaries are often found in nature and in technology. His design anticipated virtually all of the key components of the modern computer, including the central processing unit (CPU), memory storage, and integrated programs. The pair finally cracked the problem in 1977. See also: Quadratic equations Negative numbers • Algebra • Cubic equations • The algebraic resolution of equations • The algebraic resolution of equations • The complex numbers • The algebraic resolution of equations • The algebraic resolution of equations • The complex numbers • The algebraic resolution of equations • The algebraic resolution of equations • The algebraic resolution of equations • The complex numbers • The algebraic resolution of equations • The algebraic government's surveillance center, GCHQ. Octahedron A 3-D polyhedron made up of eight triangular faces. The zeroth row (the top of the triangle) is used for the binomial to the power of 0: (x + y)0 = 1. She developed a fundamental theorem of elementary game theory (see John von Neumann) in 1951, but is best known for her work on solving the tenth of David Hilbert's list of 23 mathematical problems, drawn up in 1900—whether there is an algorithm that could find a solution to any Diophantine equation (one that uses whole numbers and finite unknowns). For example, -1, 0, 19, 55, and so on. AFTER 1782 Leonhard Euler writes about Latin squares in his Recherches sur une nouvelle espèce de carrés magiques (Investigations on a new type of magic square). There are only two kinds of certain knowledge: awareness of our own existence and the truths of mathematics. Elsewhere in the world, if a comma is used for the decimal separator, a point is then used as the delimiter. He proposed the Dedekind cut, now a standard definition of real numbers, and defined concepts of set theory, such as similar sets and infinite sets. A lot of talent has been squandered over the centuries, and a lot of credit has not been appropriately given. Albert Einstein The Renaissance was a time of intellectual creativity, in which disciplines such as art, philosophy, religion, science, and mathematics were considered to be much 179 closer to each other than they are today. An image of any triangle is another triangle. Problems and might involve, for example, observing a bent coin landing on heads 13 times out of 20 and then trying to determine whether the probability of that coin landing on heads lies somewhere between 0.4 and 0.6. To show how to calculate inverse probabilities, Bayes considered two interdependent events—"event A" and "event B". This is now known as an abelian group. Positive and negative quantities were represented by rods of different colors and could be added together. Leonhard Euler used de Moivre's findings to derive the "most elegant equation in mathematics": ein + 1 = 0, also known as Euler's identity. By the 1850s, the study of heat and energy—thermodynamics—was ushering in a new model, the atomic world. The panic was caused by American banks loaning out many millions of dollars (a negative quantity) without the reserves (a positive quantity) to back this up. In 1628, he moved to Paris, where he joined Marin Mersenne's circle of intellectuals. New fields Gauss was also a pioneer of non-Euclidean geometries, which epitomized the revolutionary spirit of 19th-century mathematics. incorporated zero in his number line. The work was designed to be read by mathematicians already well versed in geometry. In medieval times, more abstract algorithmic approaches were established, and by the 1500s, mathematicians knew certain relations between the coefficients of a polynomial equation and its roots, and had devised formulas to solve cubic (highest power 3) and quartic equations (highest power 4). Key work 2004 Experiencing Geometry with David W. The machine. For example, if you combine any set of three elements with a multiplication operator, you can perform the operations in any order. The last term in the series, 263, is 2 multiplied by itself 63 times. SEEMULLER / De Agostini (br). In Central America, during the 1st millenium CE, the Mayans used a place value system, which included zero as a numeral, denoted by a shell shape. using arctan, including Leonhard Euler in the 1700s. However, d'Alembert did not prove his lemma satisfactorily; his proof was correct, but contained too many holes to satisfy his fellow mathematicians. Massive objects in the Universe bend passing light rays from distant sources in a phenomenon called gravitational lensing, creating multiple images seen through a telescope. Face A flat surface of any 3-D shape. The new religion had a high regard for philosophy and scientific enquiry, and the "House of Wisdom," a center of learning and research established in Baghdad, attracted scholars from all over the expanding Islamic Empire. It is simple... yet incredibly profound; it comprises the five most important mathematical constants. When de Roberval told Descartes of his success, the latter dismissed it as "so small a result." Descartes, in turn, discovered the tangent to a cycloid in 1638, and challenged de Roberval and Fermat to do the same. in the beehive, one of which is to mate with the queen and fertilize her eggs. Among their discoveries were polygonal numbers: these, when represented by dots, can form the shapes of regular polygons. In 1854, German mathematician Bernhard Riemann formulated criteria for which functions would be integrable or not, based on defining finite upper and lower limits for the function. The problem contains the following information: the area of a square of 100 cubits is equal to that of two smaller squares. In the late 1500s, Frenchman François Viète created more modern algebraic notation, using substitution and simplifying to reach his solutions. In China, too, c. AFTER 1965 In the US, James Cooley and John Tukey develop the Fast Fourier Transform (FFT), an algorithm that is able to speed up Fourier analysis. Other fractions become recurring decimals—meaning that they do not end. 478 Duncan Watts and Steven Strogatz showed that if you have a random network with N nodes, each of which has K links to other nodes, then the path length between two nodes is In N divided by In K (where In means the natural logarithm). The astronomical discoveries attributed to Thales include his forecast of the 585 BCE solar eclipse. For philosophers, one important principle was the formulation of cogent arguments that followed a logical progression of ideas. The problem for historians has been finding evidence for early civilizations using zero and recognizing it as such, which has been made more difficult by the fact that zero fell in and out of use over time. One of the patterns found in Pascal's triangle is the Fibonacci sequence, which lies on a shallow diagonal (see below). He simplified these by using fractions with a base system of tenth powers. If they stop at this point, each should take back his stake of 32 pistoles. The decibel scale takes the hearing threshold, defined as 0 dB, as its reference level. The chance of "success" is mr2 (the area of a circle) + 4. Many natural phenomena, such as clouds and rock formations, approximate to fractals. No ring theory from Emmy Noether, no computing from Alan Turing, and no six degrees of separation from Kevin Bacon. Mathematicians continued to work on the problem, and gradual progress was made. Yet there is no evidence that ancient architects were conscious of this ideal ratio. To make his proof, he invented a type of group theory where the order of the elements within a group is immaterial. Hipparchus's most notable contribution to astronomy was his work Sizes and Distances (now lost, but used by Ptolemy), on the orbits of the Sun and Moon, which enabled him to calculate the dates of the equinoxes and solstices. In seeking to establish the logical basis of mathematics, Bertrand Russell described a paradox that highlighted a contradiction in Georg Cantor's naive set theory, leading to a reappraisal of the subject. The formula for this particular line is y = 2x + 2 and that includes the points (0.2); (1.4); (2.6). He left behind several notebooks, which mathematicians are still studying today. Whereas the simple line, plane, and solid occupy one, two. and three dimensions respectively, these new shapes could be given non-wholenumber dimensions. A general AI that can direct its own learning in the same way as evolved intelligence (such as human intelligence) is the next goal of computer science. 1880-84 Henri Poincaré develops the concept of automorphic forms—tools that allow us to keep track of complicated groups. In 1974, he created tiles using kite and dart shapes. For example, many species of flower have a Fibonacci number of petals, and the scales of a pine cone, viewed from below, are arranged in 8 clockwise spirals and 13 counterclockwise spirals. Sir Thomas Browne English polymath An integrated philosophy In ancient Greece, mathematics and philosophy were considered complementary subjects and were studied together. Mandelbrot received many honors and prizes for his work, including France's Légion d'honneur in 1989. The work did not really need mathematics, but mathematics, but mathematics and prizes for his work, including France's Légion d'honneur in 1989. Trigonometry GASPARD MONGE 1746-1818 The son of a merchant, by the age of 17, Monge was teaching physics in Lyon, France. In the early 20th century, the Möbius strip shape was used in continuous-loop recording tapes to provide double the playback time. Sylvester taught briefly in the US, but faced similar difficulties there. In their system, the most noticeable difference from today's notation is the use of a repeated letter to indicate a power. In Egypt, Pythagoras may also have met Thales of Miletus, a keen geometrician, who calculated the heights of pyramids and applied deductive reasoning to geometry. When real (not imaginary) solutions to ax2 + bx + c = 0 exist, they will be the roots—the points where the parabola crosses the x axis. Rouse Ball, British mathematician and lawyer Just like a pocket map attributed to Peter Tait, British physicist and engineer The matrix is everywhere from the film The Matrix The music of the primes Marcus du Sautoy, British mathematician and author Some infinities are bigger than others John Green, American author Lost in that silent, measured space Paolo Giordano, Italian author MODERN MATHEMATICS Statistics is the grammar of science Karl Pearson, British mathematician and statistician Rather a dull number G. Cavalieri worked out a "method of indivisibles," which was a more rigorous method of determining the size of shapes. Proportion The relative size of something compared with something else. It covers the five regular convex solids—the tetrahedron, dodecahedron, and icosahedron, which are often called the Platonic solids—and is the first recorded example of a classification theorem (one that itemizes all possible figures given certain limitations). 1671 In Treatise on the Method of Series and Fluxions, Isaac Newton produces new analytical methods for solving problems such as the maxima and minima of functions. whether solutions exist at all. Second, the complete manipulation of algebraical symbols. The dimensions of a matrix are important, as operations such as addition and subtraction require the matrices involved to have the same dimensions. Penrose tiling consists of kites and darts, producing a nonperiodic tessellation. Cryptography is an ancient practice. In 1940, Austrian-American logician Kurt Gödel showed it could not be 430 proved that such an infinity exists, and, in 1963, American mathematician Paul Cohen showed it could not be proved that such an infinity does not exist. 495-430 BCE) FIELD Logic BEFORE Early 5th century BCE The Greek philosopher Parmenides founds the Eleatic school of philosophy in Elea, a Greek colony in southern Italy. In 1961, American mathematicians Daniel Shanks and John Wrench used arctan series to compute 100,625 digits in under eight hours. Key works 1771 Reflections on the algebraic resolution of equations 1788 Analytical Mechanics 1797 Theory of analytic functions See also: Quadratic equations • Algebra • The binomial theorem • Cubic equations • Huygens's tautochrone curve • The fundamental theorem of algebra • Group theory 317 IN CONTEXT KEY FIGURE Georges Louis Leclerc, Comte de Buffon (1707-88) FIELD Probability BEFORE 1666 Liber de ludo aleae (On Games of Chance) by Italian mathematician Gerolamo Cardano is published. While the question of whether every closed 3-manifold could be deformed to take the shape of a 3-sphere is hypothetical, Perelman has claimed that it holds the key to understanding the shape of a 3-sphere is hypothetical, Perelman has claimed that it holds the key to understanding the shape of a 3-sphere is hypothetical, Perelman has claimed that it holds the key to understanding the shape of the Universe. person in Massachusetts. He also conjectured a geometrical projection known as a Möbius net. 229 Any point P, with coordinates (x, y), on the circle's radius) that forms the hypotenuse of a right-angled triangle with sides of length x and y. Diameter A straight line touching two points on the edge of a circle and passing through the center. Apollonius viewed these in three-dimensional terms: if a circle is combined with all lines that emanate from it, above or below its plane, and those lines pass through the same fixed point—the vertex—a cone is created. The Rhind papyrus includes a calculation for the slope of a pyramid using the seked— a measure for the horizontal distance traveled by a slope for each drop of 1 cubit. In matrix algebra, switching around the order in which the two matrices are multiplication of two square matrices (A and B). Complex numbers are used to model the velocity (speed and direction) of such fluids. 598-668 CE) FIELD Number theory BEFORE c. Before graduating, however, he left to study in Europe, although few details of this time are known. Their results were published together in 1691. Obtuse angle An angle between 90 and 180 degrees. It appears in Indian texts from 450 BCE as a guide to poetic metre, by the name of "The Staircase of Mount Meru." The mathematicians of ancient India also 249 realized that the shallow diagonal lines of numbers. Alamy Stock Photo: FLHC 61 (cra). Having established that π2 = 9.8696 to four decimal places, . See also: Irrational numbers • Eratosthenes' sieve • Negative numbers • The Fibonacci sequence • The golden ratio • Mersenne primes • Pascal's triangle 47 IN CONTEXT KEY FIGURE Pythagoras (c. A rotation or reflection can send the vertex A to one of three points (including itself). Key works 1892-99 Les Méthodes nouvelles de la mécanique céleste (New Methods of Celestial Mechanics) 1895 Analysis Situs (Topology) 1903 La Science et l'hypothèse (Science and Hypothesis) See also: Euclid's Elements • Coordinates • The Möbius strip • Minkowski space • Proving the Poincaré conjecture 420 IN CONTEXT KEY FIGURE Jacques Hadamard (1865–1963) FIELD Number theory BEFORE 1798 French mathematician Adrien-Marie Legendre offers an approximate formula to determine how many prime numbers there are below or equal to a given value. To solve x + 2 = 1, however, x must be a negative integer, while solving  $x^2 + 2 = 1$ , however, x must be a negative integer, while solving  $x^2 + 2 = 1$ , however, x must be a negative integer, while solving  $x^2 + 2 = 1$ , however, x must be a negative integer, while solving  $x^2 + 2 = 1$ , however, x must be a negative integer, while solving  $x^2 + 2 = 1$ , however, x must be a negative integer, while solving  $x^2 + 2 = 1$ , however, x must be a negative integer, while solving  $x^2 + 2 = 1$ , however, x must be a negative integer, while solving  $x^2 + 2 = 1$ , however, x must be a negative integer, while solving  $x^2 + 2 = 1$ , however, x must be a negative integer, while solving  $x^2 + 2 = 1$ , however, x must be a negative integer, while solving  $x^2 + 2 = 1$ , however, x must be a negative integer, while solving  $x^2 + 2 = 1$ , however, x must be a negative integer, while solving  $x^2 + 2 = 1$ . the mathematics that lay before him. The most magical of any magic square ever made by a magician. This complex numbers. 160 Today, a cubic equation can be written in the form  $ax^3 + bx^2 + cx + d = 0$ , provided a itself is not 0. These would be key to producing a fundamental theorem of algebra (FTA) for solving all possible polynomials. 364 Leave the science of parallels alone. He merely thought it would use code with a finite set of characters. Charles Babbage, on the other hand, responded to practical demand for a means of accurate and quick calculation with his mechanical calculating device, the "Difference Engine." In so doing, he laid the groundwork for the invention of computers. The area of the rectangle is r × 1/2(2πr) = r × πr × πr<sup>2</sup> (where r is the radius of the circle and 2πr is its diameter). According to the paradox, a person in one 64 moving group can pass two people in the other moving group in a fixed time, but only one person in the stationary group. In 1958, Mandelbrot joined IBM in New York, where his role as a researcher gave him the space and facilities to develop new ideas. For 25 example, 45 was written at the age of 21 and published in 1801, was key to defining number theory. When considering the motion of an imaginary body with respect to several others, the geodesic path becomes very complicated. Alamy Stock Photo: World History Archive (tl). With one roll of a six-sided die, the probability of throwing a six is 1/6; the probability of throwing any other number is 5/6. See also: The logic of mathematics • Fuzzy logic SOFYA KOVALEVSKAYA 1850-91 Moscow-born Kovalevskaya was the first woman to join the editorial board of a scientific journal, and the first woman to be appointed a professor of math. In the course of his career, Goldbach had befriended other influential mathematicians, including Leibniz and the Bernoullis, and corresponded regularly with them about their theories. 300 BCE In Elements (Book IX proposition 20), Euclid proves that there are infinitely many prime numbers. This method was the forerunner of integration. The difference between 23 and 32 is 1, and Catalan's conjecture states that this is the only pair of squares, cubes, or higher powers that differ by 1. AFTER 1981 Dutch mathematician Nicolaas Govert de Bruijn explains how to construct Penrose tilings from five families of parallel lines. In 1873, French mathematician Charles Hermite proved that e is also non-algebraic—it is not a number with a terminating decimal that can be used in a regular polynomial equation. The addition of integers is associative, 371 and the set is also closed, because adding any of the integers together gives another integer. Pascal's use of infinitely small slices of shapes to solve the properties of cycloids would lead to the "fluxions" introduced by Isaac Newton as he developed early calculations—for example, in splitting up (partitioning) a number to discover which numbers in the particularly when infinities of different sizes or infinite collections of objects are compared. Usually, the context is sufficient for people to interpret the notation correctly, but this can go badly wrong. Probability theory Pascal's contribution to the triangle was notable because he set out a clear framework for exploring its properties. The theorem shows that when binomials are multiplied out, the results follow a predictable pattern that can be written as an algebraic expression or displayed on a 150 triangular grid (known as Pascal's triangle after Blaise Pascal, who explored the pattern in the 1600s). These included the fact that every integer can be written as a product of prime numbers or is itself a prime. That established zero as a number in its own right as opposed to simply a figurative notation or placeholder. Five hundred years before, Gersonides had proved a special case of the claim. 380 BCE In the tenth book of his Republic, Plato espouses Pythagoras's theory of the transmigration of souls. In 1744, more than 130 years after Napier produced his first logarithm table, Swiss mathematician Leonhard Euler published a full treatment of ex and its relationship to the natural logarithm. He compared two sets of results, each starting with near-identical sets of data, noting that the atmospheric conditions developed along near-identical lines at first, but then changed in completely different ways. Arc A curved line that forms part of the circumference of a circle. There, he established the Pythagorean brotherhood— a community to whom he could teach both his mathematical and philosophical beliefs. His original design had beads dropping over pegs. Brahmagupta showed that if he added zero to a negative number, the result was equal to that negative number. His findings were immortalized in Ptolemy's Almagest, where he is described as "a lover of truth." The only work of Hipparchus to survive was his commentary on the Phaenomena of the poet Aratus and the mathematician and 111 astronomer Eudoxus, criticizing the inaccuracy of their descriptions of constellations. As Napier recognized, the basic principle of calculating was simple enough: he could replace the tedious task of multiplication by the simpler operation of addition. AFTER 500 CE In India, the first trigonometric tables are used. Liouville wrote more than 400 papers and in 1836 founded the Journal de Mathématiques Pures et Appliquées (Journal of Pure and Applied Mathematics), the world's second-oldest mathematical journal, which is still published monthly. He was later made a professor of mathematics), the world's second-oldest mathematics at the University of Halle. Lorenz found that the results were chaotic. 201 By relieving the brain of all unnecessary work, a good notation sets it free to concentrate on more advanced problems. Set Any collection of numbers, or mathematical structures based on numbers. The destination of the third vertex is now determined because the triangle is rigid, so there are 3 × 2 = 6 possibilities. However, Bayes' theorem factors in the false results produced by the test's 10 percent inaccuracy—P(B). Pythagoras himself is primarily associated with the formula that describes the relationship between the sides of a right-angled triangle. Natural logarithm. Fermat also independently developed a coordinate system, but he did not publish it. It generated huge amounts of data that he had to analyze statistically. I have mathematical knowledge of his day, and organize it into a mathematical structure where the logical relationships between the various propositions were carefully explained. To enable trade, stocktaking, and taxes in uncountable goods such as oil, flour, or plots of land, systems of measurement were developed, putting a numerical value on dimensions such as weight and length. Aged 17, Sofya eloped with paleontologist Vladimir Kovalevsky to Germany, where she studied at the University of Heidelberg and then Berlin, where she received tuition from German mathematician Karl Weierstrass. Several ancient civilizations, including the Babylonians and the Sumerians, could claim to have invented zero, but its use as a number was pioneered in the 7th century CE, by Brahmagupta, an Indian mathematician. AFTER 1882 Felix Klein describes the Kleinsche Flasche (Klein bottle), a shape composed of two Möbius strips. To denote this, Bayes introduced "conditional probabilities." These are given as P(A|B), the probability of A given B, and P(B|A),

the probability of B given A. The next step up is the cubic equation, where the x3 term is an unknown volume -a three-dimensional space. 1822 French mathematician David Wolpert disproves Laplace's demon by treating the "intellect" as a computer. Language often contains remnants of this system. A two-circle Venn diagram represents propositions such as: "All A are B," while a three-circle diagram represents propositions involving three sets (such as x, Y, and Z). GEORG CANTOR Born in St. Petersburg, Russia, in 1845, Georg Cantor moved with his family to Germany in 1856. 393 Out of eight possible combinations of true and false, there are three in which it is false. iStockphoto.com: sigurcamp (bl). The prime numbers... grow like weeds among the natural numbers, seeming to obey no other law than that of chance. Ellipse A shape like a circle, but stretched out symmetrically in one direction. If interest is calculated daily, the rate is 3 ÷ 365 = 0.008... and the amount after one year is \$100 × 1.00008...365 = \$103.05. Such systems make calculation more efficient because a small set of symbols can represent a huge range of values. His family went first to Paris and then to the south of France. A simple but accurate carpenter's square can be made from a triangle with sides of three, four, and five units. Fermat wrote: "It is impossible for a cube to be the sum of two fourth powers, or in general for any number that is a power greater than the second to be the sum of two like powers." Fermat never revealed the proof he claimed to have for his theory and so it remained unsolved, becoming known as Fermat's last theorem. 88 The fact that pi ( $\pi$ )—the ratio of the circumference of a circle to its diameter, roughly given as 3.141—cannot be expressed exactly as a decimal no matter how many decimal places are calculated has fascinated mathematicians for centuries. More recently, the Higgs boson has filled another such gap. The horizontal rows of the Greek abacus became vertical columns in the Roman abacus, in which we get the word "calculation." A type of abacus was also in use in the pre-Columbian civilizations of Central America. Even a small uncertainty is enough to create chaos. 300 BCE, geometry was concerned with negative numbers, and the standard deviation is the square root of the variance. The memory keeps track of changes and feeds them back into the algorithm. This did not exist with the numbers at Bombelli's disposal, so ). Binomial probabilities arise when a fixed 298 number, n, of successes is counted. The groundwork for the growing field of abstract algebra was laid by Évariste Galois, who, although he died young, also developed group theory while determining a general algebraic method for solving polynomial equations. In 1915, this work led her to make a major contribution to physics; she proved that the laws of conservation of energy and mass each correspond to a different type of symmetry. A regular tetrahedron is one of the five Platonic solids. A dice has six faces, so the probability of it landing on any particular face when you roll it is 1/6. It might divide eggs into two sets—small eggs of 1.76 oz (50 g) or less, and large ones over 1.76 oz—and boil the former for four minutes, and the latter for six. AFTER 1994 Andrew Wiles uses a special case of Langlands' conjectures to translate Fermat's last theorem from a problem in number theory to one in geometry, enabling him to solve it. In the paradox of Achilles and the tortoise, gives the creature a head start of 100 meters in a race. The simplest Cartesian coordinate system is one-dimensional; it indicates positions along a straight line. The three rotations and the reflections give all possible symmetries of the triangle ABC. Now known as Bessel functions, they are useful to scientists and engineers. When Einstein unveiled his theory of a provide symmetries of the triangle ABC. Now known as Bessel functions, they are useful to scientists and engineers. four-dimensional reality. 531 In 2000, the Clay Mathematics Institute in the US celebrated the millennium with seven prize problems. Because of this legend, the arrangement of odd and even numbers are always in the corners of the square) were believed to have magical properties and was used as a good luck talisman through the ages. To overcome this, Lotfi Zadeh suggested a system of "fuzzy" logic, in which statements can be partly true or false, in a range between 0 (absolutely false) and 1 (absolutely true). The relationship between 0 (absolutely true). then inscribed in the squares, it creates a golden spiral. Over the years the test has had many critics, who question its ability to truly judge the intelligence of an AI effectively or see the competition as a stunt that does not advance knowledge in the field of AI. Linear transformation Also called linear mapping, a mapping between vector spaces. A clockwise rotation of 120° sends vertex A to where B was, B to C, and C to A, while a reflection in the vertical line through A swaps vertices B and C. 1812 Pierre-Simon Laplace applies probabilities). He would have written on papyrus, which is not very durable; 75 all that remains of his work are the copies, translations, and commentaries made by later scholars. Hardy was excited by Ramanujan's claim that he had found a function of x that exactly represented the number of prime numbers less than x; Ramanujan was unable, however, to offer a rigorous proof. Key work 1572 Algebra See also: Quadratic equations • Irrational numbers • Cubic equations • The algebraic resolution of equations • The fundamental theorem of algebra • The complex place value of Indian numerals spreads the place value system based on the Hindu numerals throughout the Arab world. As 12 + 12 = 2, the length of the hypotenuse is the square root of 2. In devising his axiomatic system, he began with 23 definitions for terms such as point, line, surface, circle, and diameter. Beyond his work with conic sections, Apollonius is credited with estimating the value of pi more accurately than his contemporary Archimedes, and with being the first to state that Key work c. He was also a gifted linguist, but at 16, he enrolled at the University of British Columbia, Canada, to study mathematics. The formula suggested that all polyhedra shared basic characteristics. This value appears in the Bible (1 Kings 7:23): "And he made the Sea of cast bronze, ten cubits from one brim to the other; it was completely round. The equals sign was widely adopted, and the 17th century also saw the creation of many of the other; it was completely round. The equals sign was widely adopted, and the ratio of the length of the side opposite a given angle in a right-angle to the length of the triangle to the length of the triangle to the length of the triangle in a right-angle to the length of the triangle in a right-angle to the length of the triangle in a right-angle to the length of the triangle in a right-angle in a right-angle to the length of the triangle in a right-angle to the length of the triangle in a right-angle in a righ Bovelles is the first to describe a cycloid. German astronomer Johannes Kepler had published his first two laws of planetary motion in 1605, but only after the invention of log tables was he able to make the breakthrough to discover his third law. The earliest attempt to set out the principles of logic was carried out by the Greek philosopher Aristotle around 350 BCE. Despite his success, Riemann struggled financially. The line formed by a point moving in this defined way is called a locus. In x + 4y - 2, for example, x, 4y, and 2 are all terms. He emailed Wiles, who replied, but not to Katz's satisfaction. The equations are now grouped in three classes: those with no solution, those with a finite number of solutions, and those with an infinite number of solutions. Equilateral triangle in which all three sides are the same length and all three sides are the same length are the sam worked in a similar way. According to legend, if monks at a certain temple in either India or Vietnam (depending on the version of the tale) succeed in moving 64 disks from one pole to another in line with the rules, the world will end. Although he saw the circle as one of the four conic sections, it is really an ellipse with the plane perpendicular to the axis of the cone. Future generations over the next four months can be forecast to contain 13, 21, 34, and 55 pairs of rabbits. Group theory is a branch of algebra that pervades modern mathematics. Encryption The process of converting data or a message to a secure, coded form. The Islamic world, like India, entered a "Golden Age" of learning that lasted until the 1300s, and produced a succession of influential mathematicians—such as alKhwarizmi, a key figure in the development of algebra "derives from the Arabic term for rejoining), and other scholars whose contributions to the binomial theorem and the treatment of quadratic and cubic equations were groundbreaking. However, encryption is used against all kinds of third-party "adversary"—a rival company, an enemy power, or a security service. It was never built, but was envisaged as a machine that could respond to new problems and solve them without human intervention. He died in 2010. For example, if the sides of a cube are each equal to 1 in length, what length sides do you need for a cube twice the volume? In geometry (the study of shapes and measurements), coordinates are employed to define a single point—an exact position—using numbers. 1600 English mathematician and astrologer Thomas Harriot experiments with number systems, including binary. Hipparchus died in 120 BCE. The aim of Diophantine equations is to find all the variables, but solutions must be integers or rational numbers (those that can be written as one integer divided by another, such as 8/3). Three mathematicians—Huygens, Gottfried Leibniz, and Johann Bernoulli— calculated a formula for the catenary, coming to the same conclusion. Plato found perfection in mathematical objects, believing they were the key to understanding the differences between the real and the abstract. Here, the leftmost 1 has the value of 4 ( $2 \times 2$ ), the middle 1 means one 2, and the zero means no single units: 4 + 2 + 0 makes 6. The further along the sequence this calculation is attempted, the closer the answer is to  $\phi$ . Laplace imagined an intellect that could analyze movements of all atoms in the Universe in order to accurately predict their future paths. These concentric rings show the different types of infinities. Enigma's flaw was that it could not encrypt a letter as itself. In an unpublished manuscript dated October 29, 1675, Leibniz was the first person to use the "integral" symbol f, which is used and recognized universally today. Existence proof A mathematical proof that something exists, obtained either by constructing an example or by general deduction. Key work 1907 Raum und Zeit (Space and Time) See also: Euclid's Elements • Newton's laws of motion • Laplace's demon • Topology • Proving the Poincaré conjecture 448 IN CONTEXT KEY FIGURE Srinivasa Ramanujan (1887-1920) FIELD Number theory BEFORE 1657 In France, mathematician Bernard Frénicle de Bessy cites the properties of 1,729, the original "taxicab" number. See also: The fundamental theorem of algebra • The logic of mathematics YURI MATIYASEVICH 1947- While studying for his doctorate at the Steklov Institute of Mathematics in Leningrad (now St. Petersburg), Matiyasevich became fascinated by the challenge of solving David Hilbert's tenth problem. 1970 was one checked one by one, a feat that would have been of the first computers to use virtual memory, a impossible to do manually. The 1931 theorem concluded that there will always exist some statements regarding numbers that may be true, but can never be proved. Two years later, recognizing her the first woman professor of math at any university. The division is a modulo operation (abbreviated to modn), meaning the answer is just the remainder. Higher and lower values are less frequent. Augustus De Morgan British mathematician 117 Investors rush to withdraw their money from the Seamen's Savings Bank in New York in 1857. In the late 1800s, academics deciphered the "cuneiform" (wedge-shaped) markings on clay tablets recovered from Babylonian sites in and around Iraq. His theory of transfinite numbers clashed with traditional mathematical beliefs and the criticisms of leading mathematicians damaged his career. Recording the result of dividing a number, n, by zero as n/0. Brahmagupta suggested that a number is unchanged when it is divided by zero. Each row begins and ends with the number 1. His many contributions. New century, new fields One area of exploration was the foundations of mathematics. Subsequent advances by Bernhard Riemann, Eugenio Beltrami, Felix Klein, David Hilbert, and others mean that today, non-Euclidean geometries are no longer seen as exotic, and physicists have given serious consideration to whether our Universe is indeed flat (Euclidean) or curved. The whole book should be viewed as a curated tour of mathematical highlights Brahmagupta also described subtracting zero from both a negative and a positive number, and noted again that it left the numbers and the first systems of numerals began to evolve, a 12 means of expressing operations such as acquisition of additional items, or depletion of a stock, the basic operations of arithmetic. If a group is commutative, it is known as an Abelian group. It entailed inscribing a triangle of known area to fit inside the parabola, then inscribing ever smaller triangles in all the gaps that remained. Puzzled by what seemed to be a paltry reward, the king ordered that the grains be counted out. His genius lay in combining all five milestone 308 numbers with three simple operations: raising a number to a power (for example, 54, or 5 × 5 × 5), multiplication, and addition. In the US, the use of a decimal system for measurement and coinage was championed by Thomas Jefferson. The eyeball, for example, represents 1/4, while the eyebrow is 1/8. In 1596 Johannes Kepler reasoned that the positions of the six planets then known (Mercury, Venus, Earth, Mars, Jupiter, and Saturn) could be explained in terms of the Platonic solids. A tailor's son, he went to military school, where his keen interest in mathematics led him to become a successful teacher of the subject. Key works 1666 On the Art of Combination 1684 New Method for Maximums and Minimums 1703 Explanation of Binary Arithmetic See also: The Rhind papyrus • Zeno's paradoxes of motion • Calculating pi • Decimals • The problem of maxima • The area under a cycloid • Euler's identity 273 IN CONTEXT KEY FIGURE Gottfried Leibniz (1646–1716) FIELDS Number theory, logic BEFORE c. Once he had conceptualized such a machine, however, he could theoretically take any algorithm and test it using the machine to see if it halted. To three of the conic sections he gave the names ellipse, parabola, and hyperbola. The problems, 39 recorded in this and other ancient Egyptian artifacts such as the earlier Moscow papyrus, illustrated techniques for working out areas, proportions, and volumes. In the legend, a turtle appears to the great King Yu as he faces a devastating flood. Factorization Expression in terms of factors that when multiplied together give the original number or mathematical expression. See also: Conic sections • The binomial theorem NICOLE ORESME C. 200 CE Diophantus of Alexandria writes his Arithmetica in which he lays out key issues about numbers. Some mathematicians believe algebraic geometry has great untapped potential for the future. computer software. Another account relates that the idea came to him in dreams in 1619, when he was serving as a mercenary in southern Germany. Their parents add up to eight great-grandparents, whose own parents add up to eight great-grandparents. The result is a variety of waves interfering with each other. there was a problem with his proof. This ancient Egyptian tomb painting shows surveyors using rope to calculate the dimensions of a wheat field.

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